INTELLIGENT SAMPLING OF HAZARDOUS PARTICLE POPULATIONS IN RESOURCE-CONSTRAINED ENVIRONMENTS (POSTPRINT)

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Subject Terms:
- Data sampling
- Environmental measurement
- Space radiation
- Magnetospheric plasma
- Solar energetic protons
Intelligent Sampling of Hazardous Particle Populations in Resource-Constrained Environments

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Abstract  Sampling of anomaly-causing space environment drivers is necessary for both real-time operations and satellite design efforts, and optimizing measurement sampling helps minimize resource demands. Relating these measurements to spacecraft anomalies requires the ability to resolve spatial and temporal variability in the energetic charged particle hazard of interest. Here we describe a method for sampling particle fluxes informed by magnetospheric phenomenology so that, along a given trajectory, the variations from both temporal dynamics and spatial structure are adequately captured while minimizing oversampling. We describe the coordinates, sampling method, and specific regions and parameters employed. We compare resulting sampling cadences with data from spacecraft spanning the regions of interest during a geomagnetically active period, showing that the algorithm retains the gross features necessary to characterize environmental impacts on space systems in diverse orbital regimes while greatly reducing the amount of sampling required. This enables sufficient environmental specification within a resource-constrained context, such as limited telemetry bandwidth, processing requirements, and timeliness.

1. Introduction
Space systems suffer from anomalies induced by a variety of environmental conditions (e.g., Baker, 2000). While there may be straightforward causal relationships in many cases, this is not always true. For example, Baker et al. (2009) showed that greater geomagnetic disturbance could lead to improved operations of the Aeronomy of Ice in the Mesosphere mission via an inadvertent floating input on the spacecraft receiver. Thus, it often requires significant effort to translate environmental conditions into meaningful hazard indicators. O’Brien (2007, 2009) provided a statistical framework by which these connections can be used to produce relative probabilities of four different hazards: single event effect and event total dose hazards driven by energetic protons and heavy ions, the internal charging hazard due to relativistic electrons, and surface charging driven by plasma-energy electrons. While much work has been done toward computing these hazards at Geosynchronous Equatorial Orbit (GEO) (O’Brien, 2009; Su et al., 2014), other considerations become significant in other orbit regimes.

We review these considerations so that needs in various orbit regimes can be defined. There are trade-offs between sampling the environment at high and low cadence. The ability to relate anomalies to postulated environmental drivers requires a knowledge or estimate of that driver at a cadence sufficient to capture changes between driving and nondriving environments. These changes may be temporal in general (e.g., time evolution of flux levels in a solar particle event) or merely due to spacecraft motion through spatial structure (e.g., Low Earth Orbit (LEO) satellite passage through the auroral zone).

This narrow objective of monitoring environmental drivers of anomalies does not require measurements of the highest science quality. Relative to science-quality measurements (as by missions such as the Van Allen Probes), such measurements may be relaxed in terms of energy coverage, energy/time/directional resolution, and flux dynamic range coverage. This limited scope enables use of smaller, cheaper sensors such as the Compact Environmental Anomaly Sensor (CEASE) (Dichter et al., 1998) or its follow-on, CEASE-Risk Reduction (CEASE-RR) (Lindstrom et al., 2017).

With this in mind, operational or resource constraints may demand lower cadence: minimizing telemetry demands; time integration commensurate with instrument capabilities; or, in the case of using model data, minimizing computationally intensive queries. Cadence also needs not be higher than the timescale of the
anomaly (which includes development, persistence, and recording of the anomalous state). Here we describe a method for sampling particle fluxes so that, along a given satellite trajectory, the variations from both temporal dynamics and spatial structure are largely captured, while significantly reducing the number of samples taken. The sampled fluxes may come from either on board measurements or from a model.

Sampling algorithms are defined for the three broad classes of particles mentioned above: energetic protons (5–100 MeV), energetic electrons (0.1–5 MeV), and plasma electrons (1–100 keV). Each of these classes includes multiple physical populations with distinct sources and characteristics (e.g., untrapped solar energetic particles (SEPs) and trapped protons are simply “energetic protons”).

