THE IMPACT OF TECHNOLOGIES AND MISSIONS ON CONTINGENCY BASE FUEL CONSUMPTION

by

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June 2018

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Second Reader: Hong Zhou

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Fossil fuels satisfy the bulk of the U.S. military's energy requirements for transportation and sustainment needs. In recent years, deployments to Operation Enduring Freedom (OEF), Operation Iraqi Freedom (OIF), and Operation Inherent Resolve (OIR) have highlighted inefficiencies in how the U.S. military generates electrical power. Many strategies have been proposed to unleash the U.S. military from the tether of fuel. This paper presents a mixed integer linear program to minimize the fuel needed to meet power requirements at a contingency base over a 24-hour period. The paper then assesses the impact on fuel consumption and generator run-hours of introducing energy storage systems and photovoltaic arrays to different power demand scenarios based on the mission, geographic, and seasonal parameters.

Removing the traditional requirement for spinning reserves and allowing generators to operate at 100% of their rated load resulted in substantial reductions in generator run-hours across all scenarios. The results showed that adding an energy storage system had effectively no impact on generator run-hours or fuel consumption, and that the impact of adding a photovoltaic array was highly dependent upon the latitude and season in which the contingency base was established. The author concludes that diesel and JP-8 are the best methods for storing energy at a contingency base, and that reducing energy demand is the most direct way to reduce fuel consumption and generator run-hours.
THE IMPACT OF TECHNOLOGIES AND MISSIONS ON CONTINGENCY 
BASE FUEL CONSUMPTION

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ABSTRACT

Fossil fuels satisfy the bulk of the U.S. military's energy requirements for transportation and sustainment needs. In recent years, deployments to Operation Enduring Freedom (OEF), Operation Iraqi Freedom (OIF), and Operation Inherent Resolve (OIR) have highlighted inefficiencies in how the U.S. military generates electrical power. Many strategies have been proposed to unleash the U.S. military from the tether of fuel. This paper presents a mixed integer linear program to minimize the fuel needed to meet power requirements at a contingency base over a 24-hour period. The paper then assesses the impact on fuel consumption and generator run-hours of introducing energy storage systems and photovoltaic arrays to different power demand scenarios based on the mission, geographic, and seasonal parameters.

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<th>Definition</th>
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<td>AC</td>
<td>alternating current</td>
</tr>
<tr>
<td>BCIL</td>
<td>Base Camp Integration Laboratory</td>
</tr>
<tr>
<td>CB</td>
<td>contingency base</td>
</tr>
<tr>
<td>DoD</td>
<td>Department of Defense</td>
</tr>
<tr>
<td>ESS</td>
<td>energy storage system</td>
</tr>
<tr>
<td>FP</td>
<td>Force Provider</td>
</tr>
<tr>
<td>GAMS</td>
<td>General Algebraic Modeling System</td>
</tr>
<tr>
<td>ICE</td>
<td>internal combustion engine</td>
</tr>
<tr>
<td>kWh</td>
<td>kilowatt hour</td>
</tr>
<tr>
<td>LDSS</td>
<td>load demand start stop</td>
</tr>
<tr>
<td>MILP</td>
<td>mixed integer linear program</td>
</tr>
<tr>
<td>MINLP</td>
<td>mixed integer nonlinear program</td>
</tr>
<tr>
<td>MPC</td>
<td>model predictive control</td>
</tr>
<tr>
<td>PV</td>
<td>photovoltaic</td>
</tr>
<tr>
<td>PVES</td>
<td>photovoltaic (PV) energy system</td>
</tr>
<tr>
<td>RTE</td>
<td>round trip efficiency</td>
</tr>
<tr>
<td>SOC</td>
<td>state of charge</td>
</tr>
<tr>
<td>USMC</td>
<td>U.S. Marine Corps</td>
</tr>
<tr>
<td>VRE</td>
<td>variable renewable energy</td>
</tr>
</tbody>
</table>
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CHAPTER 1:
Introduction

The Office of the Assistant Secretary of Defense for Energy, Installations, and Environment lays out three key objectives for the U.S. military’s operational energy strategy [1]:

1. Increase future warfighting capability
2. Identify and reduce logistics and operational risks
3. Enhance mission effectiveness of the current force

The Department of Defense (DoD) has identified the fuel saving benefits of tactical microgrids compared to spot generation which extends both the operational reach and operational endurance of ground forces operating at a contingency base (CB). Both the Army and the U.S. Marine Corps (USMC) have recognized the benefits of tactical microgrids and have begun the acquisition process [2]. The question of whether and to what extent to incorporate variable renewable energy (VRE) and energy storage system (ESS) technologies into tactical microgrids to achieve further fuel savings remains open.

1.1 Contingency Bases and Operational Reach
The DoD defines operational reach as “the distance and duration across which a force can successfully employ military capabilities” [3]. Modern technologies allow U.S. service members to deploy almost anywhere in the world while enjoying a high standard of living. The U.S. military will typically utilize CBs during Phases I, IV, and V of military operations when reduced enemy activity allows the establishment of more permanent support infrastructure. The establishment of CBs and support services allows the U.S. military to extend the duration of missions; however, support services are energy intensive and impose additional burdens and risk to the logistical support of the operation.

The Undersecretary of the Army has defined Contingency Bases as follows [4]:

Contingency Bases (CB) are evolving locations that support military operations by deployed units and provide the necessary support and services for sustained operations. While not permanent bases or installations per se, the longer the
duration of the supported operation, the more they require facilities similar to permanent/enduring bases and installations (e.g., enhanced infrastructure). A CB generally has a defined perimeter and established access control points.

