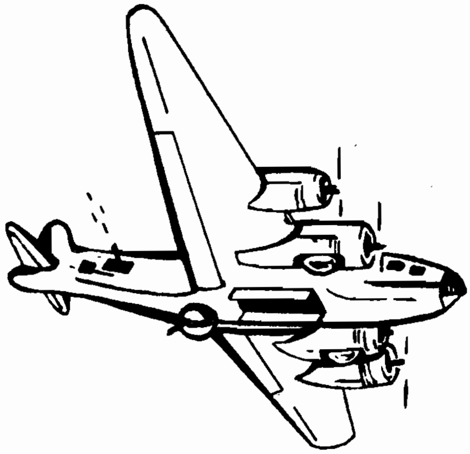
ORDNANCE CORPS

BALLISTIC RESEARCH LABORATORIES

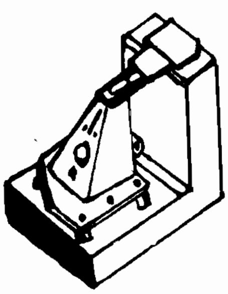
ABERDEEN PROVING GROUND, MD.



MEMORANDUM
REPORT No. 617

A Survey of Eniac Operations and Problems

1946 - 1952



W. BARKLEY FRITZ

BALLISTIC RESEARCH LABORATORIES

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August 1952

A SURVEY OF ENIAC OPERATIONS AND PROBLEMS: 1946 - 1952

W. Barkley Fritz

Project No. TB3-0007 of the Research and Development Division, Ordnance Corps

ABERDEEN PROVING GROUND, MARYLAND

TABLE OF CONTENTS

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WBFritz/gmw
Aberdeen Proving Ground, Md.
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ABSTRACT

This report summarizes the entire operating history of the Eniac since February 1946, when the machine was demonstrated to the public as the first large scale general purpose electronic digital computing machine. It brings up to date much of the material presented in the unpublished paper entitled "The Eniac - A Five Year Operating Survey" presented at the Detroit meeting of the Association for Computing Machinery on 27 March 1951.

The primary interest of the paper is the brief descriptions given of the specific unclassified problems completed during this period. It is hoped that individuals not yet in immediate contact with high speed computers will gain some insight into the type of problems which machines make accessible to computation. Pertinent reference material is given to a variety of unclassified published material related to these completed programs.

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I. HISTORICAL INTRODUCTION

The Eniac - the Electronic Numerical Integrator and Computer - was the first high speed, electronic, general purpose, digital computing machine. It was designed and constructed at the Moore School of Electrical Engineering of the University of Pennsylvania (1944 - 1946). The primary reason for the construction of the Eniac was to speed up the large quantity of numerical computations required for the Army's firing tables and bombing tables. Since the Eniac was designed to handle this particular type of problem, there was little attempt to simplify the steps necessary to prepare new problems or to change from one problem to another. The original method of Eniac control known as "direct programming" required the activation of a large number of circuits by the tedious procedure of setting switches and connecting plug wires in some forty panels in addition to setting tabulated data on the switches of the "portable" function tables, whenever a problem was to be put on the machine. The size of the problem that could be solved was limited by the number of controls on each of the machine's units. The number of problems that could be handled within a given period of time was limited by the time it took to prepare the program of control settings necessary to solve the problem.

Experience with the first problems solved on the Eniac at Philadelphia and later at Aberdeen, showed clearly that the cumbersome method of direct programming would severely limit the Eniac's use as a general purpose computer. Operating percentages were not kept during this early period, but various estimates of machine productivity have been made ranging from 5% to as much as 25%. However, during this period, the Eniac was only in operation for a normal 8 hour shift for 5 days each week. All voltages except filament voltages were turned off during the non-operative period. To take advantage of a productive machine, however, the operator occasionally overstayed the normal 1630 departure time. During this period of the machine's operation, from February 1946 to March 1948, eleven problems were completed.

Early in 1948, R. F. Clippinger and some of his associates, in the course of coding the solution of a simple system of five simultaneous hyperbolic partial differential equations, were forced to adopt a different method of using the Eniac in order to fit their problem on the machine. This method consisted of a code for parallel operation using numerical information from its function tables for logical control. The experience with this method (first discussed in reference 1), led J. von Neumann to suggest the use of a serial code for control of the Eniac. Such a code was devised and employed with the Eniac beginning in March 1948. Operation of the Eniac with this code was several times slower than either the original method of direct programming or the code for parallel operation. However, the resulting simplification of coding techniques and other advantages far outweighed this disadvantage.

II. BRIEF DESCRIPTION OF THE ENIAC

The Eniac is at present controlled by an improved version of the original one address type serial code, which incorporates a unit called the Converter as a basic part of its operation, hence the name, the Eniac Converter Code. The continual improvement of this code has been brought about primarily by changes in coding techniques and engineering additions to the original machine. The Eniac, however, is still very limited in high speed memory and somewhat awkward in comparison to the present day concept of the capabilities of a large scale digital computer. The Eniac has only fifteen high speed, storage registers in addition to the five required primarily as part of the arithmetic unit. Each register or accumulator is able to store a signed ten digit decimal number.

Of the 2400 individual 2 digit instructions of a single program, 1824 must be set by hand on the 3648 switches of the control panels. The remaining 576 instructions are wired on IBM plugboards which are prepared prior to the actual machine set up period. The average time required to place a problem on the Eniac is about 1.5 hours.