The energy boundaries of each class are somewhat arbitrary. There may be cases where a single hazard involves two particle classes. For example, shallow subsurface discharges may be driven by electrons at energies within both the “plasma” and “energetic” classes. In such a case the results of the two sampling algorithms must be reconciled to ensure that both sets of cadences are met (e.g., by sampling at the most rapid cadence of the two).

Our approach defines a small set of spatial regions for each particle class that, together, fully contain the locations where hazards from those particles may be encountered. Within each region, spatial lengths and time intervals are defined to establish the appropriate sampling intervals along a trajectory. That is, a sample is required whenever either of two things happens: the vehicle moves by an amount equal to a specified length or the elapsed time since the previous sample exceeds the specified time interval.

The remainder of this report is organized as follows: Section 2 describes the coordinates, sampling method, and specific regions and parameters employed for this study. Section 3 compares resulting sampling cadences with data from spacecraft spanning the regions of interest. Finally, section 4 provides conclusions and prospects for further development.

2. Methodology

2.1. Definitions and Assumptions

Spatial sampling requirements are driven by the spatial variability of the hazardous populations, the morphology of which is dictated primarily by the geomagnetic field. Consequently, we define spatial sampling requirements in magnetic coordinates: $L$ (McIlwain, 1961) ($L$ value), $\varphi$ (magnetic longitude), and $\lambda$ (magnetic latitude), except in some low-altitude regions where altitude, $H$, is used instead of $L$. A tilted, offset dipole (i.e., eccentric dipole) magnetic field model is used everywhere. The applicability of this model will be addressed in section 2.4, but we note here that high model accuracy is not required for our purposes that are limited to defining broad sampling regions and determining approximate gradients.

Region boundaries are specified as subscripted coordinates, e.g., $L_{A1}$, $L_{A2}$, $\lambda_{A1}$, and $\lambda_{A2}$. The sampling scales within a region are denoted by $\Delta L$, $\Delta \varphi$, $\Delta \lambda$, $\Delta H$, and $\Delta t$. The values provided are provisional; specific values may vary with application and resources.

Locations not included in one of the defined regions are places where significant hazards from that particle class are not expected. The sampling in these places is determined by generic rules that are applicable for all particle classes, e.g., with a fixed 15 min cadence. This choice is driven by the characteristic timescales of the most dynamic populations: SEPs and energetic electrons. This sampling timescale also functions as an upper limit for the defined regions.

The method of producing a “sample” differs somewhat for on board data and model representations. For on board data it will often be desirable to average data close to the requested time in order to improve counting statistics and integrate rapid temporal fluctuations. For a model, the sample would represent the model’s best estimate of the flux at exactly the requested location and time.

2.2. Sampling Method

To characterize convective changes to populations of interest, we need to determine magnetic ephemeris and gradients. We first compute the transformed locations and velocities from Cartesian Geographic (GEO) coordinates into a Cartesian eccentric dipole (ED) frame (see equations (A10) and (A11), and Table A1). In this “primed” frame, the magnetic coordinates can be written as (all units are Earth radii and radians):

$$ L = \frac{r^2}{r^2} $$

\[ (1) \]
Figure 1. (left to right) Spatial regions for the three populations of interest: energetic protons, energetic electrons, and plasma electrons. Tables 1–4 provide full definitions and sampling parameters.

\[
\lambda = \arcsin \frac{z'}{r'},
\]

\[
\varphi = \arctan \left( \frac{y'}{x'} \right),
\]

where \( \rho' = x'^2 + y'^2 \) and \( r'^2 = x'^2 + y'^2 + z'^2 \).