With this definition in mind, it is quite clear that the composition of a particular CB is entirely dependent upon the mission variables (mission, enemy, terrain and weather, troops and support available, time available, civil considerations) of the mission it is supporting. Possible support functions provided at a CB could include

- Billeting, with heating and cooling provided by environmental conditioning units.
- Personal hygiene, including shower, latrine, and laundry facilities.
- Food services, including food preparation, sanitation, and refrigeration.
- Water and waste water treatment.

The energy requirements of a particular CB will depend on the types of services it is providing, the number of personnel it is supporting, and the mission and temperature requirements under which it is operating.

1.2 Power Generation at Contingency Bases

CBs generate their power using predominantly generators because of the simplicity and reliability of generating power with this method. Energy is stored as diesel fuel (a very energy dense fuel) and combusted when needed, essentially providing on demand power generation. Diesel generators are designed for a specific load, and power generation is most efficient, on a kilowatt hour (kWh) per gallon of fuel basis, when operating at 100% of their rated loads.

The current state of the art for power generation at CBs is a technique known as spot generation. When operating under spot generation, a set of loads is connected to a singular generator sized to meet the peak demand of all loads. In practice this grid design often results in generators running at less than 50% of their rated load. In addition to sub-optimal fuel consumption, spot generation can also result in the wet stacking of diesel generators in which uncombusted fuel enters the exhaust system causing additional maintenance concerns. Under the majority of operating conditions, wet stacking can be avoided if generators are run above 60% of their rated load [5].
To improve the efficiency of power generation and avoid the potential for wet stacking generators, the Army is moving forward with acquiring a load demand start stop (LDSS) microgrid for use at CBs. The LDSS would aggregate loads across multiple generators and turns generators on or off based on power demand. During a site visit to the U.S. Army Base Camp Integration Laboratory (BCIL) the authors were shown a LDSS microgrid with six 60kW generators. The system had the following characteristics:

- Distributes load evenly among all generators necessary
- Maintains inertial "spinning" reserve to handle short term spikes in power demand
- Powers on an additional generator if the load per generator exceeds more than 80% of rated load for more than a minute
- Shuts down a generator if the load per generator is less than 40% of the rated load for more than a minute

During the site visit, Mr. Singleton from BCIL stated that under certain test scenarios, transitioning from spot generation to an LDSS microgrid resulted in upward of 30% fuel savings over a 24-hour time period [5].

### 1.3 Force Provider Base Camp

Since the range of possible base camps is infinite, this paper will center its analysis around the Army’s 150-person scalable base camp module. In addition to the variety of power consuming components found in the Force Provider (FP) 150-person base camp, the U.S. Army’s BCIL operates a FP base camp and has gathered data on the energy usage of various components. The component list and an example layout for the 150-person base camp can be seen in figures 1.1 and 1.2, respectively.
<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>150 person Module</th>
<th>50/75 person Module</th>
<th>Four 150 person Module co-located</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deployable TRICONS</td>
<td>13</td>
<td>6 (7 to support 75 personnel)</td>
<td>67</td>
</tr>
<tr>
<td>Latrine Systems</td>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Shower Systems</td>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Expeditionary TRICON Kitchen Systems</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Containerized Batch Laundry</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>TRICON Refrigerated Containers</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>60 Kilowatt Tactical Quiet Generators</td>
<td>6</td>
<td>3</td>
<td>26</td>
</tr>
<tr>
<td>TEMPER Tents (Air supported)</td>
<td>11</td>
<td>5 (6 to support 75 personnel)</td>
<td>52</td>
</tr>
<tr>
<td>400K British thermal unit water heaters</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Improved fuel distribution system</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Wastewater Evacuation Tank/trailers</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Electric Power Distribution Box</td>
<td>6</td>
<td>3</td>
<td>26</td>
</tr>
<tr>
<td>ECUs</td>
<td>12</td>
<td>6</td>
<td>56</td>
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<tr>
<td>Shower Water Reuse System</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Site Preparation Kit</td>
<td>Optional</td>
<td>Optional</td>
<td>1</td>
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</table>

<table>
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<th>Optional/Add-on Kits</th>
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<tbody>
<tr>
<td>Cold Weather Modification System</td>
</tr>
<tr>
<td>Primo Power Modification System</td>
</tr>
<tr>
<td>Electric Kitchen</td>
</tr>
</tbody>
</table>

Figure 1.1. List of 150-Person Module Components. Source: [6].

Figure 1.2. Example Layout 150-Person Module. Source: [6].
1.4 Cost of Fuel vs. Cost in Fuel

Many contemporary papers regarding optimal dispatch of microgrids express the cost function that they seek to minimize in terms of dollars. Expressing cost in dollars allows the dispatch strategy to normalize and consider many different cost factors, such as:

- Fuel
- Operation and maintenance for various systems
- Insufficient electrical power for a given load

Under such a model, the optimal dispatch solutions are driven entirely by the cost data and could potentially result in an optimal solution that burns more fuel but costs less than feasible alternatives.

