UNCLASSIFIED

AD NUMBER
AD093330
CLASSIFICATION CHANGES
TO: unclassified
FROM: secret
LIMITATION CHANGES
TO: Approved for public release, distribution unlimited
FROM: Controlling Organization: British Embassy, 3100 Massachusetts Avenue, NW, Washington, DC 20008.
AUTHORITY
DSTL, AVIA 6/17312, 5 Aug 2008; DSTL, AVIA 6/17312, 5 Aug 2008

THIS PAGE IS UNCLASSIFIED

Best
Available
Copy

AD No. 93330
ASTIA FILE COPY

TECH. NOTE
G.W.389

FC

TECH. NOTE
G.W.389

SECRET

ROYAL AIRCRAFT ESTABLISHMENT

FARNBOROUGH, HANTS

TECHNICAL NOTE No: G.W.389

WEIGHT ESTIMATES FOR LONG-RANGE, SURFACE-TO-AIR GUIDED MISSILES

by

D.G.KING-HELE and H. HILLER

CONDITIONS OF RELEASE

1. THIS INFORMATION IS DISCLOSED [illegible] OFFICIAL USE BY THE RECIPIENT [illegible] AND [illegible] CONTRACTORS, UNDER [illegible] SECRECY, AS MAY BE ENGAGED ON A DEFENCE PROJECT. DISCLOSURE TO ANY OTHER GOVERNMENT OR RELEASE TO THE PRESS OR IN ANY OTHER WAY WOULD BE A BREACH OF THESE CONDITIONS.

2. THE INFORMATION SHOULD BE SAFEGUARDED UNDER RULES DESIGNED TO GIVE THE SAME STANDARD OF SECURITY AS THAT MAINTAINED BY HIS MAJESTY'S GOVERNMENT IN THE UNITED KINGDOM.

NOVEMBER, 1955

MINISTRY OF SUPPLY

THIS DOCUMENT IS THE PROPERTY OF H M GOVERNMENT AND ATTENTION IS CALLED TO THE PENALTIES ATTACHING TO ANY INFRINGEMENT OF THE OFFICIAL SECRETS ACT, 1911-1939

It is intended for the use of the recipient only, and for communication to such officers under him as may require to be acquainted with its contents in the course of their duties. The officers exercising this power of communication are responsible that such information is imparted with due caution and reserve. Any person other than the authorised holder, upon obtaining possession of this document, by finding or otherwise, should forward it, together with his name and address, in a closed envelope to:-

THE SECRETARY, MINISTRY OF SUPPLY, LONDON, W.C.2

Letter postage need not be prepaid, other postage will be refunded. All persons are hereby warned that the unauthorised retention or destruction of this document is an offence against the Official Secrets Act.

THE RECIPIENT IS WARNED THAT INFORMATION CONTAINED IN THIS DOCUMENT MAY BE SUBJECT TO PRIVATELY OWNED RIGHTS.

Control
560135
Date 24 APR 1956

SECRET

MAY 21

56AA 19947-SD-1-6

NOTICE: THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 and 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW.

U.D.C. No. 623.451-519 : 621.454.018 : 531.565

Technical Note No. GW 389

November, 1955

ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

Weight estimates for long-range surface-to-air guided missiles

by

D. G. King-Hele

and

H. Hiller

SUMMARY

Weight estimates are made for high-altitude surface-to-air missiles having ranges between 30 and 200 n.miles, and the effects of eleven design parameters are investigated. Propulsion is by ramjet, and guidance is tacitly assumed to include a mid-course phase, followed by radar homing in the terminal phase. The 'standard' missile, after rocket boost to $M = 2$ at sea level, climbs under ramjet power, steeply at first and then more gently in the stratosphere, so that it reaches its design altitude of 70,000 ft at a Mach number M of 3 and a ground range of about 25 n.miles. This standard missile carries a payload (warhead + guidance) of 700 lb, develops a maximum lateral acceleration of 8g at design altitude, and is assumed to suffer an r.m.s. lateral acceleration of 2g in its mid-course flight. The estimated weight of the missile for 100 n.miles range at 70,000 ft altitude is about 1900 lb without boosts (see Fig.1 for sketch). Estimates are made of the changes in weight resulting from changes in design altitude, range, missile diameter, payload weight, payload density, maximum lateral acceleration, r.m.s. lateral acceleration, boost Mach number, propulsion and layout (Figs.8-17). The propulsion range of the standard missile (designed for 70,000 ft altitude) on various trajectories - up-and-along, up-along-down-along, and beam-riding, to target altitudes between 36,000 and 70,000 ft - is also given (Figs.21-29).

