M_{RG}: A MAGNITUDE SCALE FOR 1 S RAYLEIGH WAVES AT LOCAL DISTANCES WITH FOCUS ON YIELD ESTIMATION

Jessie L. Bonner and David R. Russell

Weston Geophysical Corporation
181 Bedford Street, Suite 1
Lexington, MA 02420

23 August 2016

Final Report

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION IS UNLIMITED.
Using Government drawings, specifications, or other data included in this document for any purpose other than Government procurement does not in any way obligate the U.S. Government. The fact that the Government formulated or supplied the drawings, specifications, or other data does not license the holder or any other person or corporation; or convey any rights or permission to manufacture, use, or sell any patented invention that may relate to them.

This report was cleared for public release by the PRS OPSEC Office and is available to the general public, including foreign nationals. Copies may be obtained from the Defense Technical Information Center (DTIC) (http://www.dtic.mil).

AFRL-RV-PS-TR-2016-0135 HAS BEEN REVIEWED AND IS APPROVED FOR PUBLICATION IN ACCORDANCE WITH ASSIGNED DISTRIBUTION STATEMENT.

//SIGNED//

Dr. Frederick Schult
Program Manager, AFRL/RVBYE

//SIGNED//

Dr. Thomas R. Caudill, Acting Chief
AFRL Battlespace Environment Division

This report is published in the interest of scientific and technical information exchange, and its publication does not constitute the Government’s approval or disapproval of its ideas or findings.

Approved for public release; distribution is unlimited.
# REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

<table>
<thead>
<tr>
<th>1. REPORT DATE (DD-MM-YYYY)</th>
<th>23-08-2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. REPORT TYPE</td>
<td>Final Report</td>
</tr>
<tr>
<td>3. DATES COVERED (From - To)</td>
<td>28 Sep 2009 – 30 June 2016</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. TITLE AND SUBTITLE</th>
<th>M&lt;sub&gt;Rg&lt;/sub&gt;: A Magnitude Scale for 1 s Rayleigh Waves at Local Distances with Focus on Yield Estimation</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>5a. CONTRACT NUMBER</th>
<th>FA8718-09-C-0051</th>
</tr>
</thead>
<tbody>
<tr>
<td>5b. GRANT NUMBER</td>
<td></td>
</tr>
<tr>
<td>5c. PROGRAM ELEMENT NUMBER</td>
<td>62601F</td>
</tr>
<tr>
<td>5d. PROJECT NUMBER</td>
<td>1010</td>
</tr>
<tr>
<td>5e. TASK NUMBER</td>
<td>PPM00005965</td>
</tr>
<tr>
<td>5f. WORK UNIT NUMBER</td>
<td>EF004240</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6. AUTHOR(S)</th>
<th>Jessie L. Bonner and David R. Russell</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</th>
<th>Weston Geophysical Corporation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>181 Bedford Street, Suite 1</td>
</tr>
<tr>
<td></td>
<td>Lexington, MA 02420</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8. PERFORMING ORGANIZATION REPORT NUMBER</th>
<th></th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)</th>
<th>Air Force Research Laboratory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Space Vehicles Directorate</td>
</tr>
<tr>
<td></td>
<td>3550 Aberdeen Avenue SE</td>
</tr>
<tr>
<td></td>
<td>Kirtland AFB, NM 87117-5776</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>10. SPONSOR/MONITOR'S ACRONYM(S)</th>
<th>AFRL/RVBYE</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</th>
<th>AFRL-RV-PS-TR-2016-0135</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>12. DISTRIBUTION / AVAILABILITY STATEMENT</th>
<th>Approved for public release: distribution is unlimited. (OPS-2016-12545 dtd 05 Oct 2016)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>13. SUPPLEMENTARY NOTES</th>
<th></th>
</tr>
</thead>
</table>