The magnetic gradients can then be written as

\[
\vec{\nabla} \lambda = \begin{pmatrix}
\frac{\partial \lambda}{\partial x'} \\
\frac{\partial \lambda}{\partial y'} \\
\frac{\partial \lambda}{\partial z'}
\end{pmatrix} = \begin{pmatrix}
\frac{\lambda'}{\rho'}^2 - \frac{\lambda'^2}{\rho'^2} \\
\frac{\lambda' y'}{\rho'^2} - \frac{\lambda' z'}{\rho'^2} \\
\frac{\lambda' x'}{\rho'^2}
\end{pmatrix},
\]

(4)

\[
\vec{\nabla} \varphi = \begin{pmatrix}
\frac{\partial \varphi}{\partial x'} \\
\frac{\partial \varphi}{\partial y'} \\
\frac{\partial \varphi}{\partial z'}
\end{pmatrix} = \begin{pmatrix}
-\frac{y'}{\rho'} \\
-\frac{x'}{\rho'} \\
0
\end{pmatrix},
\]

(6)

The other spatial coordinate, altitude \( H \), needs to be computed in the GEO (unprimed) frame:

\[
H = r - R_e, \quad \vec{\nabla} H = \begin{pmatrix}
\frac{\partial H}{\partial x} \\
\frac{\partial H}{\partial y} \\
\frac{\partial H}{\partial z}
\end{pmatrix} = \begin{pmatrix}
x \\
y \\
z
\end{pmatrix}.
\]

(7)

Table 1
Energetic Proton (5–100 MeV) Sampling Parameters

<table>
<thead>
<tr>
<th>Region</th>
<th>( \Delta L )</th>
<th>( \Delta \lambda )</th>
<th>( \Delta \varphi )</th>
<th>( \Delta H )</th>
<th>( \Delta t )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( \Delta L_A )</td>
<td>( \Delta \lambda_A )</td>
<td>—</td>
<td>N/A</td>
<td>( \Delta t_A )</td>
<td>SEPs</td>
</tr>
<tr>
<td>3 &lt; ( L \leq 9 ) 0.25</td>
<td>1°</td>
<td>single</td>
<td>15 min</td>
<td>(may be above cutoff)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H &gt; 2,000 km ( \varphi ) cell</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>( \Delta L_B )</td>
<td>( \Delta \lambda_B )</td>
<td>( \Delta \varphi_B )</td>
<td>N/A</td>
<td>( \Delta t_B )</td>
<td>trapped protons and possible SEPs</td>
</tr>
<tr>
<td>( L \leq 3 ) 0.1</td>
<td>2°</td>
<td>5°</td>
<td>15 min</td>
<td>to ( L \approx 2.4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H &gt; 2,000 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>( \Delta L_C )</td>
<td>( \Delta \lambda_C )</td>
<td>( \Delta \varphi_C )</td>
<td>( \Delta H_C )</td>
<td>( \Delta t_C )</td>
<td>SAA and low altitude structure</td>
</tr>
<tr>
<td>(</td>
<td>\leq 70° ) N/A</td>
<td>2°</td>
<td>5°</td>
<td>50 km</td>
<td>15 min</td>
<td></td>
</tr>
<tr>
<td>H \leq 2,000 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To avoid the computational burden of explicitly calculating magnetic coordinates along a large number of trajectory points, and then differencing them to identify “delta” changes, each subsequent sampling step is directly calculated from the current location and velocity at a corresponding time $t_i$ as follows. The elapsed times at which the various convective criteria would be met are defined to be

$$\Delta T_L = \frac{\Delta L}{|\vec{v} \cdot \vec{\nabla} L|},$$

$$\Delta T_H = \frac{\Delta H}{|\vec{v} \cdot \vec{\nabla} H|},$$

$$\Delta T_\lambda = \frac{\Delta \lambda}{|\vec{v} \cdot \vec{\nabla} \lambda|},$$

$$\Delta T_\varphi = \frac{\Delta \varphi}{|\vec{v} \cdot \vec{\nabla} \varphi|}.$$

The time of the next sample, $t_{i+1}$, is set by the first of the various criteria to be met. That is, it is the minimum of the three region-specific convective criteria along with the temporal criterion $\Delta t$. For the case considered here, with low-altitude regions defined in terms of $H$ and other regions in terms of $L$, two expressions for $t_{i+1}$ emerge:

$$t_{i+1} = t_i + \min \left( \Delta t, \Delta T_L, \Delta T_\lambda, \Delta T_\varphi \right)$$

or

$$t_{i+1} = t_i + \min \left( \Delta t, \Delta T_H, \Delta T_\lambda, \Delta T_\varphi \right).$$