56AA 19947-SD-1-6

LIST OF CONTENTS

		Page
1	Introduction	6
2	Range and design altitude	7
3	Guidance and warhead	7
4	Propulsion, speed and trajectory	8
	4.1 Choice of propulsion	8
	4.2 Missile speed	9
	4.3 Choice of engine design parameters	10
	4.4 Choice of climb path	11
5	Layout	12
6	Method	13
	6.1 General scheme	13
	6.2 Assumptions	13
	6.3 Drag and lift	14
	6.4 Procedure for estimating all-up weight, m_0	14
	6.5 Modifications of method for non-standard trajectories	15
	6.6 Boost weight	16
	6.7 Accuracy check of assumptions	16
7	Discussion of results	16
	7.1 Design altitude and range	16
	7.2 Missile body diameter, d	16
	7.3 Payload weight and density	17
	7.4 Maximum and r.m.s. lateral accelerations, Ng and ng	17
	7.5 Mach number at the end of boost	17
	7.6 Propulsion	17
	7.7 Fixed and moving wings	17
	7.8 Missile fuel/weight ratio on standard trajectory	19
	7.9 Dimensions	19
	7.10 Trajectory	19
8	Summary of results, and conclusions	20
Acknowledgement		21
References		21
Advance Distribution		22
Appendix -	Method of calculating missile speed and fuel consumption during climb	23
Detachable Abstract Cards		-

LIST OF TABLES

I	The eleven design parameters and their values	6
II	Variation of missile weight when each of the eleven design parameters departs from its standard value	28
III	The 'atmosphere functions' H and I defined by equations (7)	24

LIST OF ILLUSTRATIONS

	Figure
Standard missile of 87 n.miles range	1
Variation of weight with range for rocket, ramjet and turbojet surface-to-air missiles	2
Cover diagram, showing the effect of missile/target speed ratio on the area within which targets can be intercepted	3
Thrust coefficient and fuel consumption of ramjet motor as a function of Mach number and altitude	4
Variation of ramjet thrust and fuel consumption in the stratosphere with Mach number and fuel/air ratio	5
Variation of thrust and drag of the missile of Fig.1 with Mach number and altitude	6
Variation of maximum lateral acceleration of the missile of Fig.1 with Mach number and altitude	7
Variation of missile weight with range, for a series of design altitudes	8
Variation of missile weight with design altitude for a series of ranges	9
Variation of missile weight with diameter	10
Variation of missile weight with payload weight and density (payload = warhead plus guidance)	11
Variation of missile weight with maximum lateral acceleration for a series of r.m.s. lateral accelerations	12
Variation of missile weight with r.m.s. lateral acceleration for a series of maximum lateral accelerations	13
Variation of r.m.s. lateral acceleration with maximum lateral acceleration for a series of missile weights	14
Weight at launch, with and without boosts, as a function of Mach number at end of boost for a series of ranges	15
Variation of weight with range for turbojet, rocket and ramjet missiles	16
Variation of weight with range for moving-wing and fixed-wing missiles	17
Variation of fuel/all-up weight ratio with range on up-and-along trajectories for a series of missile design altitudes	18
Variation of ramjet combustion chamber diameter when each of the design parameters shown departs in turn from its standard value	19
Variation of missile length when each of the design parameters shown departs in turn from its standard value	20
Variation of weight with range for standard missiles (designed for 70,000 ft altitude) on up-and-along trajectories to various target altitudes	21

