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14. ABSTRACT

Filtering refers to a broad methodology for the online incorporation of data into mathematical models. It is hence of central importance in numerous application domains in the physical sciences, and in particular it is of direct relevance in a number of areas of importance to the US Navy, including oceanographic data assimilation (based on satellite data and on Lagrangian instruments) and in the monitoring of sea ice at the poles. In this work we will take the major step of developing theories of filter accuracy and stability within the context of these US Navy applications.

15. SUBJECT TERMS

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Final Report
ONR Award N00014-17-1-2079

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1 Opening Remarks

In summer of 2016 the PI Stuart took up a new position at the California Institute of Technology (Caltech). The award at the University of Warwick was closed out and a new one was initiated at Caltech, gudied by the same overall research goals. This award expired in July 2018 and this is the final report. A renewal has been applied for. The work undertaken in Caltech is DISTRIBUTION A, approved for public release: distribution unlimited.

2 Accomplishments

2.1 Major Goals

In the final report at the University of Warwick, relating to the move by the PI to Caltech, a shift in the research goals of the project was described. In particular, the wish to study climatological and oceanographic data assimilation, and the need to match empirical statistics derived through time-averaging, was described. This direction developed into one of the major thrusts of the research – thrust (a). Underpinning work is also being developed relating to the development of useable Gaussian approximations of Bayesian posterior distributions; the purpose of this thrust (b) in the research is to develop methods that can be used in the near future to make full uncertainty quantification in climatological and oceanographic models achievable.

2.2 Accomplished Under Goals

In work under thrust (a), jointly with Tapio Schneider of Geophysical Sciences at Caltech, I have developed a new ensemble Kalman based approach to learning parameters in geophysical models, based on using time-averaged statistics representing means and correlations of measurable physical variables. This resulted in publication 5. in which we trialled the methodology for the Lorenz 96 multiscale model of the atmosphere. The methodology employed should be viewed as a derivative-free optimization procedurei; uncertainty is not quantified.

In work under thrust (b) I have developed approximation methods based around Gaussian approximations of full posterior distributions; see publications 1. and 4. Although this work is formulated to apply quite generally, it is being pursued with a view to, in the near future, combining with the work under thrust (a) to extend the optimal learning of parameters in climate models. The work in papers 2. and 3. underpins what emerged in papers 1. and 4. This work broadens the impact of the ONR funded research into new applications of the methodology that has been developed.

2.3 Training Opportunities

In fall 2017 I offered a new course on Data Assimilation and Inverse Problems, at the graduate level, in Caltech. This course attracted students from across the institute, in computational and mathematical sciences, in biological modeling, in geophysics and in climate modeling; the course is informed by ONR research of the PI. A set of lecture notes from the course was placed on the arixv: `https://arxiv.org/abs/1810.06191`

2.4 Results Dissemination

- Publications. Five papers based on work relevant to ONR funded research have appeared during the reporting report. These are detailed below.
- Presentations. During the reporting period I gave four plenary lectures on ONR funded research at major international meetings: (i) Applied Inverse Problems, Hangzhou, China, June 2017; (ii) Strathclyde Numerical Analysis Meeting. Glasgow, Scotland, June 2017; (iii) Foundations of Computational Mathematics 2017. Barcelona, Spain, July 2017; (iv) 14th US National Congress on Computational Mechanics. Montreal, Canada, July 2017. I also spoke on ONR related research at workshops in Cambridge, Rice and UCSB, as well as delivering colloquia in Brown, Irvine and USC.

2.5 Plans for Future ONR Research

Paper 5. provides a blueprint for parameter estimation in climate models, by matching to empirical statistics. Next steps are to: a) generalize this work from the Lorenz 96 model to sub-grid scale cloud and ocean models; b) generalize this work to include Bayesian uncertainty quantification, using approximations based on machine learning, and leveraging information gained during the ensemble Kalman based training of parameters (see paper 6.). This proposed work is the subject of the renewal submitted earlier this year.

2.6 Honors and Awards

The PI was appointed to an endowed chair at Caltech where he is now a Bren Professor of Computing and Mathematical Sciences.

3 Participants

Andrew Mark Stuart. PI. 1 person month (involved in papers 1.–5.) Postdoctoral researchers Matt Dunlop (paper 1.) and Shiwei Lan (paper 5.) were partially supported in the reporting period.

4 Students

Nikola Kovachki (Caltech) was partially funded through this award (paper 6.) Furthermore, former student Yulong Lu (papers 3. and 4. and now a postdoc in Mathematics at Duke University) collaborated on ONR funded research which appeared in the reporting period.

5 Publications

1 D.Calvetti, M.M.Dunlop, E.Somersalo, A.M.Stuart, Iterative Updating of Model Error for Bayesian Inversion. Inverse Problems, **34**(2018) 025008.

2 C.-E. Brehier, M. Hairer and A.M.Stuart, Weak error estimates for trajectories of SPDEs under spectral Galerkin discretization. Journal of Computational Mathematics, **36**(2018) 159-182.

3 Yulong Lu, Andrew Stuart, and Hendrik Weber, Gaussian Approximations For Transition Paths In Brownian Dynamics. SIAM J. Math. Anal. **49**(2017) 3005-3047.

4 Yulong Lu, Andrew Stuart, and Hendrik Weber, Gaussian Approximations for Probability Measures on R^d. SIAM/ASA J. Uncertainty Quantification **5**(2017) 1136-1165.

5 T. Schneider, S. Lan, A.M. Stuart, J. Teixeira, Earth System Modeling 2.0: A Blueprint for Models That Learn From Observations and Targeted High-Resolution Simulations. Geophysical Research Letters **44**(2017).

6 N. Kovachki and A.M. Stuart, Ensemble Kalman Inversion: A Derivative-Free Technique For Machine Learning Tasks, `https://arxiv.org/abs/1808.03620`.