### 2.3. Specific Regions and Parameters

For the purposes of this study, we have defined spatial regions dictated by appropriate local sampling for each population of interest (see Figure 1). Tables 1–4 define the region boundaries and sampling rules, along with notes on the hazards and rationales informing the rules. For trapped particles, the relative frequency of required sampling in different directions (e.g., in $\lambda$, $\varphi$, and $H$) parallels sampling scales in the AE9/AP9/SPM...
climatological model. For example, the model developers found a separate low-altitude grid was required to represent particle gradients in LEO (Ginet et al., 2013) and recommend that model queries use different sampling timescales for different orbit regimes (Roth, 2014). Accordingly, energetic populations (Tables 1 and 2) have regions characterized by a boundary at 2,000 km separating high- and low-altitude areas. We define low-altitude regions with $H$ as a spatial parameter in lieu of $L$, since $L$ and $\lambda$ become degenerate at low altitudes.

For energetic protons, Region A corresponds to the region outside the trapped populations and may be exposed to SEPs; Region B includes trapped radiation belt protons and possibly SEPs at higher latitudes; and Region C is the low-altitude region characterized by localized structure such as the South Atlantic Anomaly (SAA). Similar to this region, energetic electrons are covered by Region D at low altitudes (e.g., where the low-altitude “horns” of the outer zone are encountered). At higher altitudes, Regions E and F encompass the inner and outer zone of the electron radiation belt, respectively. Plasma electrons are represented by two regions: Region H characterizes the plasma sheet in the magnetotail as well as near-equatorial inner magnetosphere, and Region J corresponds to the auroral population (although resolving auroral structures will not be necessary for all applications) and high-latitude extent of the plasma sheet. Untrapped auroral particles require finer latitudinal sampling due to the fine spatial structure associated with discrete aurorae. To complete the picture, we define Regions X and Y as regions that may contained trapped populations and those that do not, respectively.

Spatial and temporal scales are specified for each region. These are meant to be adjustable and are context dependent. For example, in this particular case, note that Regions E and F have identical scales. Given that those regions distinguish the inner and outer zones of the radiation belt, which are subject to different processes and dynamics, it is useful to be able to adjust the parameters for each zone independently. Note also that while the gradient of the magnetic longitude ($\varphi$) is needed for these calculations, the coordinate itself (and thus the $\text{atan2}$ function) is not required given the regions and scales in this work.

While specific values are listed in Tables 1–4, they are adjustable within the constraints of climatology. The general approach outlined above can be applied to a different set of regions designed for a particular application. For example, one could define local time-dependent regions (e.g., dawn, dusk, the midnight sector, etc.) as a function of $\varphi$. The remainder of this study will employ the regions defined in Tables 1–4.

Figure 2 shows how the algorithm samples a representative highly elliptical orbit (HEO). With the exception of the third panel showing orbit projections, the $x$ axis for the panels is time in seconds. Figure 2 (top three panels) shows orbit parameters in the course of one orbit. Perigee can be easily identified near 9,000 s of elapsed time.

In Figure 2 (bottom three panels), blue denotes energetic protons, red denotes calculations for energetic electron, and green indicates plasma electrons. The first of these shows, for each population, which regions the orbit intersects. It should be no surprise that this inclined HEO orbit traverses every region. The penultimate panel shows the sampling criterion that determines the time cadence for each population as the orbit is executed. This can be key to assessing the suitability of the parameters chosen for each region. Finally, bottom panel shows the resulting sample times for each population. The most visible feature is a general trend aligned with orbital velocity (low cadence at apogee, higher cadence approaching perigee) for all populations. Energetic electrons exhibit finer resolution away from perigee, corresponding to their most dynamic region.
Figure 2. Sample algorithm results for a specific HEO orbit. The $x$ axis for the panels is time in seconds with the exception of the third panel showing orbit projections. (top two panels) Orbit parameters in the course of one orbit. (bottom three plots) Orbit projections in three orthogonal planes. In Figure 2 (bottom three panels), blue denotes energetic protons, red denotes calculations for energetic electron, and green indicates plasma electrons. The first of these shows which region the satellite is in over the course of the orbit; next, the criterion determining sample cadence; and last, the sample times (squares) and relevant $\Delta T$ (lines).