| 14. ABSTRACT                             | The accurate estimation of yields from small chemical and nuclear explosions represents a difficult and continuing problem for the Nuclear Explosion Monitoring (NEM) community. P-wave spectra (Murphy, 1996), Pn amplitudes (Vergino and Mensing, 1990), Lg amplitudes (Nuttli, 1986), intermediate-period surface wave amplitudes (Stevens and Murphy, 2001), or coda techniques (Mayeda et al., 2003) are some of the methods that have been proposed for estimating the yields of small explosions recorded at regional distances. However, the uncertainty on seismic yield estimates can be large, and reducing the uncertainty may require a priori information about source media, knowledge of the emplacement depth, and calibrations for path and site effects. There is no general consensus as to which method for seismic yield estimation works best for all nuclear test sites. For monitoring at local and near-regional distances, we propose a methodology for yield estimation based on magnitudes of short-period, fundamental mode Rayleigh waves (Rg). At local distances, Rg can be the largest amplitude seismic arrival observed from shallow explosions, mining explosions, and shallow earthquakes. Adushkin (2001) demonstrated the ability to use Rg amplitudes at near-regional distances to provide accurate seismically-estimated yields. He corrected Rg amplitudes from Semipalatinsk underground explosions for attenuation, geometric spreading, and station-specific effects. Using the corrected Rg amplitudes, he estimated explosive yield within 20% of the true yields for the explosions. In this report, we develop a new formula for short-period (~1 s) surface wave magnitudes, called M<sub>Rg</sub>, using the methods of Russell (2006). Attenuation coefficients for short-period surface waves have been derived from diverse tectonic settings to calibrate the new M<sub>Rg</sub> formula. We demonstrate the utility of the formula at estimating magnitudes for small explosions, and then correlate the magnitudes to yield with uncertainty estimates. |

| 15. SUBJECT TERMS                        | Rg; seismic Rayleigh wave; seismic magnitude; explosion yield |

<table>
<thead>
<tr>
<th>16. SECURITY CLASSIFICATION OF:</th>
<th>17. LIMITATION OF ABSTRACT</th>
<th>18. NUMBER OF PAGES</th>
<th>19a. NAME OF RESPONSIBLE PERSON</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. REPORT</td>
<td>Unclassified</td>
<td>Unlimited</td>
<td>Dr. Frederick Schult</td>
</tr>
<tr>
<td>b. ABSTRACT</td>
<td>Unclassified</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>c. THIS PAGE</td>
<td>Unclassified</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard Form 298 (Rev. 8-98)  
Prescribed by ANSI Std. 239.18
This page is intentionally left blank.
# Table of Contents

List of Figures........................................................................................................................... ii
Acknowledgments...................................................................................................................... iii
1. SUMMARY............................................................................................................................ 1
2. INTRODUCTION.................................................................................................................. 2
3. MAGNITUDE FORMULA DEVELOPMENT ...................................................................... 3
4. $M_{RG}$ APPLICATION ...................................................................................................... 7
5. DATA AND RESOURCES................................................................................................. 12
References ............................................................................................................................... 19

Approved for public release; distribution is unlimited.
List of Figures

Figure 1. $R_g$ attenuation coefficients as a function of period for diverse lithologies in North America as well as data for Kazakhstan from Patton et al. (2005). Also shown are the period-dependent attenuation terms ($B_{att}=0.0031 \ p=1.8$) for the Russell (2006) formula as well as the proposed terms ($B_{att}=0.0031 \ p=1.96$ and $B_{att}=0.0031 \ p=2.19$) for $T=1 \ s \ R_g$ based on the measured attenuation coefficients. .......................................................... 13

Figure 2. Excitation corrections for $M_s$ to periods as low as 0.5 s for five different crustal models compared to the excitation correction used in Russell (2006). .................................................. 14

Figure 3. Processing example showing a) filtered waveforms, b) group velocity estimates for $R_g$, and c) magnitude estimates for a 101.4 kg (TNT equivalent) explosion conducted in Barre granite (Vermont, USA). ..................................................................................... 15

Figure 4. Final $M_{Rg}$ estimates (error bars show $\pm 1\sigma$) for 39 small explosions in different lithologies. The results are plotted against the TNT-equivalent yields for the explosions. a) Results using three excitation corrections (-1, -1.5, -2.5) as defined by Eq. 4. b) Results using a single excitation correction (-1) for all events. ......................................................................... 16

Figure 5. A yield-$M_{Rg}$ relationship based on regression results for chemical explosions. ....................................................................................................................................................... 17

Figure 6. Estimated yields from $M_{Rg}$ ($Y(Rg)$) in kt versus true yields for 8 Degelen and 10 Shagan Test Site nuclear explosions. .............................................................................................. 18
Acknowledgments