LIST OF ILLUSTRATIONS (Contd)

	Figure
Variation of missile weight with target altitude for standard missiles (designed for 70,000 ft altitude) on up-and-along trajectories, for a series of ranges	22
Variation of range with target altitude for standard missiles (designed for 70,000 ft altitude) on up-and-along trajectories, for a series of missile weights	23
Variation of weight with range for standard missiles (designed for 70,000 ft altitude) on up-along-down-along trajectories, with 10 n.miles flight at target altitude	24
Variation of weight with range for standard missiles (designed for 70,000 ft altitude) on up-along-down-along trajectories, with 20 n.miles flight at target altitude	25
Variation of range with missile cruising altitude for standard missiles (designed for 70,000 ft altitude) on up-along-down-along trajectories, for a series of missile weights and target altitudes	26
Beam-riding trajectories over flat earth. Target speed 0.32 nm/sec ($M \simeq 2$). Missile speed 0.4 nm/sec for altitude $y < 6$ nm, 0.45 nm/sec for $y > 6$ nm	27
Variation of weight with range for standard missiles (designed for 70,000 ft altitude) on beam-riding trajectories to various target altitudes	28
Variation of weight with range for standard missiles (designed for 70,000 ft altitude) on various trajectories to various target altitudes y_T	29
Diagram of (A) up-and-along, (B) beam-riding and (C) up-along-down-along trajectories to target altitudes of 36,000 and 70,000 ft	30
Variation of Mach number, net thrust parameter, and fuel flow with altitude for the missile of Fig.1 on its standard climb path	31
Variation of the 'atmosphere functions', H and I, with altitude y	32
Variation of missile flight Mach number with altitude for straight-line climbs from sea level to tropopause	33
Mach number at the tropopause, M^*, as a function of net thrust/weight ratios at sea level (F_0) and tropopause (F^*), for straight-line climbs	34
Variation of net thrust/weight ratio at tropopause (F^*) with that at sea level (F_0) for straight-line climbs, showing effect of Mach number at the tropopause (M^*)	35
Variation of angle of climb with F^* for straight-line climbs in the stratosphere. Mach number M^* at the tropopause. $M = 3$ at design altitude y_1	36
Variation of Mach number with altitude on straight-line climbs in the stratosphere from the tropopause to design altitude y_1. $M = 3$ at design altitude	37

LIST OF ILLUSTRATIONS (Contd)

	Figure
Ramjet fuel consumption parameters as functions of Mach number and altitude	38
Beam-riding trajectories for ranges at launch of 80 and 200 n.miles at target altitudes of 36,000 and 70,000 ft	39
$\frac{\text{Lift developed by fixed-wing missile}}{\text{Lift developed by moving-wing missile}}$ for given missile configuration for 25° incidence	40
Comparison of fixed-wing and moving-wing missiles	41
Possible boost layout for standard missile	42

1 Introduction

The British surface-to-air guided weapons now being developed were designed primarily to meet the threat from subsonic bombers flying at altitudes up to 50,000 ft and dropping conventional bombs. These 'Stage 1' missiles have maximum ranges of about 20 n.miles and their manoeuvrability is inadequate to deal with evading targets at altitudes substantially above 50,000 ft. The need to extend these altitudes and ranges has long been apparent and attention has now turned towards missiles designed for interception at ranges of about 100 n.miles and altitudes up to about 70,000 ft.

Single-stage guidance systems, as used in the Stage 1 surface-to-air weapons, will be unable to meet the new interception range requirements, even when stretched to the limit of technical possibility. Guidance must therefore be divided into two phases, a mid-course phase (of as yet unspecified type), which brings the missile near enough to the target to carry out a terminal radar homing phase. The homing may be either semi-active or active, these two alternatives being associated respectively with the 'Stage 1½' and 'Stage 2' defence systems.