### 2.4. Suitability of Approach

As noted above, the nominal boundary and sampling rule definitions for the regions are intended to be adjustable according to the user application and resource constraints. The values presented here are designed to be conservative, preferring more stringent sampling at transitions between hazard regions. An example is the fine latitudinal sampling in region J for plasma electrons, which applies down to 50° magnetic latitude even though extreme geomagnetic storms are required for aurora to reach this latitude. The rules and region definitions therefore lead to high variability in sampling rates for LEO polar satellites. Figure 3 illustrates how the requirements combine in this orbit regime. The most stringent sampling requirements for such
Figure 3. North/south sampling length scales versus magnetic latitude for polar circular orbits at (top row) 840 km and (bottom row) 2,000 km. Red lines are derived from the $\Delta \lambda$ parameter, and blue from the $\Delta L$ parameter. Thick segments are the ones that drive the sampling cadence. The latitudinal extent of hazardous populations for these orbits is indicated with gray bars.

Figure 4. Data from RBSP-A separated by population from 11:37 UT to 20:38 UT, spanning one orbit during the active period. Diamonds indicate averaged values over the sample window for each orbit.
Figure 4. Data from RBSP-A from 11:37 UT to 20:38 UT. Diamonds indicate averaged values over the sample window for the corresponding species. Regions are indicated by species-specific background colors corresponding to the colors used in Figure 1.

corresponding species. The energetic electron and proton panels show Level 2 spin-averaged flux measurements from the Relativistic Electron Proton Telescope (REPT) instrument (Baker et al., 2013), and the plasma electron panel shows Level 3 omnidirectional flux data from the Helium Oxygen Proton Electron (HOPE) instrument (Funsten et al., 2013) and Level 2 spin-averaged flux data from the Magnetic Electron Ion Spectrometer (MagEIS) (Blake et al., 2013) instrument. Regions are indicated by species-specific background colors corresponding to the colors used in Figure 1.

Inspection of Figure 4 suggests that in all parts of the orbit, the diamonds corresponding to sample values closely track the observed values. Therefore, the sample cadence is sufficient to resolve significant changes in flux level. Whereas the native time series for MagEIS had over 8,000 data points, the sample algorithm produced 141 points for energetic electrons and 58 points for plasma electrons. This indicates a significant reduction in data volume. Similarly, REPT and HOPE data saw reductions of 98% and 96%, respectively. Some of the high-frequency fluctuations in energetic electrons near apogee may be missed, but the small amplitude points to little impact on cumulative effects of exposure to this population (e.g., internal charging).
Figure 5. Data from the POES spacecraft NOAA-18 from 18:00 UT to 20:00 UT. Diamonds indicate averaged values over the sample window for the corresponding species. Regions are indicated by species-specific background colors corresponding to the colors used in Figure 1.

The sample rate increases as the vehicle nears perigee. This aligns well with the shape of the proton population (a locally confined, trapped proton belt at lower altitudes and a broad, untrapped population at higher altitudes) and the plasma population (notable decreases in the higher-energy channels near perigee). The possibility of proton contamination at lower altitudes in the energetic electron channels precludes assessment of the finer sample period there. For the MagEIS instrument, analysis suggests that below \( L = 2.5 \), all energy channels are contaminated (Claudepierre et al., 2015).
3.2. Comparison With POES Data

The Polar-Orbiting Operational Environmental Satellites (POES) are a constellation of spacecraft in Polar LEO Sun-synchronous orbits (~840 km circular, 98° inclination). They include the Space Environment Monitor (SEM-2) suite that senses the populations of interest (Rodger et al., 2010). Due to their high inclination, the POES spacecraft transit through Regions C, X, and Y for protons; D, X, and Y for energetic electrons; and H, J, and Y for plasma electrons.

