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DEPENDENCE OF Z-PARAMETERS ON THE LF TRANSISTOR T-EQUIVALENT CIRCUIT

Nicholas Kyriakopoulos

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DIAMOND ORDNANCE FUZE LABORATORIES
ORDNANCE CORPS • DEPARTMENT OF THE ARMY
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ABSTRACT

The Z-parameters of a transistor have been calculated in terms of the transistor T-equivalent circuit parameters. The calculations have been made for a frequency of 1.0 kc. In addition, each of the independent equivalent circuit parameters was halved and doubled while the rest were held constant, and the effect on the Z-parameters investigated.

The results indicate that in the common base and common emitter configurations the input impedance depends on $r_b$ only, while the forward transfer impedance depends on $r_b$ and $r_e$. The rest of the parameters are functions of $r_c$ and $C_c$ only.

1. INTRODUCTION

The purpose of this report is to investigate the dependence of the terminal characteristics of a transistor on its T-equivalent circuit parameters. This dependence is of primary importance in transistor design, since a given set of terminal characteristics can be achieved by specifying the values of the internal parameters; these values, in turn, are functions of junction area, doping, etc., and can be controlled to a certain extent.

Because of the numerous and tedious calculations involved, this work was performed with the help of an IBM 704 digital computer. This report will include the relations between the terminal characteristics and the equivalent circuit parameters, a brief discussion of the programming techniques used, and the results obtained for common base, common emitter and common collector configurations.

2. DISCUSSION

Of the various transistor equivalent circuits available, the T-equivalent was chosen for the purposes of the present investigation. Although basically a low frequency equivalent circuit, its useful frequency range is increased by including an emitter and base capacitance. On the basis of the circuit illustrated below,
the Z-parameters in the common base, common emitter and common collector configuration are found as follows:

\[ Z_{11e} = Z_{11b} = r_{bb}' + \frac{r_b}{1+(\omega C_b r_b)^2} + \frac{r_e}{1+(\omega C_e r_e)^2} - j\omega \left( \frac{C_b r_b^2}{1+(\omega C_b r_b)^2} + \frac{C_e r_e^2}{1+(\omega C_e r_e)^2} \right) \]

\[ Z_{12b} = r_{bb}' + \frac{r_b}{1+(\omega C_b r_b)^2} - j\omega \frac{C_b r_b^2}{1+(\omega C_b r_b)^2} \]

\[ Z_{21b} = r_{bb}' + \frac{r_b}{1+(\omega C_b r_b)^2} + \frac{\alpha r_c}{1+(\omega C_c r_c)^2} - j\omega \left( \frac{\alpha C_c r_c^2}{1+(\omega C_c r_c)^2} + \frac{C_b r_b^2}{1+(\omega C_b r_b)^2} \right) \]

\[ Z_{11c} = Z_{22b} = r_{bb}' + \frac{r_b}{1+(\omega C_b r_b)^2} + \frac{r_c}{1+(\omega C_c r_c)^2} - j\omega \left( \frac{C_b r_b^2}{1+(\omega C_b r_b)^2} + \frac{C_e r_e^2}{1+(\omega C_e r_e)^2} \right) \]

\[ Z_{12c} = \frac{r_e}{1+(\omega C_e r_e)^2} - j\omega \frac{r_e^2 C_e}{1+(\omega C_e r_e)^2} \]

\[ Z_{21c} = \frac{r_e}{1+(\omega C_e r_e)^2} - \frac{\alpha r_c}{1+(\omega C_c r_c)^2} + j\omega \left( \frac{\alpha C_c r_c^2}{1+(\omega C_c r_c)^2} - \frac{C_e r_e^2}{1+(\omega C_e r_e)^2} \right) \]

\[ Z_{22c} = Z_{22_e} = \frac{r_e}{1+(\omega C_e r_e)^2} + \frac{(1-\alpha) r_c}{1+(\omega C_c r_c)^2} - j\omega \left( \frac{(1-\alpha) C_c r_c^2}{1+(\omega C_c r_c)^2} + \frac{r_e}{1+(\omega C_e r_e)^2} \right) \]

\[ Z_{12c} = \frac{(1-\alpha) r_c}{1+(\omega C_c r_c)^2} - j\omega \frac{(1-\alpha) C_c r_c^2}{1+(\omega C_c r_c)^2} \]
Thus, the terminal transistor impedances are expressed in terms of the equivalent circuit parameters in the three configurations. For given values of these inherent or intrinsic parameters the impedances can be calculated.