Most of the datasets used in this study would not have been collected without excellent planning and execution by Mark Leidig and many other scientist and experimenters. We thank IRIS PASSCAL for their assistance in the collection of many of these data. We thank Steve Harder, Kate Miller, and Anne Sheehan for their assistance in obtaining the BASE data. Thanks to Xiaoning (David) Yang and Howard Patton for applying the $M_{Rg}$ estimation software to the SPE-2 near-source data. We thank Jeff Stevens and Eli Baker for access to the Degelen and Shagan nuclear explosion data. We also wish to express our thanks to Howard Patton for many fruitful discussions regarding $R_g$ over the years and an excellent review of the paper. We thank James Lewkowicz of Weston Geophysical for his enduring and patient support of this and many other projects. The project was funded by the Air Force Research Laboratory under Contract No. FA8718-09-C-0051.
This page is intentionally left blank.
1. SUMMARY

Yield estimation of small explosions at local distances represents a challenge for the nuclear explosion monitoring community. We have examined the feasibility of using short-period surface wave magnitudes, called $M_{Rg}$, to estimate explosion yields at local distances ($<100$ km). We have modified the Russell (2006) $M_s$ formula, which was derived for periods of 8-25 s for distances beyond 50 km, for application at local distances and 1 s period. We have studied short-period surface wave attenuation in diverse lithologies in order to incorporate an attenuation term in the magnitude scale, which is suitable for $Rg$. We have also incorporated multiple excitation corrections for $Rg$ based on the near source seismic velocities, which greatly affect the source region amplitudes for $Rg$. It is important to note that in the formula the excitation is estimated from the measured $Rg$ group velocity. We have also derived a new Butterworth filter cutout definition for filtering $Rg$ near 1 s period at distances between 2-100 km. We used the new formula to estimate $M_{Rg}$ for 39 small ($37 \leq Y \leq 12,270$ kg TNT equivalent) and shallow ($<120$ m) explosions detonated in North America in lithologies ranging from alluvium to granite. Regressions of the magnitudes with yield result in the equation $M_{Rg} = \log_{10}(Y)-3.03$ for chemical explosions with $Y < 12,270$ kg. An F factor with 95% confidence was determined to be 2.25, giving lower and upper bounds on the yield estimates of $Y/2.25$ and $Y*2.25$, respectively. We applied the relationship $M_{Rg} = \log_{10}(Y)-3.33$ (assuming factor of 2 equivalence between chemical and nuclear) to nuclear explosions detonated at the Degelen and Shagan, Kazakhstan, test sites. The estimated yields based on $Rg$ magnitudes were often within 20% of the true yield and had smaller F factor than the estimated yields for US chemical explosions.
2. INTRODUCTION

The accurate estimation of yields from small chemical and nuclear explosions represents a difficult and continuing problem for the Nuclear Explosion Monitoring (NEM) community. *P*-wave spectra (Murphy, 1996), *Pn* amplitudes (Vergino and Mensing, 1990), *Lg* amplitudes (Nuttli, 1986), intermediate-period surface wave amplitudes (Stevens and Murphy, 2001), or coda techniques (Mayeda et al., 2003) are some of the methods that have been proposed for estimating the yields of small explosions recorded at regional distances. However, the uncertainty on seismic yield estimates can be large, and reducing the uncertainty may require a priori information about source media, knowledge of the emplacement depth, and calibrations for path and site effects. There is no general consensus as to which method for seismic yield estimation works best for all nuclear test sites.

For monitoring at local and near-regional distances, we propose a methodology for yield estimation based on magnitudes of short-period, fundamental mode Rayleigh waves (*Rg*). At local distances, *Rg* can be the largest amplitude seismic arrival observed from shallow explosions, mining explosions, and shallow earthquakes. Adushkin (2001) demonstrated the ability to use *Rg* amplitudes at near-regional distances to provide accurate seismically-estimated yields. He corrected *Rg* amplitudes from Semipalatinsk underground explosions for attenuation, geometric spreading, and station-specific effects. Using the corrected *Rg* amplitudes, he estimated explosive yield within 20% of the true yields for the explosions.

In this paper, we develop a new formula for short-period (~1 s) surface wave magnitudes, called *M*$_{Rg}$, using the methods of Russell (2006). Attenuation coefficients for short-period surface waves have been derived from diverse tectonic settings to calibrate the new *M*$_{Rg}$ formula.
We demonstrate the utility of the formula at estimating magnitudes for small explosions, and then correlate the magnitudes to yield with uncertainty estimates.