Figure 5 shows data from the POES NOAA-18 spacecraft separated by population from 18:00 UT to 20:00 UT, spanning approximately 1.25 orbits during the active period. Diamonds indicate averaged values over the sample window for the corresponding species. The energetic electron (Figure 5, first panel) and proton (Figure 5, second and third panels) contain measurements from the Medium Energy Proton and Electron Detector (MEPED) instrument, and the plasma electron shows data from the MEPED (Figure 5, fourth panel) and Total Energy Detector (TED) (Figure 5, fifth panel) instruments. Regions are indicated by species-specific background colors corresponding to the colors used in Figure 1. The channel energy labels for the MEPED data follow CDAWeb convention and are derived from the bow-tie analysis described in Green (2013). These channel energies (and associated conversions from counts to fluxes) consequently differ slightly from the nominal energies frequently reported for these standard channels. This does not affect our discussion, as our concern is with the variability of hazardous populations over broad energy ranges.

It is important to note the apparent prevalence of cross-species contamination. This is acknowledged in section 2.2.1 of Green (2013) and suggests that the MEPED electron fluxes poleward of the auroral region are actually untrapped protons, and the flux enhancement collocated with the trapped proton belt is indeed protons. Additionally, the > 6 MeV proton channels likely exhibit electron contamination.

Inspection of Figure 5 suggests that for almost the entirety of the orbit, the diamonds corresponding to sample values closely track the observed values. Therefore, the sampling algorithm performed well. The poleward edge of Region J near 18:50 UT misses the onset of auroral plasma by approximately 1 min. This is likely due to Region J not encompassing the entire auroral oval at that time, which is the subject of ongoing study. Whereas the native time series for MEPED had over 36,800 data points, the sample algorithm produced 184 points for energetic electrons and 1,151 points for plasma electrons. This indicates a significant reduction in data volume. Similarly, MEPED Proton and TED data saw reductions of 99.5% and 97%, respectively. The energetic channels only deviate from the low-altitude high-resolution cadence near the poles, where the only population of interest is the isotropic untrapped proton population. The sample rate increases as the vehicle nears the auroral region, aligning well with the shape of the plasma population including notable enhancements and localized structure.

4. Conclusion

A method for intelligent sampling of hazardous particle populations informed by magnetospheric considerations has been developed. This should be thought of as a general framework; specific choices of region boundaries and sampling scales should be made with particular applications and constraints in mind. These constraints could include orbit characteristics, spacecraft susceptibility, bandwidth, and timeliness. Other considerations, including instrument limitations such as geometric factor, field of view, integration time, etc. should also be taken into account. With appropriate assumptions, eccentric dipole coordinates are suitable for determining sampling cadence. Defining static regions in this coordinate system based on known population climatology enabled lean implementation of the sampling algorithm.

The algorithm performed favorably against relevant satellite data spanning the defined regions. While undoubtedly “lossy,” it captures the gross features necessary to assess environmental impacts on space systems in all orbital regimes with a significant reduction in samples needed. This enables adequate environmental specification within a resource-constrained context.

An immediate next step is to incorporate a more sophisticated picture of the auroral oval in the definition of Region J. Further work includes assessment and tailoring of the algorithm for use with global models, a sensitivity study of the energy bounds on the particle populations, and examination of other hazard-driving populations.
Appendix A: Eccentric Dipole (ED) Coordinate System

Following Usoskin et al. (2010) and Fraser-Smith (1987), we define an offset vector \( \vec{d} \) and dipole axial vector \( \vec{p} \) in the GEO noninertial frame using appropriate IGRF coefficients (\( g_m^m \) and \( h_m^m \) are of degree \( n \) and order \( m \)):

\[
\vec{d} = \begin{pmatrix}
\frac{L_1 - g_1^1 E}{B_0} \\
\frac{L_2 - h_1^1 E}{B_0} \\
\frac{L_0 - g_1^0 E}{B_0}
\end{pmatrix}, \quad \vec{p} = \begin{pmatrix}
-\frac{g_1^1}{B_0} \\
-\frac{h_1^1}{B_0} \\
-\frac{g_0^1}{B_0}
\end{pmatrix},
\]

where the units are Earth radii. The parameters used above are defined as

\[
B_0^2 = (g_0^0)^2 + (g_1^1)^2 + (h_1^1)^2.
\]

\[
L_0 = 2g_0^0g_2^0 + \sqrt{3} (g_1^1g_2^1 + h_1^1h_2^1).
\]

\[
L_1 = -g_1^1g_2^1 + \sqrt{3} (g_0^0g_2^1 + g_1^1g_2^1 + h_1^1h_2^1).
\]

\[
L_2 = -h_1^1g_2^1 + \sqrt{3} (g_0^1h_2^1 - h_1^1g_2^1 + g_1^1h_2^1).
\]