While the cost of acquiring and maintaining technologies is certainly important to the design of a tactical microgrid [7], it does not factor into the dispatch. Based on his personal experience as a deployed logistics officer, the author has determined that utilizing a monetary cost function is not appropriate for a tactical microgrid at a CB. While a reasonably accurate estimate of the operations and maintenance cost of a generator can be derived from its acquisition cost and predicted life cycle, it is impossible to determine reasonable and agreed upon estimates for the fully burdened cost of fuel for a given location. Once microgrids are deployed and operational, the principal concern becomes extending the operational reach of the unit by

1. Minimizing fuel required to meet power demands of supported unit
2. Extending generator life by minimizing run hours needed to meet power demand

The model proposed by the author seeks to minimize fuel consumption and generator run hours when making dispatch decisions which removes the subjectivity associated with monetary cost based models.
CHAPTER 2:
Background Information and Literature Review

2.1 Diesel Generators

A diesel generator consists predominantly of an internal combustion engine (ICE) and an alternator that converts the mechanical energy of the ICE into alternating current (AC) power. The generator contains a control panel from which the output voltage and frequency of the power can be adjusted [8]. The efficiency with which a diesel generator converts fuel to AC power for a given set of atmospheric and maintenance conditions is principally determined by the percent at which a generator is loaded relative to its maximum output [5]. The principal factor informing generator maintenance are its run-hours, the cumulative hours the generator has run. Wet stacking is a maintenance adverse condition that occurs when uncombusted fuel from the ICE enters the exhaust system due to insufficient generator loading. Figure 2.1 shows a generator fuel efficiency curve, highlighting the operating zone in which wet stacking occurs.

![Fuel Consumption Curves of 60kW of TQG and AMMPS. Source: [5].](image)

Figure 2.1. Fuel Consumption Curves of 60kW of TQG and AMMPS. Source: [5].
2.2 Energy Storage Systems
The value of ESSs lies in their ability to store energy and deliver it at a later time period. Uncombusted fuel is a form of energy storage, and as Garcia’s research points out the round trip efficiency (RTE) of alternative ESSs is a key driver of whether or not they are utilized in a tactical microgrid [9]. While a multitude of ESS technologies exist, the author chooses to focus on lithium-ion storage devices because they offer the highest RTE compared to alternatives (e.g., lead-acid batteries, compressed air storage, hydrogenics). Tesla, Inc. is considered an industry leader in lithium-ion battery technology, and advertises storage systems with a RTE of 90% [10]. To address the nonlinearities of the lithium-ion charging dynamics [11], the author will take the following approach:

- Constrain maximum and minimum state of charge (SOC) so the ESS RTE behaves in an approximately linear manner
- Discretize ESS CHARGE-DISCHARGE rate into zones with unique RTE coefficients

2.3 Photovoltaic Energy Systems
A photovoltaic (PV) energy system (PVES) is the most likely candidate for installation at a CB because of its relative ease of use and procurement. The power output of a PVES is dependent upon the following factors:

- Surface area of installed PVES
- Efficiency of installed PV cells
- Irradiance profile (i.e., sunlight intensity at a certain time of the day), dependent upon
  - Latitude at which PVES is installed
  - Time of year
- Angle of the PV relative to the sunlight

Based on cost and availability considerations, a CB would likely be equipped with a silicon based PVES which have a conversion efficiency of less than 30% [12].
2.4 Economic Model Predictive Control

Economic model predictive control (MPC) is a framework used to obtain a discrete approximation of the solution to the open-loop optimal control problem [13]. The fundamental elements of Economic MPC are

1. Process model to govern state dynamics
2. Appropriate finite horizon approximation of infinite horizon control problem
3. State predictions; reference or target trajectories
4. Optimization problem
   (a) Cost function
   (b) Constraints

Given these inputs, Economic MPC computes an optimal control sequence across the finite horizon and the first sequence of controls are implemented. The entire time horizon is advanced one time step, state predictions are updated based on observed values, and an optimal control sequence is generated for the shifted horizon. Figure 2.2 provides a visual overview of the generalized Economic MPC process [13], [14].

![Model Predictive Control System](image)

Figure 2.2. Model Predictive Control System. Source: [14].
2.5 Optimal Dispatch of Microgrids

A large number of papers have been written about various aspects of determining the optimal dispatch strategy for a microgrid [15]–[18]. Microgrid state dynamics exhibit the following notable nonlinearities:

- Binary ON-OFF and CHARGE-DISCHARGE decisions for generators and ESSs
- Nonlinear relationship between generator fuel consumption and generator power output
- Nonlinear relationship between ESS RTE and ESS SOC and ESS CHARGE-DISCHARGE rate

Previous papers have approximated the mixed integer nonlinear program (MINLP) as a mixed integer linear program (MILP) in order to make the problem tractable by

- Utilizing a linear approximation of the generator fuel consumption.
- Constraining ESSs to operate in a range where they exhibit generally linear behavior.

These papers have also found that 15-minute time steps over a 24-hour time horizon are adequate for making optimal dispatching decisions. In his thesis work, LCDR Kevin Garcia improved upon the linear approximations of generator fuel curves by discretizing the curves into generation zones and investigating the effects of adding different sized generators to meet the needs of CBs [9].

2.6 Modeling Software and Solvers

General Algebraic Modeling System (GAMS) is a "high-level modeling system for mathematical programming and optimization" [19] that "allows its users to formulate mathematical models in a way that is very similar to their mathematical description" [19]. From the given algebraic model GAMS appropriately models and passes the optimization problem to a host of solvers. The author selected the CPLEX algorithm to solve the MILP optimization problem based on its performance in the MILP benchmark test conducted by Dr. Hans Mittelmann [20].
CHAPTER 3:
Methodology

This chapter describes the MILP developed to minimize fuel consumption and generator run-hours at a CB over a 24-hour time period.

3.1 Assumptions
This research focuses on optimally dispatching in situ operational microgrids; therefore the author assumes that informed microgrid design decisions have been made. Additionally, the model assumes the following:

- Mission and operational variables will determine the specific CB configuration and power usage.
- Generator dispatch is a tertiary controller; power quality and grid stability controllers will override optimal scheduling decisions in the event of a conflict.
- PV (or renewable) power production will never exceed power demand.
- Predicted load and PV output are 100% accurate.
- Effect of environmental conditions on generator and ESS efficiency are negligible.
- ESS self-discharge is negligible.