The prime advantage of the Eniac is its speed. The usual single instruction arithmetic operations are performed at the following approximate rates: 1) Addition - 1600/sec, 2) Subtraction - 1000/sec, 3) Multiplication - 290/sec, 4) Division and Square Rooting - 40/sec. Of course in one address type codes, a single arithmetic operation may require more than one instruction depending on the availability of the quantities. If we assume the Eniac were to perform only isolated operations unrelated to the rest of the computation, the following operation rates would be more correct: 1) Addition - 550/sec, 2) Subtraction - 450/sec, 3) Multiplication - 192/sec, and 4) Division and Square Rooting - 32/sec. Actual production figures for efficient coding, however, show rates more closely approximating the first set of figures.

The term approximate rate has been used since a variable oscillator controls the frequency of the pulse rate, hence the speed of the operations. Prior to the adoption of the code, the Eniac was always operated at a frequency of 100 KC. During the early period of the code (1948-1949), the operating frequency was often as low as 60-70 KC. At the present time, the average rate is 85 KC.

Another advantage of the Eniac is the efficient input-output devices - i.e. an IBM reader and an IBM punch. The 8 signed 10 digit numbers on an IBM card can be transferred to the high speed memory in about 0.3 seconds or essentially at the rate of 26 words per second. A similar group of 80 decimal digits from the high speed memory can be punched on an IBM card in 0.2 to 0.6 seconds depending on the frequency of the punching. This means a rate of 13 to 40 words per second, with the usual rate being 40 per second. Tabulated data or constants may be brought into the electronic memory from the hand set control panels at the rate of 715 quantities per second. Certain problems requiring a large memory have been run efficiently on the Eniac by punching intermediate results on IBM cards which in turn were read back into the machine for further computation. A complete description of the 94 arithmetic, storage, and control instructions of the Eniac Code is given in Reference 2.

III. OPERATING EXPERIENCE

Experience over the past several years has demonstrated that the code has improved the productive operation of the Eniac in the following ways: It

1. made easier the preparation of problems for the machine,

2. permitted an increase in the size of problem that can be handled by the machine,

3. reduced the time required for changing problems from several days to several hours, and

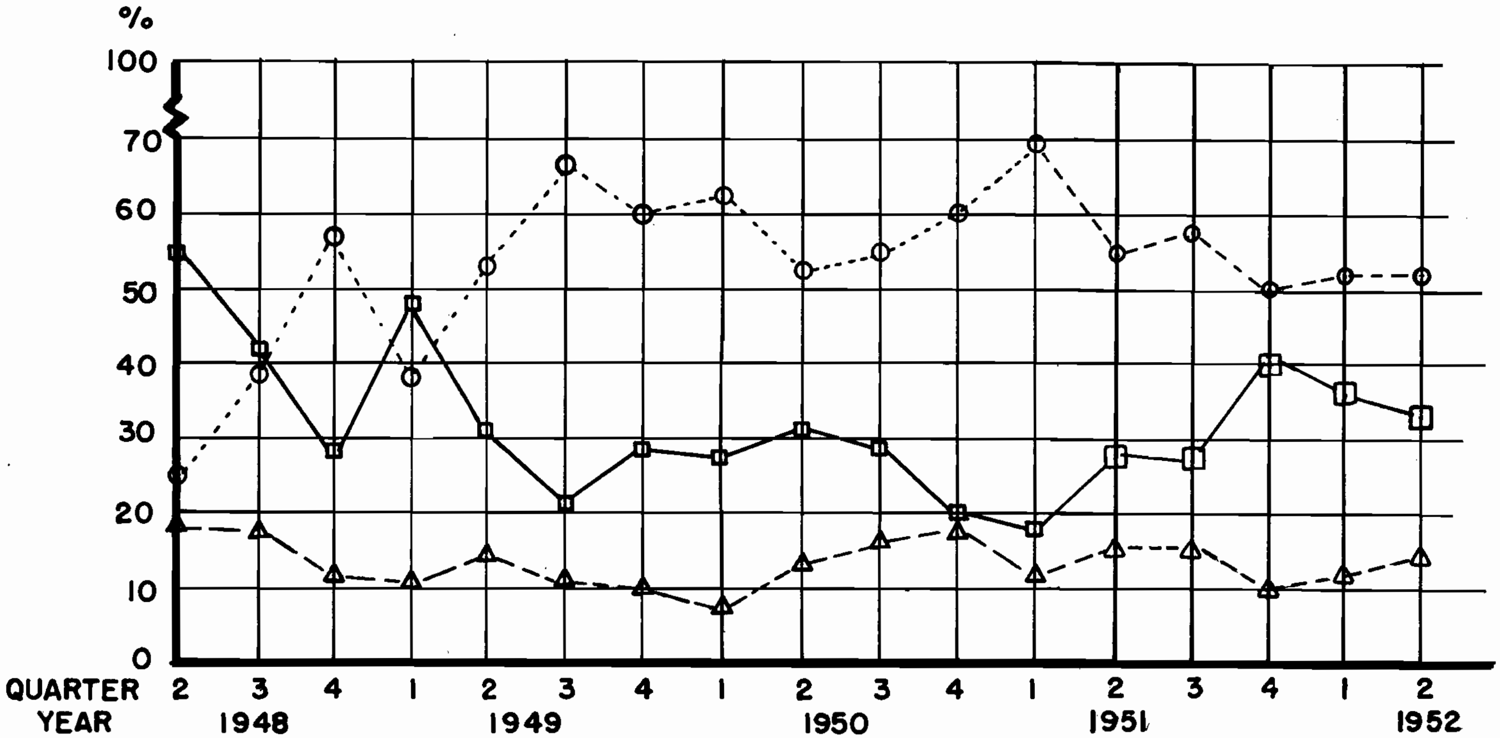
4. simplified testing procedures.

