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1. REPORT DATE (DD-MM-YYYY) 24-04-2015
2. REPORT TYPE MS Thesis
3. DATES COVERED (From - To)

4. TITLE AND SUBTITLE
DESIGN AND OPTIMIZATION OF UWB ANTENNA FOR AIR COUPLED GPR APPLICATIONS

5a. CONTRACT NUMBER W911NF-13-1-0301
5b. GRANT NUMBER
5c. PROGRAM ELEMENT NUMBER 611103
5d. PROJECT NUMBER
5e. TASK NUMBER
5f. WORK UNIT NUMBER

6. AUTHORS Amr Ahmed

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
University of Vermont
85 South Prospect Street, 340 Waterman Building
Burlington, VT 05405 -0160

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S) ARO
11. SPONSOR/MONITOR'S REPORT NUMBER(S) 63445-EL-RIP.1

12. DISTRIBUTION AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT
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15. SUBJECT TERMS
radar, ultrawideband, antenna, GPR, ground penetrating radar

16. SECURITY CLASSIFICATION OF:
a. REPORT UU
b. ABSTRACT UU
c. THIS PAGE UU

17. LIMITATION OF ABSTRACT

15. NUMBER OF PAGES

19a. NAME OF RESPONSIBLE PERSON
Dryver Huston
19b. TELEPHONE NUMBER 802-656-1922
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DESIGN AND OPTIMIZATION OF UWB ANTENNA FOR AIR COUPLED GPR APPLICATIONS

A Thesis Presented

by

Amr Ahmed

to

The Faculty of the Graduate College

of

The University of Vermont

In Partial Fulfillment of the Requirements
for the Degree of Master of Science
Specializing in Electrical Engineering

October, 2014
Accepted by the Faculty of the Graduate College, The University of Vermont, in partial fulfillment of the requirements for the degree of Master of Science, specializing in Electrical Engineering.

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Cynthia J. Forehand, Ph.D.

Date: June 13, 2014
ABSTRACT

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ACKNOWLEDGEMENTS

I would like to seize this opportunity to extent my deep gratitude to Dr. Tian Xia. For the past two years, not only he has been my academic advisor but also a mentor and a good friend. Thank you for all your guidance and support, without which I wouldn’t have made it that far. And please accept my apologies for the long time you spent revising my publication and for all the inconveniences I have caused you with my annoying travelling plans. However, it’s through hardships that one can develop great appreciation for a friend. It is my hope that this friendship doesn’t end here and that we will always keep in touch.

A special thanks to Dr. Dryver Huston, a special advisor and a friend, who introduced me to the world of ground penetrating radars. Thanks for offering me those “tips & tricks” that would always save me much struggle and confusion. To be honest, I had my doubts and was overwhelmed when I was first approached with this topic. It was only after that first meeting I attended with you and Dr. Xia, that I knew that I didn’t have to worry that much anymore. I knew that I have a strong support to lean on.

I would also like to extend my appreciation to Dr. Kurt Oughstun for accepting to take part in my defense committee. Thank you for the very helpful discussions through my thesis proposal. Your recommendations have shed plenty of light on the road to complete this work. I will always be grateful for your patience.

My thanks also go to Dr. Dylan Burns, my colleagues Anbu Venkatachalam and Yu Zhang for all their help in the antenna manufacturing, laboratory experiment
setup and field tests. It was an honor working with all of you and I really hope that our roads will cross again one day.

Special thanks to my parents, my sister, and my friends for providing me the support, understanding and encouragement to pursue more in life. The deepest gratitude goes to my mother for being there for me at good and bad times, and for her never ending love and support. The one, without her I would not have been able to get so far.

Finally I would like to express my ultimate gratefulness to my amazing wife, the best company in this journey, the one who kept supporting and encouraging me along the way. The one without her love and patience during the long nights and the stressful days, I wouldn’t have been here.
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CHAPTER 1: INTRODUCTION

The primary objective of this thesis is to focus on one of the essential blocks in any Ground Penetrating Radar (GPR) systems. We set our target on coming up with a systematic approach for antenna design, modeling, manufacturing, and field testing. A good base was already established earlier by my colleagues at the University of Vermont to investigate the feasibility of an Air-Coupled GPR system that can be used in bridge deck or highway subsurface inspections without disrupting traffic.

In the year 2000, Jing Qiong Hu led the way by first introducing the Good Impedance Matching (GIMA) antenna, which demonstrated low reflections at aperture and proved the theoretical and practical feasibility of using GIMA antennas in a GPR module. Later on, in 2012, Xianlei Xu et al. introduced the dual-band dual-channel air-launched UWB GPR to facilitate highway and bridge inspection. The new novel system achieves high pulse amplitude and low-level ringing that also improves the survey speed by using the 8-GSPS real-time data acquisition unit to accelerate reflection sampling. The promising results assure that this system can achieve good resolution while scanning on highway driving speeds. However, the new system revealed the need for a new antenna design that can achieve higher gain at a frequency band 500MHz to 6GHz with reflections at feed point S11 less than -10dB. And given that the system will ultimately be installed under a truck, we have to understand the effect of the vertical length of the antenna on the overall performance. These factors, in addition to design mechanical reliability for long term operations, exposed the need to develop a systematic approach to determine the critical dimensions that contributes to the antenna design and model the effect of each dimensional variation on the targeted performance.
In this chapter we will cover the required literature survey that will first introduce the GPR systems: their typical applications, what makes up a good GPR system, and the different types and trade-offs in the system design. Afterwards, we move to the antenna introduction including: the ultra-wide band antenna, why it is suitable for ground penetrating radars, and what types of UWB antennas are suitable for an Air-Coupled GPR. And finally we review the system our antenna will take part in and explore the system requirements imposed on the antenna structure.