3. MAGNITUDE FORMULA DEVELOPMENT

Introduction

While many surface wave magnitude scales have been developed during the past century, none were developed specifically for $R_g$. Russell (2006) developed a magnitude formula that could effectively measure surface-wave magnitudes at local, regional and teleseismic distances, at variable periods between 8 and 25 seconds. With suitable modifications, we will extend this method to be applicable to 1 sec filtered $R_g$ phases. The (2006) magnitude equation is:

$$M_{d(b)} = \log(a_b) + \frac{1}{2} \log(\sin(\Delta)) + 0.0031 \left(\frac{20}{T}\right)^{1.8} \Delta - 0.66\log\left(\frac{20}{T}\right) - \log(f_c) - C,$$

where $a_b$ is the amplitude of the Butterworth-filtered surface waves (zero-to-peak in nanometers), $\Delta$ is the distance in degrees, and $f_c$ is the filter cutoff frequency of 3rd order, zero phase Butterworth filters with corner frequencies of $1/T-f_c$, $1/T+f_c$, respectively.

For periods $8 \leq T \leq 25$, the equation is corrected to $T=20$ s, accounting for frequency-dependent source effects, attenuation, and dispersion. The constant, $C$, is determined to be 0.43, which scales the equation at $T=20$ s to be equivalent to Rezapour and Pearce (1998).

In the next few paragraphs, we discuss how Eq. 1 can be modified for short-period Rayleigh waves magnitudes, which we refer to as $M_{Rg}$. This will include a change in the filter order which results in a new constant in Eq. 1, new filter constraints to determine $f_c$, a new attenuation correction based on observed $Rg$ data in diverse geologies, and choices for the excitation correction based on near-source medium properties. These changes will also scale corrections to standard 20 sec $M_s$ as in Eq. 1.
**Filter Order.** The first modification to the Russell (2006) formula for $MR_g$ is to use a 2nd order Butterworth filter rather than 3rd order. This decision was based on multiple comparisons of filtered $R_g$ waveforms using 2nd and 3rd order filters, and preferring 2nd order filtering in the identification and processing of the waveforms at local distances. The constant ($C=0.43$) in Eq. 1 is changed to $C=0.46$ as a result of applying a 2nd order filter, due to the constant being dependent on the Butterworth gain term $\hat{b}_B$, which is a function of the Butterworth filter order (Russell, 2006).

**Filter Definition.** To estimate the time-domain amplitude of $R_g$, we filter the time series with a 2nd order, zero-phase Butterworth band pass filter with corner frequencies $1/T-f_c, 1/T+f_c$, where

$$f_c = \frac{0.1}{T\sqrt{\Delta}}. \tag{2}$$

The constant, 0.1, in Eq. 2 is based on the dispersive properties of short-period Rayleigh waves, which often exhibit strong dispersion in the $T=1$ s range. For 8-25 s Rayleigh waves (e.g., Russell, 2006), the constant was 0.6. We note that Zeiler and Velasco (2009) used a constant of 0.2 in Eq. 2 to extend Eq. 1 to short-period $R_g$ for an explosion dataset in Arizona.

**$R_g$ Attenuation.** We estimated the $R_g$ attenuation coefficient, $\gamma$, for explosion datasets in southern Arizona (igneous rocks), Colorado Plateau (sedimentary), central Alaska (metamorphic), New England (granite), Albuquerque, New Mexico (alluvium and sedimentary), Yucca Flat in Nevada (alluvium), and Wyoming (sedimentary). The period-dependent attenuation coefficients for all datasets (Figure 1) show that for most of the explosion datasets, intrinsic attenuation of $R_g$ results in an average $\gamma=0.05\pm0.03$ /km at $T=1$ s. We note that this value will not be sufficient in an area of strong $R_g$ scattering, such as near faults (see Figure 1
solid data points) or rapid topographic changes. This is not a significant problem given that identifiable \( Rg \) will likely not propagate to many stations beyond such fault zones.