At the time the Converter Code was added to the Eniac, J. V. Holberton initiated a system of record keeping which accounts for every minute of the time the machine is manned. This log of the machine's operation has been continuously kept since April 1948. The quarterly records have been summarized in Figure 1.

The increase in trouble-shooting time since 1 April 1951 is partially explained by the transfer of several key service engineers, a general emphasis on the part of the Computer Research Branch to complete the Edvac, the failure to obtain certain required replacement parts, and a general upswing in the number of problems handled by the Eniac.

Some of the obvious requirements for good machine operation are competent operating and engineering personnel, efficient testing procedures and adequate programming. Another factor affecting operating percentages is the degree of human intervention which is used in solving the problem. Frequently no human intervention is required. However, one type of problem may require that the mathematician direct the course of the computation by determining each new step on the basis of the results of the previous steps. If possible this type of machine usage is avoided but occasionally it is not possible to state numerically, ahead of time, precise criteria for various courses of action. Data

ENIAC CONVERTER CODE-
MACHINE EFFICIENCIES PER QUARTER



1. O----O CORRECTLY OPERATING ON THE SOLUTION OF REGULAR PROBLEMS.
2. □——□ LOCATING AND CORRECTING MACHINE TROUBLE IN THE ENIAC, NON DUPLICATION TIME, AND DOWN TIME ON SPECIAL PREVENTIVE MAINTENANCE.
3. Δ-----Δ PLACING NEW PROBLEMS ON THE ENIAC, CHECKING PROGRAMMING, DATA ANALYSIS, AND DOWN TIME DUE TO HUMAN OPERATING ERROR.

FIGURE 1

analysis time in a problem of this type will be large. Another type of problem that of necessity results in reduced operating efficiency is one in which the control panels have to be changed during the course of the problem. Here the memory is insufficient to handle the problem efficiently; but the Eniac, at the time, was the only means available to obtain the desired computations in a reasonable period of time.

The figures indicated in Figure 1 are percentages. The actual records show an average of over 1500 machine hours per quarter (approximately 116 hours per week). The normal work week has been 0800 Monday through 2400 Friday.

Putting this productivity another way, the machine has operated correctly on the solution of problems 55% of the time for the period 1 April 1948 through 30 June 1952.

It is interesting to note that a staff of 15 - 25 mathematicians, technicians, and engineers have been utilized to perform programming, service engineering, and to operate the Eniac.

Before giving some information on the number and type of problems handled by the machine, it might be well to define just what we mean by a "problem". A "problem" is defined as a machine program or group of machine programs related to a specific subject under investigation or a particular mission to be accomplished. Examples of single problems are the following:

1. The Doppler determination of missile position. Here the single "problem" can be adapted to any required number of individual firings, hence the problem is repeated periodically with different input data. Various improvements have been made and the program of instructions increased to make more automatic the determination of the required solutions.

2. Determination of the mole fractions of the gaseous constituents of the equilibrium composition of four component systems of hydrogen, oxygen, nitrogen, and carbon at temperatures of 1000°K to 5000°K and pressures of 0.1 atmospheres to those of interior ballistics. In this instance a number of sets of equations and a large range of parameters are involved. Several machine months have been expended on the problem in a number of short machine periods.

A large portion of the problems completed on the Eniac are discussed in the next section.

IV. GUIDE TO COMPLETED PROBLEMS

Since the Eniac was completed in February 1946, nearly 100 problems have been completed. It will be impossible to go into much detail describing these problems, but it is felt that a brief guide to the work that has been accomplished would be of general interest.

Some 70% of these problems have originated within the Ballistic Research Laboratories, while the remainder were submitted by other government agencies, or either private industrial organizations or universities with government contracts. Over half of the problems have had to do with the numerical integration of systems of ordinary differential equations. The following integration methods are typical and have been used to solve one or more problems: Heun, Euler, Runge-Kutta, Iterated Simpson, and Kutta-Simpson. One fifth of the problems have been primarily solutions of algebraic systems. Of equal frequency has been the solution of partial differential equations including systems of hyperbolic equations and single equations of the parabolic type. The remaining problems have been the solution of integral equations or number theoretic computations.

The next few pages survey a large group of problems completed on the Eniac, the problems being listed in connection with the group sponsoring the computation. It should be pointed out that security regulations prohibit a complete listing of all the work performed. If specific unclassified Eniac problems with which the interested reader is familiar are not mentioned, it is suggested that he contact the author in order that any future "guides" may correct these oversights.

The following chart summarizes the work discussed in the remainder of the report.