1.1. GPR literature survey

The Ground Penetrating Radar (GPR) technology is increasingly being used in non-destructive testing and through-wall imaging. Various commercial, governmental and military applications often require the need to detect buried objects or reveal their nature. Sometimes it is also required to determine the internal structure of materials, similar to road layer thickness, bridge construction and tunnel linings. GPR’s are very useful for investigating layers’ boundaries and can easily deliver onsite results.

1.1.1. Theory of operation

A typical surface image is captured through transmitting a short impulse, less than 1nSec in width, of electromagnetic signal into the structure under test. The energy reflected from discontinuities in material, which has different electrical impedances, is collected by a receive antenna, processed by means of signal processing, and finally presented on a display unit. Figure 1 shows a block diagram of the system.
In Figure 2, the vertical depth of a single point P₁ located at the boundary of two materials having different dielectric constants can be calculated with the aid of the electromagnetic wave propagation velocity equation. Equation (1) introduces the propagation velocity formula assuming a homogeneous, isotropic material, where \( c \) is the speed of light or the propagation velocity in free space and \( \varepsilon_{r1} \) is relative permittivity of the first material.

\[
v = \frac{c}{\sqrt{\varepsilon_{r1}}}
\]  

(1)

The depth to the target point can then be derived from equation (2), where \( d \) is the vertical depth of the point P₁, \( v \) is the propagation velocity inside the first dielectric material, \( \theta_2 \) is the refraction angle that can be easily calculated using Snell’s law given the incident wave angle \( \theta_1 \), and \( \nabla t \) is the time difference between receiving the first reflected pulse Rx₁ and the second reflected pulse Rx₂. Figure 2 explains the typical behavior of a transmitted impulse Tx given that \( \varepsilon_0 < \varepsilon_{r1} < \varepsilon_{r2} \).

\[
d = v \frac{\nabla t}{2} \cos(\theta_2) = \frac{c}{2 \sqrt{\varepsilon_{r1}}} \frac{\nabla t}{2} \cos(\theta_2)
\]  

(2)
The thickness of the second layer buried within the first material can be easily calculated if the depth of point $P_2$ is found. A cross-sectional image of the material can be constructed, point by point, given that the approximate dielectric constants of the different layers and their refractive indices are known.

Even if we don’t know the nature of the buried layer, we can still acquire some valuable information from its reflected signal. The reflection coefficient $\Gamma_{12}$ is the measure of how much energy is reflected from the second material back to the first material. It can be calculated using the characteristic impedances $Z_{o1}$ and $Z_{o2}$.

\[
\Gamma_{12} = \frac{Z_{o1} - Z_{o2}}{Z_{o1} + Z_{o2}} \quad (3)
\]
The characteristic impedance $Z_o$ is defined as the electric field over the magnetic field inside the material, which can be calculated using equation (4):

$$Z_o = \frac{E}{H} = \sqrt{-\frac{j\omega \mu}{\sigma - j\omega \varepsilon}} \quad (4)$$

Where:
- $\omega$ is the angular frequency
- $\mu$ is the material magnetic permeability
- $\varepsilon$ is the material electrical permittivity
- $\sigma$ is the material electrical conductivity

If the material is dielectric, $\sigma$ tends to be very small in value and the characteristic impedance can then be approximated to $Z_o = \sqrt{\mu/\varepsilon}$

In conclusion, firing a short impulse of electromagnetic wave towards a surface not only can detect the depths and thicknesses of its buried layers, but also can partially reveal their material characteristics.

1.1.2. GPR types and applications

Scanning the surface of the structure under test with constant velocity in linear path would develop a cross section image along the path scanned. Repeating the previous steps along parallel paths we can theoretically develop a 3-D image of the structure. One of the early successful attempts to apply this technique was developed in the UK, by EVANS (1963), to measure the polar ice thickness; probing depths of several kilometers in the Antarctic. A more recent and relative to our objective is the
system developed by Warhus (1993). He developed a GPR imaging system, using a slot line bowtie antenna, to detect the locations for a complex structure of reinforcement steel bars (REBARS) in a grid form buried in a concrete slab.

According to the diagram presented in Figure 1, and based on the antenna proximity to the surface under test, ground penetrating radars can be one of two categories; Ground-Coupled or Air-Coupled GPR. The antennas of the Ground-Coupled GPR are in direct contact, or within very close proximity, to the detection surface, which results in smaller energy losses and higher detection sensitivity. However, the scanning speed has to be low in order not to expose the antennas to possible damages if the surface in contact has a rough profile. On the other hand, Air-Coupled GPR’s do not need to be in contact with the surface under test. Therefore, they are useful in rapidly surveying long distances in relatively short time. For example, GPR’s have been flown at heights of 400m in Synthetic Aperture Radar (SAR) mode and have discovered several buried metallic mines. They have also been used from earth-orbiting satellites to image geological features buried beneath the Sahara Desert.

In general, the UWB GPR is an impulse radar system in which the receiver collects the reflected electromagnetic impulses for subsurface feature identification. According to D.J. Daniels (1996), the essential requirements that Ground Penetrating Radars should meet are:

- “Efficient coupling of the transmitted electromagnetic energy into the ground”.
- “Adequate penetration of the energy through the ground, having regard to target depth.”
- “Reception from buried objects or other dielectric discontinuities of a sufficiently large reflected signal for detection at or above the ground surface.”
- “An adequate bandwidth in the detected signal, having regard to the desired resolution and noise levels.”
- “An adequate signal-to-clutter ratio.”