Figure 1 also shows period-dependent attenuation correction Eq. 1 of the form:

\[
B_{\text{att}}(T_0/T)^p \quad (3a)
\]

where \( B_{\text{att}} \) and \( p \) were 0.0031 and 1.8 for 8-25 s surface waves. We find that to better match the \( Rg \) attenuation coefficients at \( T=0.1 \) to 2 s, the exponent \( (p) \) value should be increased to 2.19 which corresponds to a \( \gamma = 0.05 \) /km at \( T=1 \) s. We initially used this value in the development of \( MR_g \), but noted that for our small chemical explosion dataset, if \( Rg \) was observed beyond ~25 km, it was usually in a region with lower attenuation coefficients. As a result, the \( MR_g \) at distances greater than 20 km would be overcorrected for attenuation using a \( \gamma = 0.05 \) /km. Thus we chose to weight the attenuation correction in \( MR_g \) to a lower \( \gamma = 0.02 \) /km (or an equivalent exponent of \( p=1.96 \)). Returning to Eq. 3a, the attenuation term for our proposed \( MR_g \) formula at \( T=1 \) sec is:

\[
0.0031(20/T)^{1.96}\Delta, \quad (3b)
\]

or \( 1.1\Delta \). \( (3c) \)

**Rg Excitation Correction.** Eq. 1 includes a source excitation correction of \(-0.66 \log\left(\frac{T_0}{T}\right)\) in the estimation of surface wave magnitudes at \( T=8-25 \) s. The correction is used to reduce short-period Rayleigh-wave amplitudes, which are enhanced for shallow sources—particularly explosions—in order to approximate a magnitude that would be measured for the same event near the reference period of \( T=20 \) s. Figure 2 shows the theoretical magnitude correction required to extend the excitation correction to \( T=1 \) s \( Rg \) for five different crustal models. The corrections are based on explosion synthetics at 1 km distance for a source at 0.01 km depth. These corrections are normalized at \( T=20 \) s for scaling purposes. Also shown is the excitation correction from Eq.
1, which matches the theoretical magnitude correction for most models in the 8-25 s range. We note that while these corrections are for a synthetic explosion at 0.01 km depth, we find similar corrections up to about 200 m depth, except for the alluvium correction, which is slightly reduced.

Figure 2 suggests that a single excitation correction may not provide optimal results for estimating $M_{Rg}$ magnitudes in differing geological environments. The magnitude corrections range from -0.6 m.u for the global average AK135 model (Kennett et al., 1995) to -1.4 m.u. for a western United States (WUS) crustal model (Herrmann et al., 2011). If the upper km of the WUS model is replaced with velocities representing dry alluvium, the resulting excitation correction increases to approximately -2.6. The nulls in the spectrum for the alluvium model show the sensitivity of the synthetics to the low velocities. The Eq. 1 correction term is -0.9 at $T = 1$ s which is similar to the theoretical correction based on velocity models for the Korean Peninsula (Cho et al., 2007) and the central United States (Herrmann et al., 2011).

The velocities of the layer where the explosion is synthesized primarily controls the magnitude correction. For example, the $V_p$ and $V_s$ for the upper layer of the western United States model are 3.4 km/s and 2 km/s, respectively. Conversely, for the AK135 model, the $V_p$ and $V_s$ are 5.8 km/s and 3.46 km/s, respectively. For the WUS+dry alluvium model, the $V_p$ and $V_s$ for the upper layer are 2.0 and 0.8 km/s, respectively. Based on these velocities, we propose the following excitation correction, $E(T=1 s)$, for the $M_{Rg}$ formula:

$$E(T=1 s) = \begin{cases} -2.5 & U_{Rg} \leq 0.8 \text{ km/s} \\ -1.5 & 0.8 < U_{Rg} \leq 2 \text{ km/s} \\ -1 & U_{Rg} > 2 \text{ km/s} \end{cases} \quad (4)$$

where $U_{Rg}$ is the measured group velocity of $T=1$ s $Rg$ near the test site. This highlights the importance of emplacement properties on the generation of $Rg$ and provides for a larger...
magnitude correction, relative to T=20 s, for alluvium and “soft” rock test sites than “hard” rock test sites. It requires an approximate origin time and distance in order to process for $M_{Rg}$. It should be emphasized that the corrections in Eqn (4) are based on the assumption that at local source/receiver distances, the source geology is the same as for the propagation path. For more complex media where the source group velocities differ from the propagation velocities, knowledge of the source group velocity from independent information will be necessary to correct Eqn. (4).