	Sponsor	No. of Problems	% Machine Time
I	Computing Laboratory	54	35
II	Exterior Ballistic Lab.	8	12
III	Interior Ballistic Lab.	3	2
IV	Ballistic Measurements Lab.	1	1
V	Surveillance Laboratory	1	1
VI	Outside Groups	20	49

I. Computing Laboratory

The largest amount of computing for any single group was naturally enough performed for the Computing Laboratory of BRL. These computations are discussed in three categories:

A. Ballistic Tables

B. Measurements Analysis

C. Various Mathematical Investigations

A. Ballistic Tables

In recent years about 25% of the Eniac's computing time has been spent on the development of Ballistic Tables involving computations on some 41 problems. The determination of the numerical data for Firing Tables and Bombing Tables is a more extensive computational problem than the mere computation of the trajectories. The following mathematical problems have been programmed in order to determine the final form of those tables:

1. Solution of the non-linear differential equations describing the path of a trajectory.

2. Empirical determination of parameters by least square techniques to adapt theory to practice, and

3. Extensive high order interpolation in order to present the results in usable form.

A brief survey of some of the Eniac's work on ballistic tables is given in "Proceedings of the Pittsburgh Meeting of the ACM 2-3 May 1952." "Firing Table Computations on the Eniac" - H. L. Reed, Jr.

B. Measurements Analysis

The Eniac has been utilized in the reduction of data from high speed cameras in order to obtain certain information regarding the flight of guided and unguided missiles and bombs.

The reduction problem to determine the azimuth, elevation, yaw, and orientation of yaw of the missile axis of bombs and missiles is described in the following reports:

1. BRL 774 - "Attitude and Yaw Reductions of Projectiles in Free Flight" - G. R. Trimble, Jr. - 1951.

2. BRL 812 - "A Least Square Attitude Solution" - G. R. Trimble, Jr. - 1952.

The determination of space coordinates, and velocity and acceleration components of ground to ground missiles as well as bomb releases are obtained from high speed camera data. Observations from two or three camera stations provide azimuth and elevation angles of the line of sight. A least square procedure is used to minimize the distance between the lines of sight, from which the space coordinates are readily obtained.

3. BRLM 543 - "The Preparation of a Theodolite Problem for Large Scale Computing Machines" - B. Dimsdale and M. A. Powers - 1951.

4. BRL 778 - "A Recursion Relation for Computing Least-Square Polynomials over Moving Arc" - G. R. Trimble, Jr. - 1951.

C. Various Mathematical Investigations:

The following group of problems concern a variety of mathematical investigations as well as number theoretic computations.

The work of several individuals closely associated with the Computing Laboratory is included here.

1. Theory of Rounding Errors

An investigation of error accumulation was performed and reported on by Hans Rademacher as Lecture 19 in the series of lectures given at the Moore School of the University of Pennsylvania - 8 July - 31 August 1946. Hans Rademacher's paper was entitled "On the Accumulation of Errors in Numerical Integration on the Eniac" and was published in Volume II of the four volume set on the lecture series, pp 19-1 -- 19-17. The equations integrated were $y' = p$ and $p' = -y$ thus generating sines and cosines.

2. Mil Tables of Sines and Cosines

Mil tables of the sine and cosine for 0(1) 1600 ₥ to 10 decimal places using the formula $\cos(n+1)\ \theta = 2 \cos \theta \cos n\ \theta - \cos (n-1)\ \theta$. The table was reproduced locally for use in ballistic work.

3. Investigations in Prime Numbers

The computations completed during several holiday week ends by D. H. Lehmer of the University of California have been reported in three papers.

a. Math. Tables and Other Aids to Computation v. 2, p. 313, '47.

b. Amer. Math. Soc. Bulletin v. 53, 1947, pp 164-167, "On the Factors of $2^n \pm 1$" by D. H. Lehmer.

c. The American Mathematical Monthly v. 56, No. 5, pp 300-309, May 1949, "On the Converse of Fermat's Theorem II" - D. H. Lehmer.

4. Fermat's Quotient

Determination of a Table of Fermat's Quotients obtained during machine check periods. MTAC v. 5, Apr. 1951, pp 84-85.

5. Calculation of π, e, and 1/e

π was computed to 2035 places and e to 2556 places in an attempt to gain insight into the character of these numbers by obtaining the distribution of a large number of their digits as well as an illustration of machine methods for handling a large number of decimal places. 1/e was computed as a check on e. e • 1/e was obtained on an IBM tabulator.

a. MTAC IV No. 29 - Jan. 1950, "An Eniac Determination of π and e to More than 2000 Decimal Places" - George W. Reitwiesner.

b. BRL TN 381 - "A Table of Factorial Numbers and Their Reciprocals From 1! Through 1000! to 20 Significant Digits" - George W. Reitwiesner - 1951.

6. Examination of Iterated Simpson Numerical Integration

Zero and first order Bessel Functions were obtained by integrating the initial value problem $y_1 = - y_2$ $y_2 = y_1 - y_2/x$ with initial conditions $y_1(0) = 1$, $y_2(0) = 0$ using a numerical method suggested by R. F. Clippinger and B. Dimsdale. The integration method is fourth order and has the advantage over other methods in that 30-60% higher order systems can be handled with the same machine memory. Like the Runge-Kutta method, it proceeds from initial data at one point without need for data at other points. Thus the grid size can be easily changed and machine storage requirements are not unduly severe. The method is described in Sections 8 and 8' of the "Coding Notes" by Clippinger and Dimsdale and a

BRL Technical Note entitled "The Iterated Simpson Method of Numerical Integration" - W. Barkley Fritz - 1952.