These factors have an impact on the energy transmitted and the attenuation the signal is suffering from while traveling in in the media under test and consequently the received signal. The quality and quantity of the received signal is actually the key indication of a good GPR system.

1.2. Introduction to UWB

An antenna in general can be described as a transducer that guides electromagnetic signal trapped in a transmission line to radiate its electromagnetic energy in air or free space. Antennas may also be considered as impedance transformers, that provides gradual coupling between input or feed line impedance to aperture or free space impedance.

1.2.1. Brief history

The fact is there is no difference between narrow band and wide band frequency when it comes to the basic functionality of an antenna. However, narrow band antennas are only tuned to achieve the impedance coupling over a small band of frequency. In his work, “A BRIEF HISTORY OF UWB ANTENNAS”, Schantz (2004) pointed out that, ironically, Oliver Lodge (1898) has mentioned future Ultra Wide Band antenna structures as promising designs in the same patent which
introduced the concept of “syntony,” or the idea that transmitter and a receiver should be tuned to the same frequency to maximize the received signal. It wasn’t until 1939, when Carter introduced his conical monopole as an UWB antenna in his research for a wider bandwidths to satisfy the demand on video signals increase. Many advances occurred afterwards, but despite the excellent performance demonstrated, other consideration began to come to attention. As broadband receivers came into mass production, the demand on inexpensive and easy to manufacture designs increased. The triangular “bow-tie” antenna originally proposed by Lodge was then re-examined by Brown and Woodward in 1952 to demonstrate these benefits.

1.2.2. Types and applications

The frequency band of the transmitted signal is the determining factor for the GPR type of application. Higher frequencies are often needed where better resolution is required. Nevertheless, lower frequencies are preferred for deeper penetration in soil due to the increasing wave attenuation with frequency increase. Ultra Wide Band (UWB) GPR systems introduce the benefits of both high and low frequencies.

In general, several types of antennas are commonly used for UWB GPR systems, such as; resistively loaded dipole, bow-tie antenna, spiral antenna and the most widely used TEM horn antennas. The resistively loaded dipole is simple, easy-to-design and linearly polarized antenna structure. However, it mostly has a major disadvantage of poor gain. This makes other planar antennas, like Bow-tie and spiral structures, a more attractive choice.

Bow-tie and spiral antennas are mainly used in impulse GPR applications because of their non-dispersive characteristics and their relatively high gain.
However, both suffer from significant impedance mismatch at feed point when used for Air Coupled GPR applications. This also adds the ringing effect distorting the time domain waveform. Resistive loading is usually used to overcome this drawback but this technique always leads to deteriorating gain performance over the frequency wide band (Lestari et al., 2004). Figure 3 and Figure 4 show the typical structures of a bow-tie and a spiral antenna respectively.

![Figure 3: An example of Bow-Tie antenna structure](image)

1.2.3. Horan antennas

TEM Horn antenna, on the other hand, has a clear advantage over planar antennas. A typical TEM Horn antenna structure is shown in Figure 5. Its narrow beam width naturally guarantees a higher directivity gain over a wider frequency range. The main design challenge is to achieve a good impedance match in order to
minimize feed point and aperture reflections. The work in Turk et al. (2013) is an example attempt to modify the TEM horn antenna to achieve a better impedance matching. The TEM horn-fed ridged horn antenna proposed, demonstrates a high gain up to 10dB over a wide frequency band, 400MHz to 16GHz, with input reflections down to -10dB. However, simulation results show that this design suffers from relatively high reflections over frequencies less than 3GHz and greater than 7GHz.

Figure 5: An example of a Horn antenna structure

1.3. GPR system description and antenna requirements

We have been developing high speed impulse ground penetrating radar (Xianlei Xu et al., 2012) for highway pavement and bridge deck inspection. This GPR system is designed to achieve high spatial resolution, i.e. 1 cm and accurate inspection results while scanning at regular highway speed, i.e. 60 mph. In order to accomplish high detection resolution while reducing power radiation, a dual-band dual-channel UWB GPR is developed. Figure 6 presents the GPR system diagram. The transmitter consists of an UWB pulse generator fed by a tunable 1 kHz to 10 MHz oscillating signal from a programmable oscillator. The oscillating signal controls the UWB pulse repetition frequency (PRF). The UWB pulse generator provides 600ps-wide monocycle pulses to both base band and 3-4.5GHz modulated band. In the base-band operation, the monocycle pulse is directly amplified and transmitted while at the high
frequency band the monocycle pulse is fed to a mixer for frequency modulation. The selection of the operating band is remote-controlled using single-pole/dual-throw RF switches. The receiver, on the other hand, consists of the receiving antenna flowed by low noise amplifier (LNA). The amplified signal is either directly fed to the analog-to-digital (ADC), or frequency down converted through a mixer, depending on the operation band. The digital data is then stored in a memory block for further processing. To improve radar survey accuracy, two operating channels are implemented for parallel inspections. The FPGA coordinates the radar operating band selection, channel switching, sampling data collection, and storage. As an important functional unit, the antenna plays a significant role in determining GPR performance. UWB GPR antenna design is very challenging because it requires exceptional impedance matching across the whole ultra-wide bandwidth.