Because of the present uncertainties about guidance and about target behaviour, the effects of eleven design parameters were investigated in this Note. To keep the work within reasonable bounds standard values were chosen for each of these parameters and each was then varied in turn. The parameters are listed, together with standard values and the ranges of values covered, in Table I.

Table I

The eleven design parameters and their values

Parameter	Symbol	Unit	Standard value	Range of values
Design altitude	y_1	ft	70,000	50,000 - 80,000
Range	x	n.miles	87(or 100)	30 - 200
Missile body diameter	d	ft	2	1.6 - 3
Payload (warhead + guidance)		lb	700	500 - 1200
Payload density		lb/cu ft	52	25 - 100
Max. available lateral acceleration	N	g	8	5 - 12
Steady r.m.s. lateral acceleration	n	g	2	1 - 4
Mach No. at end of boost	M_0	-	2	1.7 - 2.2
	Standard type		Variants	
Propulsion	ramjet		turbojet and rocket	
Layout	fixed-wing		moving-wing	
Trajectory	up-and-along		up-along-down-along. Beam-riding	

The standard missile, powered by ramjet and boosted to $M = 2$ as Table I implies, climbs at about 75° to the horizontal as far as 36,000 ft altitude and then more gently, at a mean angle of about 15°, to its design altitude of 70,000 ft which it reaches at its design speed of $M = 3$ and at a range of 23 n.miles.

2 Range and design altitude

The expected threat, which is little better than a guess at present, must determine the choice of missile range and design altitude. So a fairly wide bracket of values of both range and design altitude had to be covered here. Some upper limits can however be set. The early-warning range will not normally exceed 250 miles and since the target may fly as fast as the missile, a missile range greater than about 150 miles is not often likely to be useful, even if the missile launching area is 50 miles behind the early-warning station. 100 n.miles has therefore been taken as a convenient round figure for missile range, and ranges up to 200 n.miles (or sometimes 400) have been covered. As ref.4 shows, missile weight tends to become excessive if 8g lateral acceleration has to be produced at altitudes above 80,000 ft by aerodynamic means. 80,000 ft has therefore been taken as an upper limit for design altitude. (Design altitude is defined as the maximum altitude at which the design lateral acceleration, here usually 8g, can be developed.) The altitude for which the Stage 1 missiles were designed, 50,000 ft, has been taken as the lower limit for design altitude. It is thought that the fastest targets likely are M = 2.5 to 3, and for these the optimum operating altitude will probably be about 60-65,000 ft. They may however fly for short periods at up to 5-10,000 ft higher. So 70,000 ft has been taken as the standard design altitude.

3 Guidance and warhead

As stated in the Introduction it is assumed that missile guidance consists of a mid-course phase, operating from launch until the missile is near enough to the target to lock-on its homing head, and a terminal homing phase. It is further assumed that for active terminal homing a common transmitting and receiving aerial can be used.

The form of mid-guidance, as yet unsettled, affects three of the missile design parameters, namely:

(1) Weight of guidance equipment. To quote two of the possibilities: inertia mid-course guidance might weigh about 200 lb; beam-riding perhaps 150 lb.

(2) Trajectory. If target altitude were known it would no doubt be possible for the missile to fly on an up-and-along trajectory or up-along-down-along (i.e. climb to optimum cruising altitude; level flight there; descent to target altitude; 10-20 miles level flight there). In the absence of information about target altitude, which might be denied by jamming, the missile might have to fly on a line-of-sight trajectory, which would lead to much higher fuel consumption. All three types of trajectory have been investigated here.

(3) Mean lateral acceleration demanded during the mid-course phase. This will depend on (a) the method of guidance (e.g. beam-riding or proportional navigation), (b) the limit chosen for missile lateral acceleration during mid-course (this limit could probably be much lower than the maximum available), and (c) the time interval between the commands to the missile to change course. Here an r.m.s. lateral acceleration of 2g has been taken as standard and values between 1g and 4g have been covered.