7. Numerical Solution of Parabolic Partial Differential Equations

A numerical solution of a linear parabolic partial differential equation with constant coefficients of the type:

$$u_{xx} + 2\,a\,u_x + b\,u = k\,u_t$$

was obtained. A three line difference equation approximation was used.

BRL 769 - "On the Numerical Solution of Parabolic Partial Differential Equations" - Werner Leutert - 1951.

8. Matrix Inversion

Inverses of matrices of orders 5 through 9 using partitioning have been obtained on the Eniac. More recently investigations on the Edvac and Ordvac have been successful in inverting matrices of higher order.

9. Maddida Analogue

Sin x and cos x were generated by a method analogous to that used by Maddida in order to determine the accuracy to be expected by that computer. The equations solved were $y' = z$ and $z' = -y$ using 0.02, 0.002, and 0.0002 as intervals for the independent variable. Two Eniac accumulators were programmed similar to the "Y" and "R" registers of the Maddida. Results showed that, using the smallest interval size, four significant figures could be obtained. The study was suggested by J. von Neumann.

10. Characteristic Trend Analysis

The special matrix

$$a_{ij} = (-1)^{n-(i-j)} \binom{2r}{r-(i-j)}$$

was diagonalized for $r = 2$ and $n = 1(1)\,34$ as part of a problem in characteristic trend analysis. More recently the Edvac has solved the same problem for $r = 3(1)\,10$ for the same values of n.

BRL 814 - "The Characteristic Roots of Certain Matrices" - B. Dimsdale and I. R. Hershner - 1952.

The total machine time spent on this group of 10 problems was of the order of 400 machine hours.

11. Miscellaneous Topics

Other mathematical techniques were studied for the Eniac, but were not directly connected with any particular machine program.

a. BRL 613 - "A Study of Fourth Order Interpolation on the Eniac" - H. B. Curry and M. Lotkin - 1946.

b. BRL 632 - "Inversion on the Eniac Using Osculatory Interpolation" - M. Lotkin - 1947.

c. BRL 615 - "A Study of Inverse Interpolation on the Eniac" - H. B. Curry and W. Wyatt - 1946.

II. Exterior Ballistic Laboratory

Problems for the Exterior Ballistic Laboratory have been of two general classes: A. - Reductions of data from various ranges, and B. - Theoretical investigations in supersonic flow and related problems.

A. Reduction of Range Data

1. Interferometric Reductions

This reduction program determines density ratios of the air surrounding free flight missiles by the reduction of measurements from interferograms obtaining solutions of the integral equation relating air density within the disturbance to the fringe shift on the interferogram.

BRL 797 - "Interferometric Analysis of Airflow about Projectiles in Free Flight" - F. D. Bennett, W. C. Carter and V. E. Bergdolt - 1952.

2. Yaw and Swerve Reductions

The yaw reduction determines values of ten parameters of the yawing motion of a free flight missile by means of an iterative differential corrections least square procedure. The swerve reduction determines eight parameters by a direct least square technique. All data that has been reduced was obtained from photographs taken of missiles in flight in the Aerodynamics and Transonic Firing Ranges at BRL.

This problem has been more successfully handled by the Bell Computers.

M.I.T. Department of Electrical Engineering, Center of Analysis, Technical Report No. 4 - "A Manual of Reduction of Spinner Rocket Shadowgrams" - Z. Kopal, K. E. Kavanagh, and N. K. Rodier - 1949.

B. Theoretical Investigations in Supersonic Flow and Related Problems

1. Supersonic Airflow

Several variations of the problem of a theoretical treatment of supersonic airflow about bodies of revolution with and without rotation have been completed. These problems deal primarily with obtaining solutions of systems of hyperbolic partial differential equations. A three point Aitken interpolation has been applied to portions of the Eniac results in order to obtain stream data for various Mach numbers not directly computed on the machine. The following large group of references indicate the scope of the work:

a. MTAC III No. 23 July 1948 - "Computation of the Airflow about a Cone Cylinder" - M. Lotkin.

b. MTAC III No. 23 July 1948 - "Airflow Problem Planned for the Eniac" - R. F. Clippinger.

c. BRL 719 - "Supersonic Flow Over Bodies of Revolution (With Special Reference to High Speed Computing)" - R. F. Clippinger and N. Gerber - 1950.

d. BRL 729 and BRL 730 - "Tables of Supersonic Flows About Cone Cylinders; Part I: Surface Flows; Part II: Complete Flows" - R. F. Clippinger, J. H. Giese and W. C. Carter - 1950.

e. BRLM 514 - "Theoretical Supersonic Pressure Distributions on Non-Yawing Cone Cylinders with Boattails" - W. C. Carter - 1950.

f. BRL 796 - "Supersonic Flow with Vorticity About a Slightly Yawing Body of Revolution" - W. C. Carter - 1952.

g. BRL 813 - "On the Numerical Solution of Hyperbolic Systems of Partial Differential Equations with Two Characteristic Directions" - W. C. Carter and G. I. Spencer - 1952.

2. Supersonic Taylor-Maccoll Flow Past Non-Yawing Cones

A system of three ordinary differential equations was solved to determine the Taylor-Maccoll flow past a non-yawing cone. The solution was obtained as a characteristic curve with the components of the velocity of air along the curve. The density and pressure of the air were determined from the velocity and terminal conditions and these density distributions and interferometer fringe shifts were calculated with $\gamma = 1.4$ for cones with semi-vertex angles of $5\,K^o$, $K = 2(1)8$, over the Mach number range of 1.3, 1.7, 2.0, 3.0, 4.0, and 5.0.