**Final $M_{Rg}$ Formula**

Based on the described changes to the Russell (2006) formula, we introduce:

$$M_{Rg} = \log(a_{Rg}) + \frac{1}{2} \log \left( \sin \left( \Delta \right) \right) - \log(f_c) + 1.1 \Delta + E(T) - 0.46$$  \hspace{1cm} (5)

where $\Delta$ is distance in degrees between 0.02° and 0.90° (2 and 100 km). The current distance limitations are constrained by the data that are discussed in this paper. Waveforms should be corrected for instrument response and converted to displacement (in nanometers) and filtered between $1-f_c$, 1+$f_c$ using a zero-phase, 2nd order Butterworth bandpass filter where $f_c$ is determined by Eq. 2. Zero-to-Peak amplitudes of $Rg$ (a$_{Rg}$) in nanometers are measured, and the time of the peak amplitude recorded and converted to group velocity ($U_{Rg}$) in order to pick an excitation correction, E(T), defined in Eq. 4. The processing can be automated using pre-selected group velocity windows for $Rg$ or manually by an analyst picking the peak amplitudes for $Rg$.

4. $M_{Rg}$ APPLICATION

**Explosions.** We have applied the $M_{Rg}$ formula to several explosion datasets recorded at local distances. Figure 3a shows the Butterworth-filtered waveforms for a 101.4 kg (TNT equivalent)
explosion detonated in Barre granite and recorded from 2 to 24 km. The Butterworth filtered waveforms (Eq. 2) highlight the T=1 s \( R_g \). Group velocities of the peak amplitude of the filtered signals are between 2.2 and 2.8 km/s and increase with distance (Figure 3b). The mean \( U_{R_g} \) is 2.6 km/s, thus the processing automatically chose an excitation correction of \( E(T)=-1 \) for \( U_{R_g} > 2.0 \) km/s, which is typical of “hard” rock emplacement media. The \( M_{R_g} \) estimates using Eq. 5 are shown in Figure 3c. The network average \( M_{R_g} \), which is estimated after discarding ±5% of high and low estimates in the population, is -1.05 with of ± one standard deviation of 0.07 m.u. To ensure that we are not biasing these results by using instrument-corrected 4.5 Hz geophone data, we calculated the magnitude using 15 broadband seismometers and obtained \( M_{R_g} = -1.06\pm0.10 \).

We also estimated \( M_{R_g} \) for 39 explosions recorded at distances between 2 and 50 km. The explosions were part of multiple active source phenomenology experiments, including the Arizona Source Phenomenology Experiment (AZ-SPE), Bighorn Active Seismic Experiment (BASE), the HUMMING ALBATROS Experiment (HAL), the Fracture Decouple Experiment (FDE), the Frozen Rock Experiment (FRE), HUMBLE REDWOOD (HR) I and II, and the New England Damage Experiment (NEDE). The explosion yields for the 40 events range from 37 kg to 12,270 kg TNT equivalent. All of the explosions were detonated below ground; however, the explosions include both fully- and partially-confined shots. All of the explosions were conducted at shallow depths of burial (DOB) less than 120 m; however, the approximate scaled DOB ranged from 43 to 271 m/kt\(^{1/3}\).

**Yield-M\( R_g \) Relationship.** Figures 4 and 5 compare the estimated \( M_{R_g} \) to TNT equivalent yield. In Figure 4a,b the results for each experiment are plotted as a different color or symbol. To show the effect of the excitation correction, we include results (Figure 4a) for \( M_{R_g} \) with three possible excitation corrections as defined in Eq. 4, and we present the \( M_{R_g} \) results (Figure 4b) using a
single correction (-1) for all events. The use of multiple excitation corrections significantly decreases the variance of the magnitude-yields than the use of a single correction. The observed magnitudes in Figure 4b confirm the theoretical results shown in Figure 2, which show that the slower velocity rocks (alluvium, sedimentary) excite larger amplitude \( T=1 \) \( s \) \( R_g \) than a similar yield explosion in faster media (e.g., granite). We plan to continue to develop this formula and the excitation corrections as more data become available, especially shots in alluvium. We regressed log yield \((Y)\) versus \( M_{Rg} \) in Figure 5 to determine the relationship:

\[
M_{Rg} = \log_{10}(Y) - 3.03 \quad \text{(Chemical)} \quad (6)
\]