Figure A1. Contours of constant \( L \) in GEO coordinate planes. The dipole center is indicated with a cross.
Figure A2. Contours of constant $L$ at an altitude of 600 km in geographic latitude and longitude.

$$E = \frac{L_0 g_0^2 + L_1 g_1^1 + L_2 h_1^1}{4B_0^2}. \quad (A6)$$

To transform positions, a translation from the origin along $\vec{d}$ is performed followed by two rotations to get to the ED frame. Velocities only need rotations applied. For the first rotation, we rotate about the GEO $z$ axis an angle $\theta_1$:

$$A_1 = \begin{pmatrix} \cos \theta_1 & \sin \theta_1 & 0 \\ -\sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} p_x & p_y & 0 \\ -p_y & p_x & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (A7)$$

Figure A3. Contours of constant $L$ at an altitude of 2,000 km in geographic latitude and longitude.
where $p_r = \sqrt{p_x^2 + p_y^2}$ is the $r$ component of $p$, in the cylindrical coordinate sense. For the second rotation, we rotate about the new $y$ axis, which should lie perpendicular to the $\rho$ direction (note that the $z$ axis is unchanged and $\hat{p}$ is a unit vector):

$$A_2 = \begin{pmatrix} \cos \theta_2 & 0 & -\sin \theta_2 \\ 0 & 1 & 0 \\ \sin \theta_2 & 0 & \cos \theta_2 \end{pmatrix} = \begin{pmatrix} p_x & 0 & -p_y \\ 0 & 1 & 0 \\ p_y & 0 & p_x \end{pmatrix}.$$  \hspace{1cm} (A8)

We can thus write the rotation matrix $A = A_2 \cdot A_1$ as

$$A = \begin{pmatrix} p_x p_z & p_x p_y & -p_y \\ -p_z & p_y & 0 \\ p_z p_y & p_x & p_y \end{pmatrix}.$$ \hspace{1cm} (A9)

$$A = \begin{pmatrix} g_i \tilde{g}_j \tilde{g}_k \tilde{g}_l & 0 & 0 \\ -g_i \tilde{g}_j \tilde{g}_k \tilde{g}_l & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$ \hspace{1cm} (A10)

This allows definition of transformations from GEO (unprimed) to ED (primed) positions and velocities as

$$\vec{r}' = A \cdot (\vec{r} - \vec{d}),$$ \hspace{1cm} (A11)

$$\vec{v}' = A \cdot \vec{v}.$$ \hspace{1cm} (A12)

We include the following figures to provide verification checks on implementing the transformations described above, using the IGRF 2015 parameters with no secular variation. Figure A1 shows how contours of constant $L$ appear in the GEO frame. Figures A2–A3 show contours of constant $L$ in geographic latitude and longitude at altitudes of 600 km and 2,000 km, respectively.

Acknowledgments
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References

Table A1
Definitions of GEO and ED Coordinate Systems

<table>
<thead>
<tr>
<th>System</th>
<th>Origin</th>
<th>z axis</th>
<th>x axis</th>
<th>y axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEO</td>
<td>Earth’s center</td>
<td>Earth’s rotation axis</td>
<td>orthogonal to z axis and intersecting the Prime Meridian</td>
<td>orthogonal to x and z axes such that they form a right-handed set</td>
</tr>
<tr>
<td>ED</td>
<td>dipole center</td>
<td>Earth’s dipole axis</td>
<td>orthogonal to z axis, in the plane of the rotation and dipole axes, chosen such that +x is in the half-plane containing +z</td>
<td>orthogonal to x and y axes such that they form a right-handed set</td>
</tr>
</tbody>
</table>

"..."


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