BRL 793 - "Densities and Interferometric Fringe Shifts for Taylor-Maccoll Flows" - J.H. Giese - 1952.

3. Prandtl-Meyer Flow

A table of values of density, temperature, and pressure ratios of supersonic Prandtl-Meyer Flow was computed. The equation:

$$\frac{d\,q^2}{d\,\theta} = \frac{q^2}{\sqrt{6\,q^2-1}}\,(2\ \sqrt{1-q^2})$$

was integrated by the method of Iterated Simpson for $\theta = 0^o\,(0.05)\,102.50^o$.

4. Mathieu's Equation

Solutions were obtained of the Mathieu Equation: $y'' + 1/4\,(n^2 + \gamma \cos x)\,y = 0$ for two sets of initial conditions:

a) $y = 1,\ y' = 0$

b) $y = 0,\ y' = 1$

The method of Runge-Kutta was used to integrate the equation from $x = 0$ to $x = \pi$ listing values at 0.1 radian intervals.

The range of parameters follow:

n^2 : 0.2 (0.2) 12.0; the interval in γ varied from 0.25 for small γ to 2.0 for large γ.

Listings of the results are available from the Computing Laboratory, BRL.

Earlier Mathieu Equation solutions were obtained by the Moore School of Electrical Engineering of the University of Pennsylvania.

5. Two-dimensional Flexible Throat Wind Tunnel Nozzles

The problem had as its object the determination of two-dimensional nozzle shapes for a flexible throat wind tunnel. The results were used to determine cam shapes for an automatic jack control system for test section Mach numbers ranging from 1.25 (0.25) 5.50. The problem involves selection of suitable combinations of three parameters to produce nozzles which have the prescribed test section Mach numbers and which satisfy three additional conditions imposed by the physical characteristics of the tunnel and by the desire to have reasonable cam shapes. Since the choice could not be determined readily by theoretical analysis, a trial and error procedure, requiring several computations for each nozzle, was employed.

6. Shock Curvature vs Nose Curvature

An extension of some work by C. C. Lin relating to a comparison of shock curvature and body curvature about the nose of pointed objects in supersonic flow.

III. Interior Ballistic Laboratory*

A. Interior Ballistic Trajectories

Various Interior Ballistic Tables for powder of constant burning surface based on an interior ballistic system described by Laidler and Hershey have been computed. The integration methods of Kutta-Simpson and Runge-Kutta have been utilized for various areas of the tables. Extensive additional computations have recently been completed on the Ordvac.

BRLM 552 - "Interior Ballistic Integrations for Powder of Constant Burning Surface" - S. Kravitz - 1951.

B. Mathematical Theory of Thermal Ignition

A theoretical investigation of ignition and burning of powder by obtaining solutions of a non-linear parabolic partial differential equation using six grid sizes to aid in analyzing error.

$$u_T = u_{\xi\xi} + e^{-1/u}$$

in the semi-infinite region $\xi > 0$.

* See also VI, B, 1

The boundary conditions are

$$u_{\xi} = -H(u\,g - u), \ \xi = 0, \ T = 0$$

$$u \to U \quad \text{as} \quad \xi \to \infty$$

Initial conditions are $u = U$, $\xi \geq 0$, and $T = 0$.

BRL 756 - "Mathematical Theory of the Ignition Process Considered as a Thermal Reaction" - B. L. Hicks, J. W. Kelso and J. Davis - 1951.

C. Crystal Structure

A numerical method for the determination of the Crystal Structure of Banfield's or Kenyon's free radical was investigated. The numerical problem was to determine a sum of 22 products of an expression involving sines and cosines of the form $\cos 2\pi H x_i \cdot \cos 2\pi K y_i \cdot \sin 2\pi L z_i$ where H, K, and L are positive integers and x_i, y_i, z_i are the space coordinates of the 22 carbon, nitrogen and oxygen atoms in Kenyon's radical obtained by the methods of crystallography.

There were 4 sums for each set of the integers, the exact form of the product being determined by whether K is odd or even. In all, 819 cases involving the computation of over 100,000 sines or cosines were completed. The problem was also used as a trial problem to gain experience with the operation of the Univac of the Eckert-Mauchly Computer Corporation.

IV. Ballistic Measurements Laboratory

The main problem handled for BML has been the reduction of Doppler Data from three or five simultaneous radio-Doppler observations. The program has been used to reduce data for about 20 firings to date and determines:

A. rectangular coordinates and probable error,

B. velocity and acceleration by numerical differentiation, and

C. reduction of rectangular coordinates to transverse mercator coordinates of the missile.

The method of least squares is used. The three station problem is discussed in the first reference, the five station problem in the second, and additional information is contained in the third.

1. BRL 638 - "Doppler Determination of Position" - Boris Garfinkel - 1947.

2. BRL 707 - "Least Square Determination of Position from Radio Doppler Data" - Boris Garfinkel - 1949.

3. MTAC III No. 25 January 1949 - "A Comparison of Various Computing Machines Used in the Reduction of Doppler Observations" - Dorrit Hoffleit.