3.2 Component Modeling
This section describes the manner in which key components of the CB tactical microgrid were modeled and discusses the advantages and disadvantages of those modeling decisions.

3.2.1 Generators
To model the generator, the author improves upon Garcia’s technique of utilizing uniform step sizes for each power generation zone [9] by partitioning the power generation spectrum relative to the peak efficiency ($e_{max}$) achieved by the generator. The generation spectrum is then partitioned into six zones, as shown in Figure 3.1:

- Zones 1 and 2 (0 - 77.5% $e_{max}$) constitute the low end of fuel efficiency curve...
• Zones 3, 4, and 5 (77.5% - 92.5% $e_{max}$) provide finer resolution around the battery RTE points (80%, 85%, and 90%, respectively)
• Zone 6 (92.5% - 100% $e_{max}$) represents the upper end of the fuel curve where the generator operates most efficiently

The pointwise approximation of the fuel efficiency for a particular zone is the average value of the fitting function in that zone. Compared to a uniform step size of 1kW (resulting in 60 zones per generator), this methodology reduced MILP solve time by a factor of 500 while remaining within 1% of the objective function value.

Figure 3.1. AMMPS Fuel Data Fitting and Approximation. Source: [5].
3.2.2 Energy Storage Systems

Li et al. documented the nonlinear dependence of lithium-ion round trip charging efficiencies on the SOC of the ESS as well as the rate at which charging occurs. The effects of SOC on RTE were addressed by constraining the ESS to maintain minimum and maximum charge levels such that RTE as a function of SOC remains constant. Similar to the approach taken with generator modeling, the ESS was partitioned into distinct zones based upon the rate at which charge or discharge occurs. In keeping with the nonlinearities observed by Li et al. [11] the greater the magnitude of the charge/discharge rate, the lower the RTE of the ESS while operating in that zone, as shown in Table 3.1.

<table>
<thead>
<tr>
<th>Zone</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
<th>$f_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RTE</td>
<td>80%</td>
<td>85%</td>
<td>90%</td>
<td>100%</td>
<td>90%</td>
<td>85%</td>
<td>80%</td>
</tr>
<tr>
<td>$\frac{P_{\text{ESS}}}{P_{\text{GRID}}}$</td>
<td>0.894</td>
<td>0.922</td>
<td>0.949</td>
<td>1</td>
<td>1.054</td>
<td>1.085</td>
<td>1.118</td>
</tr>
<tr>
<td>$rate_{\text{max}}$ [kW]</td>
<td>-25</td>
<td>-12</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>$rate_{\text{min}}$ [kW]</td>
<td>-35</td>
<td>-25</td>
<td>-12</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 3.1. ESS Zone Characteristics

3.2.3 Photovoltaics

The model overestimates power production from PVESs by assuming that all installed panels will be capable of 2-axis tracking and that panels will operate at a 30% conversion efficiency, which exceeds observed efficiencies. These overestimations are not considered to be significant since this is a generalized study and the amount of PVES supplied power will be varied throughout the study. The author recommends that for real world implementations PV predictions be generated through stochastic methods rather than the deterministic approach taken in this study.

3.2.4 Load Data

Representative load data was constructed by analyzing load data received from multiple sources [5], [21] as well as the author’s personal experience with CB operations for a company sized (150 Soldier) element. Mission profiles represent the following:

- **Mission 1**: Wide area security/train-advise-assist, majority of unit activity during duty hours occurs off the CB
• **Mission 2:** Sustainment and support operations, majority of unit activity during duty hours occurs on the CB

Mission profiles are combined with seasonally based heating and cooling profiles as well as irradiance profiles for a given latitude to provide a representative load profile for a broad category of mission parameters. Representative loads profiles for the mission and seasonal components of the net load can be seen in Appendix A, Figures A.3 and A.4, respectively. Figure 3.2 shows a load profile for Mission 1 being conducted in the summer with 400m² of installed PV. Note the increase in demand as personnel wake up in the morning, drop when they depart to conduct their mission, and increase when they return in the evening.

![Graph of Load Profile](image)

**Figure 3.2. Load Profile for Specified Mission and Equipment Parameters**
3.3 Optimization Model

The Optimization Model is a discrete-time MILP approximation of the optimal dispatch problem over a 24-hour time horizon. The optimization model has two goals:

- Primary: Minimize fuel required to meet the power demand at a CB over a 24-hour period
- Secondary: Minimize total generator run hours over a 24-hour period

3.3.1 Indices

- \( t \in T \) set of all time periods
- \( g \in G \) set of all dispatchable generators
- \( h \in H \) set of all generator operating zones
- \( e \in E \) set of all Energy Storage Systems (ESS)
- \( f \in F \) set of all ESS operating zones

3.3.2 Parameters

- \( l \) length of time period [hours]
- \( pveff \) efficiency of installed PV panels
- \( pvins \) surface area of installed PV panels \([m^2]\)
- \( \varepsilon \) arbitrarily small value

3.3.3 ESS Data

- \( socmin(e) \) minimum charge of ESS \( e \) [kWh]
- \( socmax(e) \) maximum charge of ESS \( e \) [kWh]
- \( soc0(e) \) initial charge of ESS \( e \) [kWh]
- \( soc(t) \) end of horizon charge of ESS \( e \) [kWh]
- \( cdef(e,f) \) efficiency of charging/discharging ESS \( e \) in zone \( f \)
- \( cdmin(e,f) \) minimum charge/discharge rate of ESS \( e \) in zone \( f \) [kW]
- \( cdmax(e,f) \) maximum charge/discharge rate of ESS \( e \) in zone \( f \) [kW]
3.3.4 Generator Data