Figure 6: Block diagram of the high-speed UWB dual-band GPR system (Xianlei Xu et al., 2012)
CHAPTER 2: ANTENNA STRUCTURE

2.1. Antenna sections

Impedance matching is one of the most critical parts in UWB antenna design. Unfortunately, it is difficult to achieve an exact match along the whole wide band. Basically, there are two main points needing intensive considerations, one is the feeding port of the transmitter, and the other is the interface between antenna aperture and air. For the air coupled GPR system, the loading impedance of the aperture is equal to the characteristic impedance of air which can be calculated using equation (4). However, given that the air electric conductivity $\sigma$ is very small in value and both $\mu_r$ and $\varepsilon_r$ can be approximated to 1, the characteristic impedance can then be approximated to:

$$Z_o = \sqrt{\mu/\varepsilon} = \sqrt{\mu_0/\varepsilon_0} \approx 377 \, \Omega$$

On the other hand, the impedance of the feeding coaxial cable is 50 Ohms. A good antenna design should resolve this mismatch by gradually converting the impedance to minimize reflections and therefore minimize the Voltage Standing Wave Ratio (VSWR). In return this leads to maximizing radiation efficiency. Figure 7 and Figure 8 show that our new design consists basically of three sections; feed line, Vivaldi shaped section and rounded shaped aperture. Essentially our proposed design combines the benefits of both Vivaldi and horn antennas.
2.2. Analytical model

Both feed line and the linear Vivaldi section can be modeled as combination of N parallel plate transmission line segments. Each segment consists of two metal plates, with varying width and space, separated by a dielectric.
Figure 9: Feed line as parallel-plate transmission line

Starting with the feed line, the relationship between input and output impedances is controlled by equation (5).

$$Z_{in} = Z_0 \frac{Z_{out} + jZ_0 \tan(\beta l)}{Z_0 + jZ_{out} \tan(\beta l)}$$

(5)

Where:

- $Z_{in}$: input impedance of the parallel plate wave guide
- $Z_{out}$: output impedances of the parallel plate wave guide
- $l$: The length of the feed line.
- $Z_0$: The characteristic impedance
- $\beta$: The wave number of the parallel plate

The characteristic impedance $Z_0$ and the wave number $\beta$ of the transmission line are defined by equations (6) and (7) where $d$ is the dielectric height that separates the two parallel plates and $w$ is the width of the feed line.

$$Z_0 = \frac{d}{w \sqrt{\varepsilon}}$$

(6)

$$\beta = \omega \sqrt{\mu \varepsilon}$$

(7)
In order to get a better understanding of the parallel plate structure, the MATLAB script, presented in APPENDIX section, is developed to get the 3D plot of the variations of input impedance versus the change in both frequency and the length of our feed point. Given an output impedance of 50 ohms and a dielectric of air, Figure 10 and Figure 11 show the real and imaginary parts of the input impedance respectively.

![Figure 10: Real part of input impedance versus length and frequency](image1)

![Figure 11: Imaginary part of input impedance versus length and frequency](image2)
Tabulated results are shown in Table 1, where the input impedance is reported for each length value along its corresponding frequency points. The analysis shows that when the feed line has dimensional size of \( L_0 = 6 \text{mm}, \) and \( d_0 = 3 \text{mm} \) and \( W_0 = 12 \text{mm} \), the feed line demonstrates minimal impedance variation across the wide frequency band.

### Table 1 - Input impedance for different length values over frequency band 0.5GHz to 6.0GHz

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<thead>
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<th>( f ) (GHz)</th>
<th>( L_0 = 6 \text{mm} )</th>
<th>( L_0 = 9 \text{mm} )</th>
<th>( L_0 = 12 \text{mm} )</th>
<th>( L_0 = 18 \text{mm} )</th>
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<td>50.32 + 6.380i</td>
<td>50.57 + 8.510i</td>
<td>51.30 + 12.78i</td>
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<td>51.30 + 12.78i</td>
<td>52.33 + 17.06i</td>
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<tr>
<td>3.0</td>
<td>55.40 + 25.66i</td>
<td>63.01 + 38.58i</td>
<td>75.40 + 50.90i</td>
<td>121.48 + 63.23i</td>
</tr>
<tr>
<td>3.5</td>
<td>57.50 + 29.98i</td>
<td>68.50 + 44.89i</td>
<td>87.23 + 57.94i</td>
<td>153.4 + 49.82i</td>
</tr>
<tr>
<td>4.0</td>
<td>60.02 + 34.29i</td>
<td>75.40 + 50.90i</td>
<td>102.55 + 62.72i</td>
<td>175.7 + 49.82i</td>
</tr>
<tr>
<td>4.5</td>
<td>63.01 + 38.58i</td>
<td>83.96 + 56.34i</td>
<td>121.48 + 63.23i</td>
<td>170.46 - 28.93i</td>
</tr>
<tr>
<td>5.0</td>
<td>66.53 + 42.81i</td>
<td>94.43 + 60.72i</td>
<td>142.84 + 56.65i</td>
<td>142.46 - 56.84i</td>
</tr>
<tr>
<td>5.5</td>
<td>70.63 + 46.94i</td>
<td>106.96 + 63.35i</td>
<td>162.97 + 40.38i</td>
<td>111.26 - 63.66i</td>
</tr>
<tr>
<td>6.0</td>
<td>75.40 + 50.90i</td>
<td>121.48 + 63.23i</td>
<td>175.7 + 46.5i</td>
<td>87.00 - 57.83i</td>
</tr>
</tbody>
</table>

The other perspective that should be of interest is the propagation modes that travel through the parallel plate structure. Given that the desired antenna will operate at a wide range of frequencies, it is important to understand the modes traveling through the antenna at various frequencies. The TEM propagation mode is the dominant as it has a cutoff frequency of zero. However, TE and TM modes also exist but with higher cutoff frequencies. Equation (8) shows that for each propagation mode \( n \), the cutoff frequency \( f_{cn} \) is inversely proportional to the dielectric thickness \( d \), the square root of magnetic permeability \( \mu \) and the square root of the electrical permittivity \( \varepsilon \).
\[ f_{cn} = \frac{n}{2d} \left( \frac{1}{\sqrt{\mu \varepsilon}} \right) \quad \forall \ n = 0, 1, 2, \ldots \quad (8) \]