V. Surveillance Laboratory

Although several problems were originally initiated by this group including the Characteristic Trend Analysis Problem, the only one we shall discuss in this section is one in probability distribution. This problem had to do with the probability distribution of the difference between the largest observation and the sample mean (or the difference between the sample mean and the smallest observation) for normally distributed random variates. The table of probabilities for these statistics is very useful in determining whether the greatest or smallest observations in a sample of n observations can be judged "outlying" observations. For normally distributed variables which are ranked after the sample is drawn, i. e. into the form $u_1 \leq u_2 \leq \dots \leq u_n$, then the functions H_n (nk) give the probability that the extreme deviation from the sample mean, i.e. $u_n - \bar{u}$ or $\bar{u} - u_1$, will not exceed k times the standard deviation of the normal population from which the random sample of size n was drawn. The functions H_n (k) are defined below and were computed on the Eniac for n = 2(1) 25

$$H_1(x) = 1$$

$$\vdots$$

$$H_n(x) = \sqrt{\frac{n}{n-1}} \int_0^x \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-t^2}{2n(n-1)}\right) H_{n-1}(t)dt$$

Tables of the above functions which have use not only for the problem of outlying observations but also for the more general field of Order Statistics have been published in the March 1950 issue of the Annals of Mathematical Statistics, pp 27-58. "Sample Criteria for Testing Outlying Observations" - Frank E. Grubbs.

VI. Outside Groups

A. Atomic Energy Commission

Eight problems have been completed for the A. E. C. The information given is limited to unclassified reports published by the individual sponsors of the particular problems.

1. Calculations in the Liquid-Drop Model of Fission

The liquid drop model of fission as developed by Bohr, Wheeler, and others was investigated. Classical deformation energies were calculated for many shapes not all near spherical. The shapes considered were axially and primarily, but not exclusively, bilaterally symmetric, and single valued in polar representations. Agreement was obtained with observed fission thresholds and spontaneous fission rates obtained.

"Calculations in the Liquid-Drop Model of Fission" - S. Frankel and N. Metropolis - Physical Review Vol. 72 No. 10, Nov. 1947, pp 914-925.

2. Fermi-Thomas-Dirac Equation

Solutions of the Fermi-Thomas-Dirac equation:

$$\frac{d^2 \psi}{d x^2} = x (\epsilon + \psi^{1/2} \; x^{-1/2})^3$$

where $\epsilon^3 = 3 \cdot 2^{-5} \cdot \pi^{-2} \; z^{-2}$

were obtained in terms of the variable $w = (2x)^{1/2} \cdot 2 \psi$ is tabulated to 5 decimal places at intervals of 0.08 extending until ψ becomes negative. The parameter z takes on 24 values 6(4) 14, 16, 18(4) 26, 29(4) 81, 84(4) 92 and there are 8 different initial slopes ψ'.

"Solutions of the Fermi-Thomas-Dirac Equation" - N. Metropolis and J. R. Reitz, Journal of Chemical Physics Vol. 19, 1951, pp. 555-573.

3. Table of Atomic Masses - Program run on card control.

"Table of Atomic Masses" published by Argonne National Laboratory - March 1950.

4. Miscellaneous Problems

The following references indicate the scope of further unclassified work:

a. Seminar on Scientific Computation - Nov. 1949 - IBM Corp. "Modification of the Monte Carlo Technique" - H. Kahn.

b. Proceedings of the Monte Carlo Conference - "Estimation of Particle Transmission Through Plane Slabs" - T. E. Harris and H. Kahn.

c. Journal of Applied Physics Vol. 21 No. 3, pp 232-237 - March 1950 - "A Method for the Numerical Calculation of Hydrodynamic Shocks" - J. von Neumann and R. D. Richtmyer.

d. Nat. Bur. Stand. Appl. Math. Ser. No. 12, 36-38 (1951) - "Various Techniques Used in Connection with Random Digits" - J. von Neumann. This is a paper presented at the Monte Carlo Method Symposium describing a method of producing random numbers according to an assigned probability law on the Eniac, using random numbers from a rectangular distribution in the interval (0,1).

e. Nat. Bur. Stand. Appl. Math. Ser. No. 12, 19-20 (1951) - "Report on a Monte Carlo Calculation Performed with the Eniac" - M. Mayer. Here the absorption of neutrons in a composite cylindrical body was investigated.

B. U. S. Bureau of Mines

1. Equilibrium Composition and Thermodynamic Properties

This problem has as its object the determination of the mole fractions of the gaseous constituents of the equilibrium composition of the four component system of hydrogen, oxygen, nitrogen, and carbon at temperatures of 1000° K to 5000° K and pressures of 0.1 atmospheres to those of interior ballistics. In addition various thermodynamic properties have been evaluated. Approximately 7% of the Eniac's time has been spent on this problem and the work is continuing.

a. Chemical and Engineering News, American Chemical Society, Vol. 27, 5 Sept. 1949. "Combustion Gases ... Equilibrium Composition and Thermodynamic Properties" - Stuart R. Brinkley, Jr. and Bernard Lewis.

b. Bureau of Mines Report of Investigations 4806 - "The Thermodynamics of Combustion Gases: General Considerations" - Stuart R. Brinkley, Jr. and Bernard Lewis - U.S. Department of the Interior - April 1952.

2. Optimum Spacing and Performance of Gas Wells

The non-linear parabolic equation:

$$\frac{\partial p}{\partial t} = e^{-2u} \frac{\partial^2 (p^2)}{\partial u^2}$$

was solved for specific boundary conditions relating to gas well performance and optimum spacing of such wells. A single solution has been obtained using a six point finite difference procedure. Additional machine studies are contemplated on Edvac and Ordvac.