- \( \text{powmin}(g, h) \) minimum power out of generator \( g \) operating in zone \( h \) [kW]
- \( \text{powmax}(g, h) \) maximum power out of generator \( g \) operating in zone \( h \) [kW]
- \( \text{fuel}(g, h) \) fuel consumption of generator \( g \) operating in zone \( h \) [gal/kWh]

3.3.5 Predicted Data - Reference Trajectories

- \( \text{load}(t) \) predicted average load during period \( t \) [kW]
- \( \text{insol}(t) \) average irradiance during period \( t \) [kW/m²]

3.3.6 Continuous Decision Variables

- \( \text{POWGEN}(g, h, t) \) dispatch-able power supplied by generator \( g \) in zone \( h \) during period \( t \) [kW]
- \( \text{POWESS}(e, f, t) \) power supplied to the grid from ESS \( e \) in zone \( f \) during period \( t \) [kW]
- \( \text{RUN} \) total run time of generators over the entire time horizon [run-hours]

3.3.7 Binary Decision Variables

- \( X(g, h, t) \) operate generator \( g \) power in zone \( h \) during period \( t \) (0=NO, 1=YES)
- \( Y(e, f, t) \) operate ESS \( e \) in zone \( f \) during period \( t \) (0=NO, 1=YES)

3.3.8 Objective Function

\[
\text{minimize} \quad l \sum_T \sum_G \sum_H \text{fuel}(g, h) \cdot \text{POWGEN}(g, h, t) + \varepsilon \cdot \text{RUN}
\]

The objective function seeks to minimize the total fuel needed to meet the power demand at a given CB over a 24-hour period. The \( \varepsilon \cdot \text{RUN} \) term is a tiebreaker that instructs the solver to choose the solution with the lowest run-hours in the event of multiple optimal solutions.
3.3.9 Constraints

The objective function described in Section 3.3.8 is subject to the following constraints:

\[
\sum_{G} \sum_{H} POWGEN(g, h, t) + \sum_{E} \sum_{F} POWESS(e, f, t) + \text{insol}(t) * p\text{vins} * p\text{veff} - \text{load}(t) = 0 \quad \forall t, d, p \tag{1}
\]

\[
\text{soc}0(e) - l * \sum_{s=1}^{t} \sum_{F} cdff(e, f) * POWESS(e, f, s) \geq \text{socmin}(e) \quad \forall e, t \tag{2}
\]

\[
\text{soc}0(e) - l * \sum_{s=1}^{t} \sum_{F} cdff(e, f) * POWESS(e, f, s) \leq \text{socmax}(e) \quad \forall e, t \tag{3}
\]

\[
\text{cd\text{min}}(e, f) * Y(e, f, t) \leq POWESS(e, f, t) \quad \forall e, f, t \tag{4}
\]

\[
\text{cd\text{max}}(e, f) * Y(e, f, t) \geq POWESS(e, f, t) \quad \forall e, f, t \tag{5}
\]

\[
\sum_{F} Y(e, f, t) \leq 1 \quad \forall e, t \tag{6}
\]

\[
\text{soc}0(e) + l * \sum_{T} \sum_{F} POWESS(e, f, t) = \text{soct}(e) \quad \forall e \tag{7}
\]

\[
p\text{ow\text{min}}(g, h) * X(g, h, t) \leq POWGEN(g, h, t) \quad \forall g, h, t \tag{8}
\]

\[
p\text{ow\text{max}}(g, h) * X(g, h, t) \geq POWGEN(g, h, t) \quad \forall g, h, t \tag{9}
\]

\[
\sum_{H} X(g, h, t) \leq 1 \quad \forall g, t \tag{10}
\]

\[
\sum_{G} \sum_{H} l * X(g, h, t) = RUN \tag{11}
\]

Constraint (1) enforces that the power demand is met at every time period. Constraints (2) and (3) ensure that ESS charge level remains above the minimum and below the maximum levels. Constraint (4) and (5) individually control the charge and discharge rates of each zone of the ESS, and constraint (6) enforces that an ESS is only operating in at most one zone during a given time period. Constraint (7) enforces that our desired end of horizon energy storage is met. Constraint (8) and (9) set the upper and lower power generation limits for each zone, and constraint (10) enforces that each generator operations in no more than one zone during a given time period. Constraint (11) calculates the total run-hours required to meet power demand over a 24-hour period.
CHAPTER 4: Results

This chapter analyzes the impact of various technologies on the fuel and generator run-hour savings at a different CB power demand scenarios over a 24 hour period.

4.1 Impact of ESS
Incorporating a lithium-ion ESS with 80-90% RTE into a tactical microgrid had little impact on the 24-hour fuel consumption of the microgrid. The cost (in fuel consumption and generator run hours) for various scenarios is compared with and without an ESS being a component of the microgrid. The following are the ranges of fuel and run-hour savings observed across all scenarios:

- Fuel: 0 to 1.5% savings
- Run-hours: 0 to 3% savings

Maximal fuel and run-hour savings were observed on scenarios where a substantial amount of CB power is provided from PV arrays.

4.2 Impact of Spinning Reserves
The current practice utilized by the LDSS microgrid is to maintain 80% of each generators capacity as a spinning reserve to address sudden increases in power demand. This research analyzed the impact of removing this constraint and allowing generators to operate at up to 100% of their rated load. The following are the ranges of fuel and run-hour savings observed across all scenarios:

- Fuel: 0 to 3%
- Run-hours: 6.5 to 14.5%

Maximal fuel and run-hour savings were observed on scenarios without a PV array contributing to the CB power generation.
4.3 Breakeven Load

During the process of analyzing various load profiles, I began to observe a possible link between the mean and variance of load profile and whether or not the solver chose to utilize the ESS. For every ESS and generator configuration we have the following characteristics:

- \( \text{powmax} \) is the maximum generator output [kW].
- \( \text{RTE} \) is the round trip efficiency of ESS.
- \( f(\text{load}) \) is a function that outputs generator efficiency [kWh/gal] given an input load [kW].