for \( 37 \leq Y \leq 12270 \) kg (TNT equivalent). The slope from the analysis was 0.99, thus statistically representing a slope of unity between \( M_{Rg} \) and log \( Y \). Slopes for historical \( M_s \)-log yield relationships typically fall somewhere between 0.75 to 1.2 (Murphy, 1977; Bache, 1982; Evernden and Marsh, 1987). Slopes of 1 have been reported by Stevens and Murphy (2001) for the Shagan Test Site in Kazakhstan and Bache (1982) for multiple datasets. The 95% confidence intervals were also determined for the data and plotted in Figure 5. Note that the \( M_{Rg} / \) yield formula is expressed in kilograms of yield; to change to kilotons add 6 to the right hand side of Eqn (6).

Another way of determining approximate confidence bounds on yields of explosions is to estimate F factors. For 95% confidence interval, the F factor is defined as:

\[
F = 10^{(1.96\sigma/\alpha)}, \quad (7)
\]

where \( \sigma \) is the standard deviation of the magnitudes about the regression line with slope \((\alpha) = 1\). The regression in Figure 5 has \( \sigma = 0.18 \) resulting in an \( F = 2.25 \). The confidence bounds on the yield are defined as:

\[
Y_{\text{lower}} = Y/F \quad (8a)
\]
The yield-$M_{Rg}$ relationship (Eq. 6) was developed for a series of chemical explosions. The source function for the chemical explosion is equivalent to that of a nuclear explosion with twice the yield (Denny et al., 1995). Thus for nuclear explosions, Eq. 6 would be modified by subtracting 0.3 from the constant:

$$M_{Rg} = \log_{10}(Y) - 3.33 \quad \text{(Nuclear)} \quad (9)$$

Thus for a 1 kiloton nuclear explosion, we would expect an $M_{Rg}=2.7$. This is at least 0.5 m.u. higher than most historical yield-$M_s$ relationships would predict (e.g., Stevens and Murphy, 2001).

**Validation of the Yield-$M_{Rg}$ Formula**

**Kazakhstan.** We have validated the new yield-$M_{Rg}$ formula on additional datasets that were not included in the development processes. Validation included estimating magnitudes for 8 Degelen, Kazakhstan, Test Site (DTS) and 10 Shagan, Kazakhstan, Test Site (STS) nuclear explosions and recorded at local distances by the Institute for Dynamics of the Geosphere (IDG). The datasets are described in Stevens et al. (2007) and has excellent examples of $Rg$ recorded at local distances. The test sites both have $Rg$ velocity $>2\text{ km/s}$ resulting in a choice for the excitation correction, $E(T)=-1$. The $M_{Rg}$ estimates for all 18 events were converted to yield using Eq. 9.

The comparison of the true and estimated yields is shown in Figure 6. Because of the small number of stations recording these events (typically 4-6 stations), we did not trim the mean for these results. The highest percentage error between the estimated and true yield is 64%, which suggests a smaller $F$ factor for DTS and STS that what was determined for the North
America chemical explosions used to derive Eq. 6. For 12 of the 18 events, the estimated yield percentage error is 20% or less, which is similar to Adushkin’s (2001) yield estimates for Shagan River Test Site nuclear explosions using $R_g$ with stations corrections. Considering that these DTS and STS nuclear explosions have yields that are orders of magnitude larger and significantly greater depths than the events used to develop Eqs. 6 and 9, these results are very promising.

Over-buried explosions may provide difficulties for $M_{Rg}$ due to moment decrease and surface wave eigenfunction depth decay for $R_g$. As an example, Figure 5 shows the $M_{Rg}$ for a 25 ton chemical explosion detonated at 550 m depth (scaled DOB = $\sim$1452 m/kt$^{1/3}$) as part of the 1997 STS DOB Experiment (Myers et al., 1999; Patton et al., 2005). The $M_{Rg}$ estimate sits well below our observed data population of events with sDOB ranging from 43 to 271 m/kt$^{1/3}$ and results in a yield estimate of only 5 tons for this 25 ton event. However, after correcting the observed $M_{Rg}$ estimate for over-buried moment using the Denny and Johnson (1991) source model and surface wave depth excitation using synthetic seismograms, the corrected $M_{Rg}$ is consistent with the regression equation for $M_{Rg}$ and log $Y$.