At the feed point, \( d \) is at its minimum value, and equals to \( d0 \) where \( d0 = 3 \)mm. Equation (8) can directly be used to calculate \( f_{cn} \). However, the local dielectric thickness \( d_A \) at the aperture, the end of the Vivaldi section, is calculated in equation (9) where \( V_{length} \) the length of the Vivaldi section assumed to be 180mm and \( D_{angle} \) is set to 5.5°.

\[ d_A = d_0 + V_{length} \times \sin(D_{angle}) \quad (9) \]

By substituting in equation (8), the cutoff frequency at aperture \( f_{cnA} \) is calculated by equation (10).

\[ f_{cnA} = \frac{n}{2 \left( d_0 + V_{length} \times \sin(D_{angle}) \right)} \left( \frac{1}{\sqrt{\mu \varepsilon}} \right) \quad (10) \]

Table 2 shows the calculated values of each cutoff frequency at both ends of the antenna.

<table>
<thead>
<tr>
<th>Fc</th>
<th>Feed</th>
<th>aperture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fc1</td>
<td>49.97 GHz</td>
<td>3.99 GHz</td>
</tr>
<tr>
<td>Fc2</td>
<td>99.93 GHz</td>
<td>7.97 GHz</td>
</tr>
<tr>
<td>Fc3</td>
<td>149.90 GHz</td>
<td>11.96 GHz</td>
</tr>
<tr>
<td>Fc4</td>
<td>199.86 GHz</td>
<td>15.93 GHz</td>
</tr>
<tr>
<td>Fc5</td>
<td>249.83 GHz</td>
<td>19.92 GHz</td>
</tr>
</tbody>
</table>
For the Vivaldi section, it can be represented by staircase model shown in Figure 13, which consists of N segment. Each segment can be assumed homogenous when the segment length is small in comparison with signal wavelength.

Similarly, the input impedance $Z_{in}$ of each stage can then be calculated using equation (11), where $\beta_i$, $Z_{0i}$ and $l_i$ are the wave number, the characteristic impedance and the length of the $i$'th segment respectively. $Z_{in+1}$ is the input impedance of the $i+1$ stage that loads the $i$'th segment. The point where $i = 0$, represent the touching point of the feed line and Vivaldi sections. Where $i = N$ represent the other end of the Vivaldi section where it touches the arc section.
\[ Z_{m_i} = \begin{cases} \frac{Z_{0i}Z_{i+1} + jZ_{0i}\tan(\beta_i l_i)}{Z_{0i} + jZ_{i+1}\tan(\beta_i l_i)} & \forall \ i = 0, \ldots, (N - 1) \\ \\
\frac{Z_0 Z_L + jZ_0 \tan(\beta_i l_i)}{Z_0 + jZ_L \tan(\beta_i l_i)} & \forall \ i = N \end{cases} \] (11)

\[ S_{11} = \frac{Z_L - Z_0}{Z_L + Z_0} \] (12)

An analytical model developed in MATLAB is created based on Equations (11) and (12) to find suitable values of design angels; D_angle and W_angle shown in Figure 7 and Figure 8 respectively. The code is available in the APPENDIX section. In the model, the loading impedance of the last stage $Z_L$ is equal to free space characteristic impedance (377 Ohms). The $N^{th}$ segment of the Vivaldi section is chosen to interface directly with air, dropping out the arc section for simplicity. Equation (12) is then used to plot S11 behavior at the feed point.

$D\_angle$ is swept from 2.75 to 5.5° with a step of 0.25° and $W\_angle$ is swept from 13° to 20° with a step of 0.5°. Figure 14 shows the S11 results at the start and end points of each sweep range for simplicity. It shows that when $D\_angle$ at its largest value 5.5° and $W\_angle$ at its smallest value 13°, promising S11 results across the wide frequency range is achieved.
Figure 14: S11 results for a preliminary MATLAB model of feed point and Vivaldi sections
CHAPTER 3: FIELD SOLVER SIMULATION

3.1. Model construction

Starting by building the structure in SolidWorks, the basic design parameters are initially plugged to create the mechanical model. This helped later in manufacturing this antenna structure to perform lab measurements. The second step is to import the model into HFSS for design parameter optimization. Feed port is added and the simulation setup is prepared to obtain the far field simulation results. HFSS simulation results guides us through tuning up different parts of the antenna to obtain the optimum performance of this structure.

Figure 15 illustrates that the exact model of the coaxial signal probe with 0.5 inch flange is first imported in SolidWorks to remove the outer threading and to shorten the probe length. The threading removal helps later when the model is imported in HFSS for simulation. The thread is a complex feature that requires more convergence timing in electromagnetic field simulation.

![Coaxial feed modeling](image)

Figure 15: Coaxial feed modeling

After importing the antenna structure in HFSS, we place a vacuum box around. The walls of the box are selected as radiation boundaries for intercepting the far field radiation. The size of the box has to be greater than the longest feature in the antenna.
structure, in our case it’s the antenna total length of 300mm. This is to satisfy the far field radiation conditions.

![Figure 16: HFSS 3D antenna model](image)

The excitation is then assigned to face of the dielectric material of the coaxial probe. The input impedance is set to 500mm. This matches the standard connector impedance that connects our antenna to the impulse generator when building up the GPR system.