Errors in missile position and heading at the end of mid-course guidance, again not yet known at all accurately, affect the required homing-head lock-on range, which in turn depends on dish diameter. (Ref.6 studies the effects of homing range and mid-course errors on interception probability.) Here the standard missile diameter has been taken as 2 ft, corresponding to a dish diameter of perhaps 20 inches, and missile diameters between 1.6 ft and 3 ft have been considered.

Two other design parameters, the maximum lateral acceleration and the warhead weight, depend on the expected behaviour of missile and target in the final phase (assuming missile manoeuvrability is adequate to correct mid-course errors). The values which ought to be chosen for these two parameters remain uncertain, if only because the evasive manoeuvre of the target, and its radar-reflecting and structural properties are unknown. For maximum lateral acceleration, 8g has here been taken as standard, values between 5g and 12g being covered.

To deal with the uncertainties about warhead and guidance weights, payloads* between 500 and 1200 lb and payload densities between 25 and 100 lb/cu ft have been covered. The standard payload is 700 lb, and the standard density is 52 lb/cu ft, implying roughly equal division of the 700 lb between guidance and warhead.

4 Propulsion, speed and trajectory

4.1 Choice of propulsion

References 1-5 provide comparable weight estimates for surface-to-air missiles powered by rocket, ramjet or turbojet engines. From these References it is possible to obtain, either directly or with minor modifications, weight-versus-range curves for missiles carrying a payload of 340 lb, having a design Mach number of 2 and a maximum lateral acceleration of about 10g, flying on either up-and-along trajectories to 45,000 ft altitude (refs.1-3) or beam-riding trajectories to 65,000 ft altitude (ref.4). These comparable curves are plotted in Fig.2, the full lines referring to up-and-along trajectories to 45,000 ft and the broken lines to beam-riding trajectories to 65,000 ft.

Fig.2 shows that, for ranges greater than 30 miles, ramjet missiles are considerably lighter than rockets for 45,000 ft design altitude, and that this superiority increases under the severer conditions of beam-riding to 65,000 ft. Also, for ranges up to about 150 n.miles, ramjet missiles are appreciably lighter than turbojet missiles for 45,000 ft altitude - though it should be emphasized that quite a small reduction in turbojet specific weight could cancel out this advantage.

When speed is increased above $M = 2$, ramjet efficiency improves relative to turbojet and rocket. In the missiles of this Note therefore, where speeds up to $M = 3$ are used, the rocket is not likely to show to advantage. From Fig.2 it would appear that the turbojet might be competitive with the ramjet at the longest ranges, although increasing speed to $M = 3$ and altitude to 70,000 ft will tilt the scales in favour of the ramjet. For at $M = 3$ the turbojet's fuel consumption is only a little less than the ramjet's and the higher engine weight leads to higher empty weight and hence greater wing and control surface area to maintain a given lateral acceleration and control time-lag. A few weight estimates have been made for rocket and turbojet missiles to give some idea of the weight increases which would occur, but most of the results are for ramjet missiles.

A fixed-geometry ramjet motor has been assumed because it was thought that the extra weight of a variable intake or exit nozzle would be greater than the fuel saved.

---

* Payload is defined as warhead (including fuse) plus guidance (including guidance power supplies, but excluding radome).

4.2 Missile speed

4.2.1 Design speed

The missile design Mach number M_1 (i.e. the Mach number in level flight at design altitude, 70,000 ft) has here been taken as 3. The arguments which point towards this value are discussed below under four headings.

(a) Aerodynamic lift

The wing area needed to produce a given lateral acceleration, and the control surface area needed to secure a given control time-lag both decrease as speed increases. As a result there would be a sharp reduction in missile weight as M_1 was increased from 2 to 3 (other things being equal), though little would be saved by going above $M_1 = 3$ because the wings and controls would then already be fairly small. If motors, fuel and body structure formed a fixed percentage of all-up weight, 50%, the missile weights for $M_1 = 2$, 3 and 3.5 would be about 2400 lb, 1850 lb and 1750 lb respectively for a payload of 700 lb. If turning circle, and not lateral acceleration, were the quantity which ought to be kept constant as speed increases, these weight differences would be less.