In the present study, the IBM 704 digital computer of the National Bureau of Standards was used to calculate the terminal parameters. The program was written in the FORTRAN (Formula Translation) language. FORTRAN is a system by which a program written in a relatively simple language can be translated into the language that the computer understands. This technique enables a non-professional programmer to use the computer for a wide range of problems with a minimum of difficulty (ref 1).

It was desired to find the effect on the terminal characteristics of the transistor when one of the equivalent circuit parameters was varied while the others were held constant. The data used in this report were taken from the data sheet for a 2N220 transistor. In the equivalent circuit representation C was assumed to be very small, so the capacitive reactance in parallel with $r$ gave an effective impedance approximately equal to $r$. In view of the low frequency being considered, this appears to be a valid assumption. The values of the equivalent circuit parameters used as basis for the calculation of the terminal characteristics appear in table I. Since $C_b$ was not given in the original data, the value of 8850 pf was taken from measurements that were performed on a 2N180 transistor (ref 2).

\[
Z_{2le}^e = \frac{r_c}{1 + (\omega C_c r_c)^2} - j \omega \frac{C_c r_c^2}{1 + (\omega C_c r_c)^2}
\]

<table>
<thead>
<tr>
<th>$r_{bb}$</th>
<th>$r_b$</th>
<th>$C_b$</th>
<th>$r_e$</th>
<th>$r_c$</th>
<th>$C_c$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ohms</td>
<td>ohms</td>
<td>pf</td>
<td>ohms</td>
<td>megohms</td>
<td>pf</td>
<td></td>
</tr>
<tr>
<td>190</td>
<td>1085</td>
<td>8850</td>
<td>37.7</td>
<td>2.86</td>
<td>50</td>
<td>.985</td>
</tr>
</tbody>
</table>

TABLE I
The values given in table I and the terminal parameters corresponding to these values (table II) were used as bases for normalizing the results. Each value of the equivalent circuit parameters was halved and doubled while the others were kept constant. For each set of values the terminal characteristics were computed at a frequency of 1.00 kc. The results of these computations appear in figures 1 through 13. In the graphs, the ordinate represents the normalized terminal characteristics while the abscissa represents the normalized equivalent circuit parameters.

Table II

| $Z_{11e}$ | $1.31 \times 10^3 - j6.52 \times 10^1$ | $Z_{12e} = 3.77 \times 10^1 + j0.0$ |
| $Z_{21e}$ | $-1.60 \times 10^6 + j1.40 \times 10^6$ | $Z_{22e} = 2.38 \times 10^4 - j2.13 \times 10^4$ |
| $Z_{11b}$ | $1.31 \times 10^3 - j6.52 \times 10^1$ | $Z_{12b} = 1.27 \times 10^3 - j6.52 \times 10^1$ |
| $Z_{21b}$ | $1.56 \times 10^6 - j1.40 \times 10^6$ | $Z_{22b} = 1.58 \times 10^6 - j1.42 \times 10^6$ |
| $Z_{11c}$ | $1.58 \times 10^6 - j1.42 \times 10^6$ | $Z_{12c} = 2.37 \times 10^4 - j2.13 \times 10^4$ |
| $Z_{21c}$ | $1.58 \times 10^6 - j1.42 \times 10^6$ | $Z_{22c} = 2.38 \times 10^4 - j2.13 \times 10^4$ |

3. **RESULTS**

The greatest effect on most of the terminal characteristics of a transistor, at the frequency considered, is brought about by the variation of $r_c$ and $C_c$. The only exceptions are the parameters with which $r_c$ and $C_c$ are not associated. Thus, in the common emitter and common base configurations the input impedance is largely a function of $r_b$ and $C_b$ while $r_{bb}$ has little effect and $r_e$ plays an insignificant role. In fact, the only terminal characteristic which is affected by $r_e$ is the common emitter forward transfer impedance $Z_{12e}$ which for all practical purposes is directly proportional to $r_e$. In this investigation, the emitter capacitance was assumed negligible, thus making the forward transfer impedance equal to $r_e$. 

8
Figure 1. Real part of $Z_{11}$ versus equivalent circuit parameters.
Common base, emitter configuration.
Figure 2. Imaginary part of $Z_{11}$ versus equivalent circuit parameters.
Common base, emitter configuration.
Figure 3. Real part of $Z_{11}$ versus equivalent circuit parameters.
Common collector configuration.
Figure 4. Imaginary part of $Z_{11}$ versus equivalent circuit parameters.
Common collector configuration.
Figure 5. Real part of $Z_{12}$ versus equivalent circuit parameters. Common base configuration.
Figure 6. Imaginary part of $Z_{12}$ versus equivalent circuit parameters.