![Figure 17: Assigning excitations in HFSS](image)

### 3.2. Parameters optimizations

One of the critical parameters of this antenna design is the length of the Vivaldi section. Parametric simulations are performed to evaluate the effects on Vivaldi section length on S11 across the wide band frequency. To perform a
frequency sweep in HFSS, we need to “Add Solution Setup” from “Analysis Setup” menu and then we select “Add Frequency sweep”. Figure 18 shows the sweep setup of a frequency range of 100MHz to 6GHz with a linear step 1MHz.

![Figure 18: Frequency Sweep Setup in HFSS](image)

S11 results can be displayed through creating a report using “Create Model Solution Data Report” from “Results” menu and then choosing “Rectangular Plot”. The dialog in Figure 19 is used to plot S11 in dB units over the frequency sweep that is called “Sweep1”.

![Figure 19: Create a rectangular plot for S11 parameters in HFSS](image)
Figure 20 illustrates the simulation results when the length is linearly changed starting from 90mm to 180mm with a step size of 30mm. S11 is plotted for each length value correspondingly. The results show that 180mm length results in steady S11 curve below -10dB across the whole frequency band.

The shape and length of arc section also stands as a critical design variable. Given that the lowest frequency is 600MHz, the full wave length is 500mm. The total length of the antenna is chosen to be about half the wave length. Hence, the arc length and the length of the Vivaldi section, 180mm, should add up to 250mm to match the half-wavelength of the lowest end frequency of the frequency band. Simulation result confirms that the initial arc length value of 70mm is proven to be optimum.

Figure 20: S11 vs. Frequency for different antenna lengths
Enhanced performance is achieved by rounding the edges at the feed point. Figure 21a and Figure 21b show the traditional sharp edges versus the edge rounding at feed point respectively. Results in Figure 22 demonstrates that the structure in Figure 21b has lower S11 results, dashed line, over the solid line that represents simulation results of the traditional sharp cornered design.

Figure 21: Antenna design, feed line tune-ups

Figure 22: S11 simulation results for antenna structures in Figure 21a (solid line) vs. Figure 21b (dashed line)
Rounding the flare edges of the antenna also demonstrates an enhancement to the final S11 results. Figure 25 shows that rounding the sharp corners, as in Figure 24, has lower S11 results (dashed line) versus the sharp edges, as in Figure 23, over the low frequency range 600MHz to 1GHz. Although the upper end, 5GHz to 6GHz, has higher S11 results, it is still well below -10dB which is acceptable. In conclusion the structure in Figure 24 achieved better performance.

Figure 23: Antenna design tune-ups. Feed line modifications

Figure 24: S11 simulation results for antenna structure in Figure 23a
3.3. Simulation results

Figure 25 shows the radiation pattern for the final antenna design at the midrange 2.8GHz frequency and $\phi = 0$ degrees. Figure 12b shows the 3D polar plot at the same frequency with a high peak gain of 10.7dB and a radiation efficiency reaching up to 98%.

Figure 26: Radiation pattern in the x-y plan (2.8 GHz)
Figure 27: 3D polar plot (2.8 GHz)
CHAPTER 4: EXPERIMENTAL RESULTS

4.1. Prototype manufacturing

After updating the SolidWorks mechanical model with the optimum design parameters verified by HFSS, a first prototype is then manufactured. Figure 28 shows the exact dimensions of one of the antenna flattened copper plates. Copper sheet with 0.5mm thickness is cut to the pattern below. The two plates are then bended at the end of the Vivaldi section to form an arc with a 120mm length as a part of a complete circle with 45mm radius. Rounding the edges of both the feed line and aperture was also performed based of the SolidWorks mode. The coaxial connector is welded between the two plates, maintaining the 3mm gap in the antenna design and providing some robustness at the feed line.

![Figure 28: Antenna’s right copper plate](image)

To enhance the robustness of the structure, the antenna is surrounded by a material with dielectric constant equals to approximately 1, in order not to affect
performance. Figure 29 shows the final shape of the prototype manufactured in UVM laboratory. The structure is then placed in a regular cardboard box as shown in Figure 31 for field measurements.

![First prototype antenna](image)

**Figure 29: First prototype antenna**

### 4.2. Lab results

S11 measurement is then performed using HP 8753D network analyzer. The magnitude of the reflection coefficient is measured over a frequency span from 100MHz to 6GHz with a 20MHz step. Figure 30 shows the plot is perfectly matching the simulation results swept over the same frequency range with a 1MHz step.

The test setup in Figure 31 uses a pair of transmitting and receiving antennas to measure the time domain response. The transmitted signal is a Gaussian monocycle pulse; that has pulse width of approximately 600ps and amplitude of approximately 1.8V. Our second prototype, the receiver, is placed exactly 1 meter away from the transmitter and connected to the spectrum analyzer to capture the waveform of the signal received.
Figure 30: S11 measurements vs. simulation results of the prototype antenna

Figure 31: Time domain waveform test setup
The same experiment is repeated using a pair of A.H. Systems’ double ridge guide horn antennas model SAS-571 for performance comparison. Figure 31 shows that using our antennas, the received pulse features smaller ringing effect than using the double ridge guide horn whose pulse signal received is shown in Figure 30. Many factors can contribute to this better performance; including linear phase and steady high gain across the wide frequency band. However, we believe that the major reason is the good matching achieved at lower frequencies, less than 1 GHz.

Figure 32: Transmitted impulse
Figure 33: Received signal from A.H. Systems’ Double Ridge Guide Horn Antennas (model SAS-571)

Figure 34: Received signal from our antenna
4.3. Field results

In order to measure the far field radiation pattern, the test setup in Figure 35 uses an isotropic antenna as a receiver placed 1m apart from our transmitter. The transmitting antenna is placed over a rotating table that allows us to measure radiating power over E-plan angles from 0° to 180° with a step of 10°. The open area test site is equipped with buried grounded mesh that acts as a perfect ground plane. However, for more accurate readings, the presented results reflect the same experiment conducted in an electromagnetic compatibility (EMC) lab. The laboratory is a concealed chamber that has electrostatic discharge, magnetic and radiation immunity capabilities. The input signals are fed using standard 50 Ohms cables through chamber outlets.