**Western United States.** We have experienced difficulties with estimating yields for Nevada Test Site (NTS) explosions using the $M_{Rg}$ method. Analysis of near-source recordings (Brian Stump, pers. comm. 2011) of NTS explosions such as Hunter’s Trophy, Mineral Quarry, and the NPE showed little, if any, observable $R_g$ energy at nearby stations, perhaps due to significant topographic and structural scattering. We have also examined recent chemical explosions conducted at NTS (now called the Nevada National Security Site). Because 1 s $R_g$ was difficult to identify at distances less than 2 km, we modified Eq. 5 for application on 3 Hz $R_g$ recorded at 1-2 km from the 1000 kg Source Physics Experiment shot 2 (SPE-2; Snelson et al., 2012).
Results of the $M_{Rg}$ methodology varied significantly depending on path effects, as we estimated a viable yield (892 kg) for three stations north of the explosion, while yield estimates to the northeast ranged from 486 kg at 1.1 km to 295 kg at 1.6 km. These results demonstrate that there are regions where the scattering and attenuation of $Rg$ is significant and/or highly variable making application of the proposed yield-$M_{Rg}$ methods problematic or implausible. With publication of this paper and public availability of $M_{Rg}$ estimation software, we hope other researchers will apply the formula to additional explosion datasets and inform us when the method fails or succeeds.

5. DATA AND RESOURCES

The software developed to estimate $M_{Rg}$ is available upon request from the authors. Data used in this project is derived from numerous small-scale explosion experiments conducted by various organizations. Most of the datasets used in this project, which are older than 2 years from the publication date, can be found at the Incorporated Research Institutions in Seismology (IRIS) Data Management Center (DMC). The data are from experiments with code names SPE, FRE, NEDE, HR, BASE, and FDE. For data that are not located in the IRIS DMC repository, please contact author Bonner for details on how to obtain those data.
Figure 1. $R_g$ attenuation coefficients as a function of period for diverse lithologies in North America as well as data for Kazakhstan from Patton et al. (2005). Also shown are the period-dependent attenuation terms ($B_{att}=0.0031 \ p=1.8$) for the Russell (2006) formula as well as the proposed terms ($B_{att}=0.0031 \ p=1.96$ and $B_{att}=0.0031 \ p=2.19$) for $T=1 \ s \ R_g$ based on the measured attenuation coefficients.
Figure 2. Excitation corrections for $M_s$ to periods as low as 0.5 s for five different crustal models compared to the excitation correction used in Russell (2006).
Figure 3. Processing example showing a) filtered waveforms, b) group velocity estimates for $R_g$, and c) magnitude estimates for a 101.4 kg (TNT equivalent) explosion conducted in Barre granite (Vermont, USA).
Figure 4. Final $M_{Rg}$ estimates (error bars show ±1σ) for 39 small explosions in different lithologies. The results are plotted against the TNT-equivalent yields for the explosions. a) Results using three excitation corrections (-1, -1.5, -2.5) as defined by Eq. 4. b) Results using a single excitation correction (-1) for all events.
**Figure 5.** A yield-$M_{Rg}$ relationship based on regression results for chemical explosions. The data points are shown as solid circles, the best fit regression line is solid, and the dashed lines describe the 95% confidence intervals. The open circles and arrow represent results from an over-buried Shagan DOB explosion which was not considered in the regression. The arrow points from the observed $M_{Rg}$ to a value corrected for depth of burial effects due to moment and $R_g$ excitation.
Figure 6. Estimated yields from $M_{Rg}(Y(Rg))$ in kt versus true yields for 8 Degelen and 10 Shagan Test Site nuclear explosions. The dashed line represents a 1:1 relationship. The vertical error bars represent an F factor of 2.25, which is based on the North American chemical explosion datasets used to develop the yield-$M_{Rg}$ relationship. When only the Shagan and Degelen nuclear explosions are considered, the F factor is estimated as 1.6.
References


DISTRIBUTION LIST

DTIC/OCP
8725 John J. Kingman Rd, Suite 0944
Ft Belvoir, VA 22060-6218 1 cy

AFRL/RVIL
Kirtland AFB, NM 87117-5776 2 cys

Official Record Copy
AFRL/RVBYE/Dr. Frederick Schult 1 cy

Approved for public release; distribution is unlimited.
This page is intentionally left blank.