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AUTHORITY

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STEPLESS AND STEPPED PLAINING HULLS-

GRAPHS FOR PERFORMANCE PREDICTION AND DESIGN

by

Eugene P. Clement and James D. Pope, LTJG USN

HYDROMECHANICS LABORATORY
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January 1961 Report 1490
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Report 1490
SR 009 0101
**NOTATION**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Aspect ratio, $b/l_m$</td>
</tr>
<tr>
<td>b</td>
<td>Beam of planing surface, ft</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Skin-friction coefficient</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Lift coefficient based on principal wetted area, $\Delta/2^1/2 \rho V^2 b^2$; also, $C_{l,l}$ equals $C_{l,B} + C_{l,C}$</td>
</tr>
<tr>
<td>$C_{l,B}$</td>
<td>Lifting line term in expression for $C_{l,B}$</td>
</tr>
<tr>
<td>$C_{l,C}$</td>
<td>Cross-flow term in expression for $C_{l,B}$</td>
</tr>
<tr>
<td>$C_{l,B}$</td>
<td>Lift coefficient based on beam of planing surface, $\Delta/2^1/2 \rho V^2 b^2$</td>
</tr>
<tr>
<td>$C_{l,P}$</td>
<td>Lift coefficient based on center-of-pressure location, $\Delta/2^1/2 \rho V^2 l_{cp}^2$</td>
</tr>
<tr>
<td>$F_{v}$</td>
<td>Froude number based on volume of water displaced at rest, $V/\sqrt{gV^{1/3}}$</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity, $32.16 \text{ ft/sec}^2$</td>
</tr>
<tr>
<td>$l_m$</td>
<td>Mean wetted length (distance from aft end of planing surface to the mean of the heavy spray line), ft</td>
</tr>
<tr>
<td>$l_{cp}$</td>
<td>Center-of-pressure location (measured from aft end of planing surface), ft</td>
</tr>
<tr>
<td>$l_{cp}/l_m$</td>
<td>Nondimensional center-of-pressure location</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance of planing bottom, lb</td>
</tr>
</tbody>
</table>
\( \text{Re} \) Reynolds number, \( \frac{V}{\nu} \)

\( S \) Principal wetted area (bounded by trailing edge, chines, and heavy spray line), sq ft

\( S_s \) Area wetted by spray, sq ft

\( V \) Horizontal velocity, ft/sec

\( V_m \) Mean water velocity over pressure area, ft/sec

\( \beta \) Angle of deadrise, deg

\( \rho \) Mass density of water, slugs/ cu ft

\( \gamma \) Trim (angle between planing bottom and horizontal), deg

\( \nu \) Kinematic viscosity, \( \nu \), ft/sec

\( \Delta \) Gross weight (equals planing lift), lb

\( \Delta \lambda \) Effective increase in friction as a length-beam ratio due to spray contribution to drag

\( \nabla \) Volume of water displaced at rest, cu ft
ABSTRACT

This report presents graphs by means of which the high-speed resistance and trim of conventional and stepped planing boats of a wide range of sizes and proportions can be determined. Graphs which give guidance in selecting parameters which will result in optimum planing performance are also presented. Values for the graphs were obtained from equations for the lift, center of pressure, and resistance of prismatic planing bottoms which were previously developed by the National Aeronautics and Space Administration and the David Taylor Model Basin.

INTRODUCTION

Reference 1 by the National Aeronautics and Space Administration, presented semiempirical equations for the pure planing lift and center of pressure on flat and V-bottom planing surfaces. This reference showed that there was good agreement between results from the equations and data from extensive tests of prismatic planing surfaces. Subsequently, in Reference 2, the Taylor Model Basin presented equations (utilizing the NASA equations for lift coefficient and center of pressure) by means of which the resistance of planing boats at high speeds can be calculated. Comparisons of calculated values of resistance with values obtained from tests of a model of a representative planing boat showed good agreement.