For a single frequency, the reading of the received RF voltage $V_{RF}$ is captured in (Volts) and converted to (dB V) using equation (7).

$$V_{RF} \text{ (dB V)} = 20 \log_{10} [V_{RF} \text{ (V)}] \quad (7)$$

The losses due to cables and connectors are determined by connecting the cable under test between the signal generator and the spectrum analyzer at the frequency of interest, and examine how much attenuation the signal suffered.
On the other hand, the antenna calibration process starts with replacing our antenna with A.H. Systems’ (SAS-571) Double Ridge Guide Horn Antenna and measure the power received at the same frequency points. The readings are then compared to the tabulated expected readings documented in the SAS-571 datasheet. The difference is then subtracted from our antenna measurements to acquire the calibrated radiation pattern. Figure 36 and Figure 37 show that our design has demonstrated a very good match relative to simulation results over the frequencies of 1.9 GHz and 2.8 GHz. The solid line on both figures represents the simulated radiation pattern over E-plan angles from 0° to 360° with a step of 1°.
Figure 36: Measured (dashed) versus simulation (solid) radiation pattern at 1.9GHz

Figure 37: Measured (dashed) versus simulation (solid) radiation pattern at 2.8GHz
CHAPTER 5: CONCLUSION

5.1. Goals achieved

The design procedure from initial concept to manufacturing the first prototype has been demonstrated. Both targets of compact design dimensions and wideband operation have been achieved. Throughout the design process we have identified the critical physical features to tune in order to achieve the optimum results. The antenna characteristics have been simulated and measured. Its good performance has been verified in several test setups. Our antenna is suitable for the detection of subsurface structure with good resolution.

During the course of this thesis, a design methodology has been developed to test any horn antenna structure by first changing the contributing dimensions in the analytical MATLAB model to get a proof of concept if these dimensions will have enough potential to proceed in the next design step. The second step is to match the SolidWorks model to the desired dimensions and finally import the structure in HFSS for field simulation. Design iterations will guarantee the design readiness for production.

5.2. Future work

The modeling technique used, opens the door to many possibilities. The first and foremost near objective is to integrate the antenna developed with the GPR system for production. All the field testing is already performed using the blocks from the system proposed, namely the UWB impulse generator, the LNA and the mixer for high frequency band operation. However, good packaging and handling the system reliability is still requires a follow up.
This thesis provides a bundle of Swiss-knife set of software tools that can be helpful to perform task specific operation. Many enhancements can be added to each software module for shorter simulation time, better accuracy or a more user-friendly interface. The set of MATLAB scripts included represents preliminary analytical models that can be enhanced to improve result accuracy. More models can also be added to this set to include 2D antennas in addition to the current only supported Horn antennas. A graphical user interface can also be developed to point out the different input features to the model, so it becomes more user-friendly.

One of the most challenges in this work is the communication link between SolidWorks and HFSS. The increase usage of 3D printers and the powerful capabilities of SolidWorks as solid-modeling CAD software, demands an easy-to-use link to powerful software in electromagnetic simulation similar to HFSS. A program that runs both tools in the background and provide the result in GUI format is definitely a nice to have.
REFERENCES


APPENDIX

The following MATLAB module is used to calculate input impedance for a parallel-plate transmission line with a length \( l \) and output impedance of 50 Ohms.

```matlab
% Start a clean session
clear;
close all;
clc;

zout = 50; % output impedance
d = 0.003; % fixed dielectric height
w = 0.012; % fixed dielectric width
meu = 4*pi*10^-7; % meu0 of free space
eps = 8.854187817 * 10^-12; % eps0 of free space
z0 = (d/w) * sqrt(meu/eps); % characteristic impedance of free space

% Loop on frequency range starting at 1MHz and ends at 6GHz,
% with a step of 10MHz
j=0;
for f = 1*10^6 : 10^7 : 6*10^9
    j=j+1;
    freq(j) = f/10^6;
k = 0;
    % Loop on transmission line length, starting from 6mm to 18mm,
```
% with a step of 1mm

for l = 0.006 : 0.001 : 0.018

    k=k+1;

    beta = 2*pi*f*sqrt(meu*eps);       % Calculate beta

    % Calculate input impedance
    Zin = z0 *((zout+1i*z0*tan(beta*l)) / ((z0+1i*zout*tan(beta*l))));

    length(k) = l*10^3;

    RZin(j,k) = real(Zin);

    IZin(j,k) = imag(Zin);

end
end

% Plot real part of input impedance
meshc (length,freq,RZin);
xlabel ('Length (mm)');
ylabel ('Frequency (MHz)');
zlabel ('Real Impedence (Ohms)');

% Open a new figure
figure;

% Plot imaginary part of input impedance
meshc (length,freq,IZin);
xlabel ('Length (mm)');

ylabel ('Frequency (MHz)');

zlabel ('Imaginary Impedence (Ohms)');
The following is the MATLAB code is used to calculate the final input impedance when combining the feed line with the Vivaldi section designed.

```matlab
% Start a clean session
clear;
close all;
clc;

d0 = 0.003;       % feedline dielectric hight
w0 = 0.012;       % feedline dielectric width
meu = 4*pi*10^(-7);   % feedline dielectric meu = meu0
eps = 8.854187817*10^(-12); % feedline dielectric eps = eps0
c0 = 3*10^8;       % speed of light in free space
Z0_air = sqrt(meu/eps);   % ~ 377 Ohms

D_angle = 5.52*pi/180;   % Vivaldi section D_angle
W_angle = 12.95*pi/180;  % Vivaldi section W_angle

AnLength = 0.180;       % Assuming a lenght of 180mm for Vivaldi section
j = 0;

% Loop on frequency range starting at 1MHz and ends at 6GHz,
% with a step of 10MHz
for f = 1*10^7 : 1*10^7 : 6*10^9
    lamda = c0/f;  % Calculate lamda
```
\( l = \text{lamda}/1000; \)