From these characteristics, it is possible to define the breakeven point (i.e., load) below which utilizing the ESS could yield fuel savings:

\[
\text{breakeven} = f^{-1}(\text{RTE} \times f(\text{powmax}))
\]

For the 60kW LDSS microgrid the breakeven point is \( \approx 30 \) kW. Figure 4.1 shows how different generator configurations and changes to the RTE of the ESS affect the breakeven load for a microgrid.

![Figure 4.1. Impact of Generators and ESS RTE on Breakeven Loads](image)
4.4 Impact of Power Demand Variance

Various load profiles, shown in Figures A.5 and A.6, were generated to determine how the mean and variance of the load throughout the day impact fuel consumption and generator run hours. The equations for the respective load profiles are as follows:

- 35kW: \( \text{load}(t) = 35 \).
- 25kW: \( \text{load}(t) = 25 \).
- 24 hour period: \( \text{load}(t) = 32 - 25 \times \cos(\frac{t \pi}{48}) \).
- 2 hour period: \( \text{load}(t) = 32 - 25 \times \cos(\frac{t \pi}{2}) \).

The optimal dispatch results for each of these load profiles, with and without a ESS, are shown in Table 4.1. The optimal dispatch strategy for a 25kW constant load is shown in Figure 4.2. The following observations can be made from these results:

- Time periods can be classified as charge or discharge periods based on whether the net load is above (charge) or below (discharge) the breakeven load
- Load profiles where the load never falls below the breakeven load do not utilize the ESS in their optimal dispatch solution

<table>
<thead>
<tr>
<th>Load Scenario</th>
<th>No ESS Fuel (gal)</th>
<th>Gen-hours</th>
<th>ESS Fuel (gal)</th>
<th>Gen-hours</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>35kW constant</td>
<td>64.69</td>
<td>24</td>
<td>64.69</td>
<td>24</td>
<td>-</td>
</tr>
<tr>
<td>25kW constant</td>
<td>52.62</td>
<td>24</td>
<td>47.80</td>
<td>18.25</td>
<td>9.16%</td>
</tr>
<tr>
<td>High var. 2 hr period</td>
<td>64.61</td>
<td>24</td>
<td>59.93</td>
<td>18</td>
<td>7.24%</td>
</tr>
<tr>
<td>High var. 24 hr period</td>
<td>64.32</td>
<td>24</td>
<td>59.86</td>
<td>18.25</td>
<td>6.93%</td>
</tr>
<tr>
<td>Low var. 2 hr period</td>
<td>61.56</td>
<td>24</td>
<td>59.79</td>
<td>22.75</td>
<td>2.88%</td>
</tr>
<tr>
<td>Low var. 24 hr period</td>
<td>61.41</td>
<td>24</td>
<td>59.82</td>
<td>22.50</td>
<td>2.59%</td>
</tr>
</tbody>
</table>

Table 4.1. Impact of Load Variance on Fuel Consumption
Figure 4.2. Optimal Dispatch Strategy for 25kW Constant Load
CHAPTER 5: 
Future Work and Conclusions

Reducing the fuel consumption of generators used to meet the power demand at CBs will continue to remain a DoD priority. This thesis removed the subjectivity associated with the procurement and operations cost of technologies relative to the cost of fuel by focusing on the impact that those technologies have on fuel consumption.

5.1 Future Work

5.1.1 Extension of Breakeven Load to Time of Use Pricing
The principle of the break even load could be extended to permanent, grid connected DoD facilities to determine if an ESS makes sense based on the RTE of the ESS relative to the price difference between peak and off-peak electricity. With additional data, power demand loads could be dis-aggregated based off end use (e.g. building heating and cooling demands) and thermal ESSs could be investigated alongside electrical ESSs.

5.1.2 Removing Spinning Reserve Requirements
Removing spinning reserve requirements could result in a substantial reduction in generator run-hours and a modest reduction in the fuel needed to meet CB power demand. The impacts of short duration overloading (generating more than 100% of rated load) generators on the generator life-cycle could be studied to determine if it is a worthwhile trade off. Additionally, ESSs with high power density could be studied to determine if they could provide the same intertial reserve to the microgrid as spinning reserves.

5.1.3 Negative Net Load
Negative net load scenarios would require the presence of an ESS in order to fully utilize all power produced throughout the day. Different power producing technologies (wind, solar, tidal, etc.) and ESS technologies have distinct characteristics. The particular power production technology will determine the amount and time at which power is produced,
and different ESSs will have different storage capacities, RTEs, and charge and discharge rate limits. Different power production and storage technologies pairings could be studied to determine the optimal mix of technologies for a particular site.