Common base configuration.
Figure 7. Real part of $Z_{12}$ versus equivalent circuit parameters.
Common emitter configuration.
Figure 8. Real part of $Z_{12}$ versus equivalent circuit parameters.

Common collector configuration.
Figure 9. Imaginary part of $Z_{12}$ versus equivalent circuit parameters.
Common collector configuration.
Figure 10. Real part of $Z_{21}$ versus equivalent circuit parameters.

Common base, collector, emitter configuration.
Figure 11. Imaginary part of $Z_{21}$ versus equivalent circuit parameters.

Common emitter, collector base configuration.
NORMALIZED EQUIVALENT CIRCUIT PARAMETERS

Figure 12. Real part of $Z_{22}$ versus equivalent circuit parameters. Common emitter, collector, base configuration.
Figure 13. Imaginary part of $Z_{22}$ versus equivalent circuit parameters
Common collector, base, emitter configuration.
The base capacitance \( C_b \) contributes only to the imaginary components of \( Z_{11e}, Z_{11b} \) and \( Z_{12b} \). From table II it is seen that these components are negligible in comparison to the real parts. Of course, as the frequency is increased, \( C_b \) will play a significant role on the terminal characteristics. If only the magnitude of the impedance is required, the base capacitance can be neglected. However, if phase angle information is needed \( C_b \) must be included. In the common emitter and common base input impedance the phase angle is approximately \( 2^\circ \). By varying \( C_b \) from one-half to twice its nominal value, the phase angle could be changed from \( 1^\circ \) to \( 6^\circ \) respectively. Thus, the phase angle of the input impedance can be controlled to a certain extent by adjusting \( C_b \), while its absolute value will remain constant.

In the common collector configuration, all four elements of the Z-matrix depend only on \( r_c \) and \( C_c \). Of particular interest is the effect of \( r_c \) on the real parts and of \( C_c \) on the imaginary parts of the elements of the matrix. For values of \( r_c \) less than its nominal value, the real part of the impedance increase according to some exponential functional of \( r_c \). As \( r_c \) reaches a value slightly higher than its nominal, the real part of \( Z, \text{Re}[Z] \), begins to decrease and it will continue to decrease as \( r_c \) increases. This rather unusual behavior can be traced to the expression

\[
\frac{r_c}{1 + (\omega C_c r_c)^2}
\]

where for large values of \( r_c \) the term containing \( r_c^2 \) begins to influence the fraction more than the numerator \( r_c \). The maximum value of the real part is given for a value of \( r_c \) slightly higher than its nominal value. Similarly, the imaginary components behave in almost identical manner for the variation in \( C_c \). The term responsible for this effect of \( C_c \) is

\[
\frac{\omega C_c r_c^2}{1 + (\omega C_c r_c)^2}
\]
Since \( r_c \) behaves in similar fashion in both the numerator and denominator, the imaginary terms of the Z-parameters, \( \text{Im}[Z] \), are affected by \( r_c \) according to some proportionality function. Thus \( \text{Im}[Z] \) increases with increasing \( r_c \) while \( C_c \) causes it to increase initially and then reverse slope. The points of inflection were calculated after the data indicated that either halving or doubling \( r_c \) and \( C_c \) the impedance was less than that of the nominal values.

A few remarks can also be made as to the dependence of the absolute value of the Z-parameters on \( r_c \) and \( C_c \). It is obvious that any increase in the collector capacitance will cause the impedances which depend on \( C_c \) to decrease, perhaps slowly in the beginning but rather rapidly as \( C_c \) attains values higher than its nominal value. The effect of \( r_c \), however, is not as easily predictable. For values of \( r_c \) less than nominal, the absolute value of the impedance will increase since both the real and the imaginary parts increased with \( r_c \). As the value of \( r_c \) becomes much larger than nominal, the real part begins to decrease while the imaginary part continues increasing. To the extent for which information is available, \(|Z|\), which depends almost equally on the real and imaginary parts (table II), will continue to increase with increasing \( r_c \) but at an ever-decreasing rate. The phase angle will experience a rapid increase since the imaginary part increases and the real part decreases with \( r_c \). It should be kept in mind that all the parameter variations are for a frequency of 1.0 kc. For any other frequency the curves might assume a completely different form.

4. ACKNOWLEDGEMENT

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5. REFERENCES


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