3. Well Depletion

A preliminary study was made of the rate of withdrawal of oil from a well consisting of two pressure areas. The method of Runge-Kutta was used to integrate three ordinary differential equations in one independent variable. Additional computations have been completed on the Edvac.

C. Universities (including Government Contractors)

1. Compressible Laminar Boundary Layer (University of Cambridge)

An examination was conducted by D. R. Hartree of compressible laminar boundary layer flow for the case of a flat plate at zero incidence.

The mathematical method is described in the Philosophical Transactions of the Royal Society of London - Series A - Math and Physical Sciences No. 827, Vol. 241, pp 1-69, 22 June 1948 - "The Laminar Boundary Layer in Compressible Flow" - W. F. Cope and D. R. Hartree.

A briefer discussion is given in MTAC III No. 23, July 1948 - "Laminar Boundary Layer Flow in a Compressible Fluid" - J. V. Holberton.

2. Zero - Pressure Properties of Diatomic Gases - (University of Pennsylvania)

J. A. Goff, Dean of the Towne Scientific School, determined the zero-pressure properties of certain diatomic gases using the equations:

$$\left(\frac{h-\bar{u}}{R\,T}\right)_{p\,=\,o} = 3.5 + B/\alpha$$

$$\left(\frac{C_p}{R}\right)_{p\,=\,o} = 3.5 + \frac{\gamma}{\alpha} - (B/\alpha)^2$$

where $B = T\,\frac{d\,\alpha}{d\,T}$ and $\gamma = T^2 \cdot \frac{d^2\,\alpha}{d\,T^2}$

The computations were based on the best available spectroscopic data.

An additional problem for the University of Pennsylvania is discussed in I, C, 1.

3. Refraction of Plane Shock Waves (Institute for Advanced Study)

Numerical solutions were obtained of the equations governing the position of the various relevant quantities in an assumed shock configuration for the refraction of plane shocks. The assumed shock configuration consisted of an incident shock, a reflected shock, and a transmitted one. It was further assumed that the pressure is constant in the angular domains between these shocks and that across the density discontinuity responsible for the refraction, the pressure is continuous as is the deflection of the flow.

a. "Reflection of Plane Shock Waves" - A. H. Taub, Physical Review, Vol. 72, pp 51-60 (1947).

b. "Some Numerical Results on Refraction of Plane Shocks" - A. H. Taub - University of Washington.

4. Axisymmetric Wind Tunnel Nozzle Design (M. I. T.)

Determination of axisymmetric wind tunnel nozzles solving a system of five first order partial differential equations.

BRL 794 - "Supersonic Axially Symmetric Nozzles" - R. F. Clippinger - 1952.

5. Mie Theory of Light Scattering Functions for Spherical Particles (University of Michigan)

Evaluation of electromagnetic wave scattering functions for spherical particles to be used in connection with experimental work in the examination of small particles according to the Mie Theory. Two books have appeared giving the Eniac results:

a. "Tables of Riccati Bessel Functions for Large Arguments and Order", and "Light Scattering Functions for Spherical Particles Part I, Part II", both by R. O. Gumprecht and C. M. Sliepcevich, Engineering Research Institute, University of Michigan, 1951.

b. Journal of the Optical Society of America, Vol 42, No. 4, pp 226-231, "Angular Distribution of Intensity of Light Scattered by Large Droplets of Water" - R. O. Gumprecht, Neng-Lun Sung, Jin H. Chin, and C. M. Sliepcevich - University of Michigan - April 1952.

6. Investigations in Weather Forecasting (Institute for Advanced Study)

A method was adapted to the Eniac for the numerical solution of the barotropic vorticity equation over a limited area of the earth's surface.

"Numerical Integration of the Barotropic Vorticity Equation" - J. G. Charney, R. Fjörtoft, J. von Neumann - Tellus, Vol. 2, No. 4 - November 1950.

The problem discussed in I, C, 3 was performed by D. H. Lehmer of the University of California.

D. Other Outside Agencies

1. Heat Flow (Armour Research Institute)

A group of 96 one dimensional heat flow problems with a single source and a new set of boundary conditions for each problem was computed. An error study and 16 of the one and two slab cases were successfully completed before the project was cancelled.

2. Guided Missile Trajectories (General Electric Co.)

Two problems in guided missile trajectory computations have been completed on the Eniac.

a. Power flight computation for the two dimensional case with cartesian coordinates, a time and position dependent control system, the moment equation, and a time and atmosphere pressure dependent thrust. The integration of all quantities, essentially a sixth order differential equation, was computed by the iteration of the trapezoidal rule.

b. Free flight trajectories for the two dimensional case with intrinsic coordinates, i. e. velocity and path angle, and a control system determined by a second order differential equation and by a time dependent signal. The integration of the three second order derivatives was computed by an iterated use of the trapezoidal rule and first derivatives by the Euler MacLaurin rule.

W. Barkley Fritz

W. Barkley Fritz

GENERAL REFERENCES

1. Report on the Eniac - Technical Report I, Volume I and Volume II.

2. BRLM 582 - "Description of the Eniac Converter Code" - W. Barkley Fritz - 1951.

3. BRL TN 104 - "Preparation of Problems for the BRL Calculating Machines" - Harrison, Holberton and Lotkin - 1949.

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* Index of authors of previously published reports mentioned in the text.

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