\( \beta = 2\pi f \sqrt{\mu e \varepsilon}; \quad \% \text{Calculate beta} \)

\% Construct a vector with antenna Vivaldi section length
\( V\_\text{length} = \text{AnLength}:\text{-}1:0; \)

\% Construct a vector with antenna Vivaldi section local dielectric thickness
\( V\_d = d0 + V\_\text{length}. \times \sin(D\_\text{angle}). \times 2; \)

\% Construct a vector with antenna Vivaldi section local dielectric width
\( V\_w = w0 + V\_\text{length}. \times \sin(W\_\text{angle}). \times 2; \)

\( z\text{out} = Z0\_\text{air}; \quad \% \text{Initialize loading impedance with air intrinsic impedance} \)

\% Loop over each stage of the staircase model
\( \text{for } k = 1 : \text{length}(V\_\text{length}) \)

\( \text{if } (k==\text{length}(V\_\text{length})) \)

\( \% \text{If it’s the last stage towards the feed line:} \)
\( \quad d = (d0+V\_d(k))/2; \quad \% \text{get the average of the d value} \)
\( \quad w = (w0+V\_w(k))/2; \quad \% \text{get the average of the w value} \)
\( \quad l = V\_\text{length}(k); \)

\( \text{else} \)

\( \% \text{If it’s any other stage starting from the aperture towards the feed line:} \)
\( \quad d = (V\_d(k+1)+V\_d(k))/2; \quad \% \text{get the average of the d value} \)
\[ w = (V_w(k+1)+V_w(k))/2; \quad \% \text{get the average of the w value} \]

end

\% Calculate intrinsic impedance for each stage
\[ z_0 = (d/w) \times \sqrt{\frac{\mu_e}{\epsilon_s}}; \]

\% Calculate input impedance for each stage
\[ Z_{in}(k) = z_0 \times \frac{(z_{out} + 1i \times z_0 \times \tan(\beta \times l))}{(z_0 + 1i \times z_{out} \times \tan(\beta \times l))}; \]

\% Assign current input impedance as the output impedance for the next iteration
\[ z_{out} = Z_{in}(k); \]

end

\% This is the input impedance of the last stage in Vivaldi section
\[ Z_{in\_vivaldi} = z_{out}; \]

\% Local parameters for the feed line
\[ \text{feed}_d = 0.003; \]
\[ \text{feed}_w = 0.012; \]
\[ \text{feed}_l = 0.012; \]
\[ \text{feed}_z0 = (\text{feed}_d/\text{feed}_w) \times \sqrt{\frac{\mu_e}{\epsilon_s}}; \]

\% Calculate input impedance for the feed line
\[ Z_{in} = \text{feed}_z0 \times \frac{(Z_{in\_vivaldi} + 1i \times \text{feed}_z0 \times \tan(\beta \times \text{feed}_l))}{(\text{feed}_z0 + 1i \times Z_{in\_vivaldi} \times \tan(\beta \times \text{feed}_l))}; \]
j = j + 1;

freq(j) = f;

% Final input impedance for this frequency node
ABS_Zin(j) = abs(Zin);
ANG_Zin(j) = angle(Zin)*180/pi;

% Final S11 value for this frequency node
S11(j) = 20*log10(abs((Zin-50)/(Zin+50)));

end

% plot input impedance in absolute and angle
[AX, H1, H2] = plotyy(freq,ABS_Zin,freq,ANG_Zin);
grid on;

% set axes limits and label
axes (AX(1))
ylim([-450 550])
ylabel ('Zin Absolute');

axes (AX(2))
ylim([-90 90])
ylabel ('Zin Angle');
xlabel ('Frequency');

title(['Vivaldi length is ', num2str(AnLength*1000), ' mm']);

% Open a new Figure
figure;

% Plot S11 results
plot (freq,S11,'r');
ylabel ('S11 (dB)');
xlabel ('Frequency (Hz)');
grid on;
The following MATLAB code is used to calculate the cutoff frequency for propagation modes in a parallel-plate transmission line.

% Start a clean session
clear; close all; clc;

d0 = 0.003; % fixed dielectric height
w = 0.012; % fixed dielectric width
meu = 4*pi*10^-7; % meu0 of free space
eps = 8.854187817 * 10^-12; % eps0 of free space

V_length = 0.180; % Assuming a length of 180mm for Vivaldi section
D_angle_variable = 5.52; % Vivaldi section D_angle
D_angle = D_angle_variable*pi/180; % D_angle in radians

% aprature dielectric thickness
d1 = d0 + V_length*sin(D_angle)*2;

% loop over propagation modes "n" from 1 to 5
for n = 1 : 5

% cutoff frequency at feed line
Fon_0(n) = n/(2*d0*sqrt(meu*eps)*10^9);

% cutoff frequency at aperture

\[ \text{Fon}_1(n) = \frac{n}{(2d1 \sqrt{\mu \varepsilon}) \times 10^9}; \]

end

% round values to the second decimal point

final1 = (round(Fon_0*100)/100)

final2 = (round(Fon_1*100)/100)