5.1.4 Three-Stage Solver
Building upon the ideas laid out by Cañizares, Kazerani and Olivares [22], a three-stage solver could be introduced to determine the optimal generator dispatch strategy. This concept would introduce the notion of schedulable loads, such as laundry services or designated shower times, which are major power consumers whose timing can be relatively easily controlled. The following is a reference for how such a model could be built:

- **Stage I (MILP - Schedulable Load Commitments)**
  - Inputs: 48-hour net load forecast
  - Outputs: 24-hour schedulable load commitments

- **Stage II (MILP - Generator Commitments)**
  - Inputs: 24-hour net load forecast, 24-hour schedulable load commitments
  - Outputs: 24-hour generator commitments [ON/OFF]

- **Stage III (NLP - Generator Set Points)**
  - Inputs: 24-hour net load forecast, 24-hour schedulable load commitments, 24-hour generator commitments [ON/OFF]
  - Outputs: 2-hour generator set point targets

5.2 Conclusions
The author has reached the following conclusions regarding the impact of missions and technologies on fuel consumption at CBs:

- Diesel and JP-8 remain the most effective way to store energy at a CB.
- Reducing net energy demand at a CB is the most direct and effective way to reduce fuel consumption and generator run-hours.
- Under typical load scenarios, a CB would not benefit from the addition of a ESS.
  - ESS would not significantly reduce fuel consumption or generator run-hours.
  - ESS would introduce additional complexity and maintenance requirements for grid operators.
• Removing spinning reserve requirements could provide substantial generator run-hour savings.

• CB fuel and generator run-hour savings from variable renewable energy resources are effectively independent from whether or not a ESS is present.
APPENDIX A: Data

![Table of AMMPS Fuel Data](image)

Figure A.1. AMMPS Fuel Data. Source: [5].

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Figure A.2. Best PV Research-Cell Efficiencies. Source: [12].
Figure A.3. Representative 24 hour mission based load components.

Figure A.4. Representative 24 hour seasonal load components.
Figure A.5. High Variance Load Profiles

Figure A.6. Low Variance Load Profiles
APPENDIX B:
GAMS and MATLAB Code

B.1 GAMS Code
OPTIONS
  SOLPRINT = OFF,
  LP = XA,
  MIP = CPLEX,
  RMIP = XA,
  NLP = CONOPT,
  RMINLP = MINOS,
  MINLP = DICOPT,
  optcr = 0.01

SETS
  t  time step /t1*t96/
  g  generator /g1*g6/
  h  generation zone /h1*h7/
  e  ESS /e1/
  f  ESS operating zone /f1*f7/

ALIAS (t,s);

SCALARS
  lt  length of time step [hours] /.25/
  gas price of a gallon of diesel fuel [\$/gal] /1/
  pveff efficiency of installed PV panels /0.30/
  pvinst surface area of installed PV [m^2] /0/
  epsilon small value /0.00001/

PARAMETER socmin(e) minimum battery charge level [kWh] /
$ondelim offlisting
$include ess_socmin.csv
$offdelim onlisting
/

PARAMETER socmax(e) max battery charge level [kWh] /
$ondelim offlisting
$include ess_socmax.csv
$offdelim onlisting
/

PARAMETER soc0(e) initial battery charge level [kWh] /
$ondelim offlisting
$include ess_soc0.csv
$offdelim onlisting
/

PARAMETER soct(e) terminal battery charge level [kWh] /
$ondelim offlisting
$include ess_soct.csv
$offdelim onlisting
/

TABLE cdeff(f,e) efficiency of charging ESS e in zone f
$ondelim
$INCLUDE ess_cdeff.csv
$offdelim

TABLE cdmin(f,e) minimum charge rate of ESS e in zone f [kW]
$ondelim
$INCLUDE ess_cdmin.csv
$offdelim

TABLE cdmax(f,e) maximum charge rate of ESS e in zone f [kW]
$ondelim
TABLE fuel(h,g) fuel consumption of generator g in zone k [gal\kWh]

PARAMETER load(t) projected demand at time t /

PARAMETER insol(t) projected insolation at time t [kW\m^2] /

FREE VARIABLES
OBJ objective function
RUN total run time
BAT total battery usage
POWESS(e,f,t) power output of ESS e in zone f at time t

POSITIVE VARIABLES
POWGEN(g,h,t) power output of generator g in zone h at time t
P(t) power generated during time t
BD(t) battery discharge rate during time t
BC(t) battery charge rate during time t

BINARY VARIABLES
X(g,h,t) operate generator g in zone h at time t
Y(e,f,t) operate ESS e in zone f during time t

EQUATIONS
objfun objective function
one(t) meet demand at every time t
two(e,t) maximum battery level at every time t
three(e,t) minimum battery level at every time t
four(e,f,t) battery charge rate
five(e,f,t) battery discharge rate
six(e,t) terminal battery storage
7. minimum power in zone $k$ during time $t$
8. maximum power in zone $k$ during time $t$
9. power generated at time $t$
10. power generated at time $t$

\[
\text{OBJ} = \text{lt} \times \sum ((g,h,t), \text{fuel}(h,g) \times \text{POWGEN}(g,h,t)) + \epsilon \times \text{RUN} + \epsilon \times \text{BAT}
\]

\[
\text{one}(t) : \quad \sum ((g,h), \text{POWGEN}(g,h,t)) + \sum ((e,f), \text{POWESS}(e,f,t)) - \text{load}(t) + \text{insol}(t) \times \text{pvef}\}
\]

\[
\text{two}(e,t) : \quad \text{soc0}(e) - \text{lt} \times \sum ((f,s) \{ \text{ord}(s) \geq \text{ord}(t) \}, \text{cdeff}(f,e) \times \text{POWESS}(e,f,s)) = \text{socmin}\}
\]

\[
\text{three}(e,t) : \quad \text{soc0}(e) - \text{lt} \times \sum ((f,s) \{ \text{ord}(s) \geq \text{ord}(t) \}, \text{cdeff}(f,e) \times \text{POWESS}(e,f,s)) = \text{socmax}\}
\]

\[
\text{four}(e,f,t) : \quad \text{cdmin}(f,e) \times \text{Y}(e,f,t) = \text{POWESS}(e,f,t)
\]