The graphs of Reference 2 presented values of lift coefficient, center of pressure, and resistance-displacement ratio (R/Δ) for trims from 1 degree to 4 degrees and for values of aspect ratio from 0.3 to 0.6. The values of lift coefficient and center-of-pressure ratio are applicable for boats of any size. The values of R/Δ were computed for a gross weight of 100,000 lb. By means of the graph of Reference 1 it is possible to make estimates of the high-speed resistance and trim of large stepless planing hulls.

* References are listed on page 10
In the present report, graphs of lift coefficient and center of pressure are presented for a more extensive range of trim angles and aspect ratios (trim angles as high as 10 degrees, and values of aspect ratio as high as 2.3). Values of resistance-displacement ratio are given for a similar extended range of trim angles and aspect ratios. Furthermore, values of $R/\Delta$ have now been calculated for a number of gross weights from 1000 lb to 100,000 lb, and graphs are presented in this report which make it possible to predict the high-speed resistance for any gross weight within this range. The graphs of Reference 1, on the other hand, make it possible to make accurate prediction of resistance only for gross weights in the region of 100,000 lb.

The graphs have been prepared for deadrise angles of 5, 10, and 15 degrees. This is considered to be the practical range for most purposes. The high-speed resistance will be the least with the least angle of deadrise. However, in order to attain good directional stability and good turning characteristics, some deadrise is necessary. Accordingly, deadrise angles less than 5 degrees are considered to be outside of the practical range.

By means of these graphs it is possible to make estimates of the performance of small, medium-sized, and large planing boats of both the conventional and stepped types. As before, the calculations for the graphs were made by an electronic digital computer.

It was found possible, from the graphs of the calculated values of $R/\Delta$, to determine planing conditions which would give minimum resistance. Accordingly, auxiliary graphs were prepared which make it possible to solve several planing hull design problems in such a way as to achieve optimum performance.
DEVELOPMENT OF GRAPHS FOR PREDICTING PERFORMANCE

The equations given in Reference 1 for the pure planing lift and center of pressure of flat and V-bottom planing surfaces having straight sections are as follows:

Lift

Lifting line term $C_{LL}$

$$C_{LL} = 0.5 \pi \frac{AZ^*}{1 + A} \cos^2 \gamma (1 - \sin \beta) + \frac{1}{3} \sin^2 \gamma \cos^3 \gamma \cos \beta$$

Cross-flow term $C_{LC}$

Center of Pressure

$$\frac{1_{cp}}{1_m} = 0.875 \frac{C_{LL}}{C_{LS}} + 0.50 \frac{C_{LC}}{C_{LS}}$$

Values of $C_{LS}$ were calculated for a range of values of $\beta$, $\gamma$, and $A$, using the first equation. These values are presented in the form of ratios of $C_{LS}$ to $\gamma$ (in degrees) in Figure 1. Presentation of the lift coefficient data in this form, rather than in the usual form of $C_{LS}$ versus $\gamma$, results in graphs which yield greater accuracy when the graphs are used for wing performance predictions.

Values of $1_{cp}/1_m$ were calculated using the second of the above equations, and are plotted as ordinates in Figure 2, with the ratio $1_{cp}/b$ as abscissa. The values of $1_{cp}/b$ were determined from the selected values of aspect ratio and the calculated values of $1_{cp}/1_m$ by means of the relationship:

$$\frac{1_{cp}}{b} = \frac{1_{cp}}{1_m} \cdot \frac{1_m}{b} = \frac{1_{cp}}{1_m} \cdot \frac{1}{A}$$

Equations from which the resistance can be calculated were developed in Reference 2. The final equations are as follows:

$$R/\Delta = \tan \gamma + \frac{C_L}{C_{LS}} \left[ \left( \frac{W}{V} \right)^2 + A \Delta \right]$$

* $\gamma$ in radians
$C_{18}$ is given by the first equation in the report, and $C_f$ is given as a function of Reynolds numbers by the 1947 ATTC friction formulation, as follows:

$$\frac{0.242}{\sqrt{C_f}} = \log_{10} \Re \cdot C_f$$

Reynolds number is given by

$$\Re = \frac{\frac{2.3}{2} \cos \beta}{0.18 \cos \beta \left(1 - \frac{C_{18}}{C_{18} \cos \beta \cos \beta}ight)}$$

Both a mathematical expression for, and a graph of, $\Delta \lambda$ are given in Reference 3. An expanded version of the graph is presented in Figure 7. The negative values of $\Delta \lambda$ correspond to the case where the velocity of the spray has a forward component with respect to the planing bottom, and therefore tends to reduce rather than increase the drag. However, we are concerned here with deadrise angles greater than 5 degrees, and in general, with trim angles less than 6 degrees; and in this region the magnitudes of the negative values of $\Delta \lambda$ are not large enough to have important effects on the resistance.

Values of $R/\Delta$ were calculated for a range of values of $\beta$, $\lambda$, and $A$ (as was the case for the calculations of $C_{18}$ and $l_{CP}$). However, the ratio of resistance to displacement is a function not only of $\beta$, $\lambda$, and $A$, but also of the gross weight, $\Delta$. Therefore, values of $R/\Delta$ were calculated for gross weights of 1000, 5000, 10,000, 50,000, and 100,000 lb. The values of $R/\Delta$ for a gross weight of 10,000 lb are presented in Figure 3. These curves will be put to further use later in the report. The values of $R/\Delta$ for the range of gross weights from 1000 lb to 100,000 lb are presented in Figures 4, 5, and 6.
SAMPLE PERFORMANCE PREDICTION

Values of the resistance and trim angle of a planing boat at several speeds in the planing region can be readily determined by means of the graphs which have been presented. The following example illustrates the process of evaluating the performance of a typical boat. The dimensions assumed are as follows:

- Displacement \((\Delta)\) = 13,000 lb
- Length of boat = 30 ft
- Maximum beam over spray strips \((b)\) = 9.5 ft
- Average deadrise angle for after half of length \((\beta)\) = 10 deg
- Distance of c.g. forward of transom \((l_{cp})\) = 13.0 ft

The numbered columns below indicate the sequence of the process of determining the planing performance:

<table>
<thead>
<tr>
<th>(l_{cp} )</th>
<th>A</th>
<th>( R/\Delta )</th>
<th>( R/) lb</th>
<th>( CD_{LS} )</th>
<th>( C_{LS} )</th>
<th>( S_n ) b/A</th>
<th>13,000 lb</th>
<th>( V, ) fps</th>
<th>( V, ) knots</th>
<th>( P_{Vc} )</th>
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</thead>
<tbody>
<tr>
<td>1.0</td>
<td>.858</td>
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</tr>
<tr>
<td>5.0</td>
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</tr>
</tbody>
</table>

First a number of trim angles are assumed and entered in Column 1.

Next, the ratio \(l_{cp}/b\) is determined. This is:

\[
\frac{l_{cp}}{b} = \frac{13.0}{9.5} = 1.37
\]

5
Then values of the ratio $l_{cp}/l_m$ for the different trim angles are read from Figure 2 and entered in Column 2. The values of $l_{cp}/l_m$ are then divided by the constant value of $l_{cp}/b$ to give the aspect ratio. These values are entered in Column 3. Next, values of $R/A$ are read from Figure 5, and entered in Column 4. Then, multiplying the values of $R/A$ by the boat displacement (13,000 lb) will give the boat resistance in pounds. These values have been entered in Column 5.

The resistance is now known, and the remaining calculations are for the purpose of determining the corresponding values of speed. The speed is determined by solving for $V$ in the expression $C_{ls} = \frac{1}{2} \rho \frac{S}{\sqrt{V}}$. $\frac{1}{2} \rho$ is assumed equal to 1. Then $V^2 = \frac{\Delta}{C_{ls} S}$.