\[
\text{five}(e,f,t) : \quad \text{cdmax}(f,e) \times \text{Y}(e,f,t) = \text{POWESS}(e,f,t)
\]

\[
\text{six}(e,t) : \quad \sum ((f), \text{Y}(e,f,t)) = 1
\]

\[
\text{seven}(e) : \quad \text{soc0}(e) - \text{lt} \times \sum ((f,t), \text{cdmin}(f,e) \times \text{Y}(e,f,t)) = \text{socmax}\}
\]

\[
\text{eight}(g,h,t) : \quad \text{powmin}(h,g) \times \text{X}(g,h,t) = \text{POWGEN}(g,h,t)
\]

\[
\text{nine}(g,h,t) : \quad \text{powmax}(h,g) \times \text{X}(g,h,t) = \text{POWGEN}(g,h,t)
\]

\[
\text{ten}(g,t) : \quad \sum ((h), \text{X}(g,h,t)) = 1
\]

MODEL gridmodel

SOLVE gridmodel USING MIP MINIMIZING OBJ;

display OBJ.l;
display RUN.l;
display BAT.l;

evaluate unload "results.gdx" OBJ.l P.l Y.l POWESS.l;
evaluate 'gdxxrw.exe results.gdx o=results.xls var=P.l rng=A1 var=y.l rng=A46 var=powess.l rng=A81'
B.2  60kW Curve Fitting to BCIL Data
clear;
cdf; close all
%
% for 30KW generator
%
xd_30=30*[1, 0.75, 0.5, 0.25];
data_30=[2.41, 1.76, 1.23, 0.88;
        2.47, 1.82, 1.25, 0.91;
        2.49, 1.82, 1.20, 0.90];
yd_30=xd_30./mean(data_30,1);
%
c2a=0.05;
c2b=5;
[ss_30, c0_30, c1_30, c2_30]=optimal_fit(xd_30,yd_30,c2a,c2b);
%
% for 60KW generator
%
xd_60=60*[1, 0.75, 0.5, 0.25];
data_60=[4.54 3.52 2.49 1.61;
        4.33 3.44 2.42 1.56;
        4.53 3.56 2.49 1.59];
yd_60=xd_60./mean(data_60,1);
%
c2a=0.05;
c2b=5;
[ss_60, c0_60, c1_60, c2_60]=optimal_fit(xd_60,yd_60,c2a,c2b);
%
x2=60*[5/60:0.02:7/6];
y_fit_30=c0_30+c1_30*exp(-c2_30*x2);
y_fit_60=c0_60+c1_60*exp(-c2_60*x2);
%
h1=plot(xd_30,yd_30,'rs','markersize',10,'markerfacecolor','r');
hold on
h2=plot(x2,y_fit_30,'b-','linewidth',1.5);
h3=plot(xd_60,yd_60,'gd','markersize',10,'markerfacecolor','g');
h4=plot(x2,y_fit_60,'k-','linewidth',1.5);
%
set(gca,'fontsize',16)
xlabel('X')
ylabel('Y')
legend([h1,h2,h3,h4],['30KW data','fitting curve','60KW data',
     'fitting curve','location','SE'])
%
axis([0 70 -2 15])
grid on
B.3 Optimal Fit Sub Routine
function [ss, c0, c1, c2]=optimal_fit(xd, yd, c2a, c2b)
%
% This code finds the optimal fitting of the form
% y = c0+c1*exp(-c2*x)
% to data (xd(i), yd(i), i=1, ..., N)
%
% Input:
%   xd: x coordinates of the data
%   yd: y coordinates of the data
%   [c2a, c2b]: an interval large enough such that it
%      contains the optimal value of c2
%
% Output:
%   ss: sum of squares of the difference
%   c=[c1,c2,c3]: optimal values of parameters
%
% We find the optimal fitting by minimizing ss(c2)
% c2=golden_search(@(w) Ls_linear(w,xd,yd),c2a,c2b);
% [ss, c0, c1]=Ls_linear(c2,xd,yd);
%
function [ss, c0, c1]=Ls_linear(c2,xd,yd)
%
% This code finds the optimal fit of the form
% y = c0+c1*exp(-c2*x) with c2 GIVEN
% to data (xd(i), yd(i), i=1, ..., n)
% Then it calculates the sum of squares of the difference between
% the data and the optimal fitting (as a function of c2)
%
% Input:
%   xd: x coordinates of the data
%   yd: y coordinates of the data
%
% Output:
%   ss: sum of squares of the difference (as a function of c2)
%   c0, c1: optimal values of parameters
%
u=exp(-c2*xd);
u_avg=mean(u);
u2=u-u_avg;
c1=mean(yd.*u2)/mean(u2.*u2);
c0=mean(yd)-c1*u_avg;
% dif=c0+c1*u-yd;
ss=sum(dif.^2);
%
function [z]=golden_search(f,a,b)
tol=1.0e-10;
n=0;
g=(sqrt(5)-1)/2;
r1=a+(b-a)*(1-g);
f1=f(r1);
r2=a+(b-a)*g;
f2=f(r2);

while (b-a) > tol,
  n=n+1;
  if f1 < f2,
    b=r2;
    r2=r1;
    f2=f1;
    r1=a+(b-a)*(1-g);
    f1=f(r1);
  else
    a=r1;
    r1=r2;
    f1=f2;
    r2=a+(b-a)*g;
    f2=f(r2);
  end
end
z=(a+b)/2;
%
List of References


Initial Distribution List

1. Defense Technical Information Center
   Ft. Belvoir, Virginia

2. Dudley Knox Library
   Naval Postgraduate School
   Monterey, California