Values of $C_{ls}/\tau$ are read from Figure 1 and entered in Column 6. Multiplying by $\zeta$ in degrees gives $C_{ls}$ which is entered in Column 7. Next $S$ is calculated from the relationship $S = \frac{b^2}{A}$ and entered in Column 8. The quantity $10,000/S C_{ls}$ is then computed and entered in Column 9. The square root of Column 9 gives the velocity in feet per second (Column 10). Speed in knots has been entered in Column 11, and the dimensionless speed coefficient $F_v$ in Column 12. The resulting values of trim and hydrodynamic drag are plotted against $F_v$ in Figure 8.

The graphs for predicting performance which are presented in this report are valid for the planing region, where most of the load is supported by dynamic lift. However they do not give accurate predictions of performance at speeds where an appreciable portion of the load is supported by buoyancy. Previous comparisons of predicted performance with performance as determined from model tests have shown close agreement beginning at a speed slightly above the point corresponding to the minimum value in the curve of predicted resistance. This point at which the predicted values become valid is indicated in Figure 8.
The curves of Figures 1 through 3 have been used to construct some auxiliary graphs which provide guidance for solving planing hull design problems. It can be seen that there is a minimum-resistance point on each of the curves of Figure 3. These minimum-resistance values have been plotted in Figure 9 as a function of aspect ratio. $R/\Delta$ has been inverted, however, to give $\Delta/R$, or lift-drag ratio. The values of $\tau$ corresponding to the minimum-resistance points are also plotted in Figure 9. Figure 9(b) shows that a 10-degree deadrise planing hull can attain a lift-drag ratio of about 8.5 at values of aspect ratio greater than about 1.3. The trim angle corresponding to this optimum condition is a little less than 4 degrees. A hull of the stepped type is required in order to obtain values of aspect ratio as high as 1.3. Such a hull is under development at the Model Basin. The resistance of this hull at low speed is only slightly greater than that of the conventional, stepless planing boat, and at high speed its efficiency approaches the optimum as indicated by Figure 9. Furthermore, this hull gives promise of having improved seaworthiness and maneuverability, in addition to reduced high-speed drag.

The superiority of the optimum stepped hull over a conventional planing hull can be seen by comparing the optimum drag value from Figure 9 with the drag of a conventional hull, which is shown in Figure 8. The value of $\Delta/R$ of 8.5 referred to above corresponds to an $R/\Delta$ value of 0.12. The performance values presented in Figures 3 and 9 are not restricted to particular values of speed, but can be attained at any speed in the planing range. As an example, the optimum $R/\Delta$ value of 0.12 can be achieved, by appropriate design, at a speed corresponding to $F_V = 5.0$. (The corresponding speed in knots for a boat of any particular size can be determined from Figure 11.) Figure 8 indicates the value of $R/\Delta$ for the conventional planing boat at $F_V = 5.0$ is 0.172. This resistance value is 43 percent higher than the resistance value for an optimum stepped hull having the same deadrise angle.

Several auxiliary functions are also plotted in Figure 9, by means of which a number of interesting design problems can be solved. Two of these
functions are forms of the lift coefficient. One is $C_{Lb}$, which equals $\frac{1}{2} \rho \frac{V^2}{b^2}$, and the other is $C_{LP}$, which equals $\frac{1}{2} \rho \frac{V^2}{l_{cp}^2}$. There is a unique value of each of these functions for each of the associated pairs of values of aspect ratio and $\gamma$, which correspond to the minimum-resistance points of Figure 3. The steps involved in obtaining the values for preparing Figure 9 are indicated in Table I. The auxiliary graphs of Figure 10 were drawn in order to obtain the values of $l_{cp}/l_m$ needed for the calculation of $C_{LP}$. The values of $l_{cp}/b$ and $C_{LS}$ corresponding to the minimum-resistance condition are also plotted in Figure 9.

One of the design problems which can be solved by means of Figure 9 is the determination of the width of a planing hull which will give minimum resistance when the weight, speed, deadrise, and distance of the center of gravity forward of the transom are known. From the known quantities, the value of $C_{LP}$ can be calculated (the distance of the center of gravity forward of the transom is identical to $l_{cp}$). Figure 9 can then be entered with this value of $C_{LP}$ and the corresponding value of $l_{cp}/b$ determined (this is the value at the same aspect ratio). The value of the beam, $b$, can now be calculated. This procedure will be found to be a useful guide in selecting the width of each of the pontoon of a planing catamaran. In this case, of course, the weight to be used in the calculation is the weight carried by one pontoon.

If the ratio $l_{cp}/b$ is known for a design, together with the weight, deadrise, and speed, Figure 9 can be entered, and $b$ then calculated from the corresponding value of $C_{lb}$.

A third design problem which can be solved is the determination for a single-step planing boat of the location of the step for minimum resistance. It is assumed that the known quantities are the speed, deadrise, width of planing bottom at the step, and the weight carried by the main forebody planing surface (this will ordinarily be about 90 percent of the total weight of the boat). From the known quantities the value of $C_{lb}$ can be calculated. Figure 9 can then be entered to obtain the corresponding value of $l_{cp}/b$. $l_{cp}$ in this case is the distance between the step and the center of pressure on the forebody, and knowing the value of $l_{cp}$ will now make it possible to position the step in the optimum location.
Table I
THE STEPS INVOLVED IN CALCULATING THE VALUES
PRESENTED IN FIGURE 9 \((\ell = 10^2)\)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>R/\Delta</td>
<td>C_{IE}</td>
<td>C_{IS}</td>
<td>C_{IB}</td>
<td>\Delta/R</td>
<td>l_{cp}/l_m</td>
<td>l_{cp}/b</td>
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<td></td>
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</tr>
<tr>
<td>(Aspect ratio)</td>
<td>(Minimum)</td>
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</table>

Notes:

Values for Col. D are from Figure 1.

Col. F = C_{IB} = \Delta \sqrt{2} \rho V^2 b^2 = C_{IS}/A

Values for Col. H are from Figure 10.

Col. I = l_{cp}/b = l_{cp}/l_m \cdot l/A

Col. K = C_{Lp} = C_{IS}/Col. J
REFERENCES

1. Shuford, C.L., Jr., "A Theoretical and Experimental Study of Planing Surfaces Including Effects of Cross Section and Plan Form," National Aeronautics and Space Administration Report 1355 (1958)


Figure 1a

Figure 1 - Lift-Coefficient/Trim Angle Ratio versus Trim Angle

\[ \beta = 5^\circ \]
Figure 1b
Figure 2a

Figure 2b

Figure 2 - Center-of-Pressure/Mean-Wetted-Length Ratio versus Center-of Pressure/Beam Ratio
Figure 3 - Resistance/Displacement Ratio versus Trim Angle with Aspect Ratio as Parameter

Displacement 10,000 lb
Salt Water 59°F
Figure 3c

\[ \beta = 15^\circ \]
Figure 4 - Resistance/Displacement-Ratio versus Displacement with Aspect Ratio as Parameter
$\beta = 5^\circ$

Figure 4b
Figure 5a

Figure 5 – Resistance/Displacement Ratio versus Displacement with Aspect Ratio as Parameter
Figure 6

Figure 6 - Resistance/Displacement Ratio versus Displacement with Aspect Ratio as Parameter
Figure 5c

\[ \beta = 15^\circ \]
Figure 8 – Predicted Values of Trim and Hydrodynamic Drag for a Typical Planing Boat
Figure 9 - Optimum Values of Lift/Drag Ratio ($\Delta/R$) for Planing Hulls and Corresponding Values of $\tau$, $C_{L,b}$, $C_{L,p}$, $C_{p/b}$, and $C_L$.
Figure 10 -- Center-of-Pressure/Mean-Wetted-Length Ratio versus Trim Angle
These are auxiliary graphs which were needed in the preparation of Figure 9.
Figure 11 - Variation of Volume Froude Number with Speed and Displacement