(b) Engine performance

Boost weight tends to become excessive if a Mach number much greater than 2 is demanded at boost separation. The ramjet motors should therefore be capable of accelerating the missile from a Mach number near 2 to its design Mach number. Now it is difficult to design a fixed-geometry ramjet with good performance over a wide range of Mach number: with the lowest M near 2, $M = 3.5$ can probably be regarded as an upper limit if a reasonably efficient design is sought. No advantage is gained, however, by going above $M = 3$, for with a simple conical-centrebody intake, specific fuel consumption tends to fall as design Mach number increases from 2 to 3 and then to rise again for $M_1 > 3$ (see e.g. Fig.1(a) of ref.8).

(c) Heating

Aerodynamic heating becomes much more severe as speed is increased between $M = 2.5$ and 3.5. The equilibrium boundary-layer temperatures at $M = 2.5$, 3 and 3.5 are about 190°C, 280°C and 390°C respectively. The missile skin, though it would only locally reach these temperatures during the comparatively short time of flight, would probably have to be of steel for $M = 3$; and materials for the radome present a more serious and as yet unsolved problem. Increasing speed from $M = 3$ to $M = 3.5$ would seriously aggravate the difficulties.

(d) Defence tactics

Tactically, high speed is an advantage. For a given guidance range interceptions can be made further from the launcher and a single group of launchers can defend a wider front. Fig.3 shows the areas within which interceptions can be made, for various values of missile/target speed ratio V_M/V_T. A missile boosted to $M = 2$ might have a mean ground speed of 2600 ft/sec if its design Mach number M_1 were 3, or 2200 ft/sec if $M_1 = 2.5$, i.e. against a 2500 ft/sec target $V_M/V_T = 1.04$ and 0.88 in the two cases. So reducing M_1 from 3 to 2.5 reduces the lateral cover 30 miles ahead of the launcher from about 160 to about 125 miles (see Fig.3). The higher the missile speed too, the less danger there is from feint attacks, when the enemy approaches the defences to draw their fire, and then turns and retreats.

Individually, these four arguments are not cogent enough to justify any precise choice of Mach number. Taken together, however, they seem to point to a value not far from 3 as the best compromise.

4.2.2 Boost speed

A similar compromise must be sought in choosing the Mach number at the end of boost, M_o. Since a ramjet can provide a given thrust at a much lower fuel consumption than a solid-fuel rocket for Mach numbers between 1.5 and 3, the acceleration in this phase can best be done by ramjet, provided of course that the thrust-drag margin at the lowest speeds is great enough. Heating problems too are eased if the Mach number is kept as low as possible during the early part of the climb, where, for given M, heating rates will be highest. Thus, if the ramjet motor design is fixed, it would seem best to find the Mach number $M_{o\ min}$ at which the motors can just accelerate the missile on some suitable climb path, and to choose for M_o a value very little above $M_{o\ min}$ - only enough to provide a safety margin. If the motor design is not fixed however, the question which arises is: "How far should the motor design be biassed to give low values of $M_{o\ min}$, i.e. high thrust at low Mach number?" Only a grossly quantized answer can be given. With a design Mach number of 3 it is certainly worth modifying ramjet design to give good thrust down to $M = 2.5$, since this can be done with little loss in thrust or increase in fuel consumption. It is probably worth further modifying the design to give good thrust down to $M = 2$, since the losses in design thrust etc. are not prohibitive and the saving in boost weight is large (about 0.75 x missile weight). It is almost certainly not worth striving for high thrust at $M = 1.5$, since (1) the ramjet is fundamentally less efficient at this Mach number, (2) a Mach number range of 1.5-3 is too wide for a fixed geometry ramjet, and (3) less is saved in boost weight in going from $M_c = 2$ to $M_o = 1.5$ (about 0.45 x missile weight).

Again, these arguments are by no means conclusive, but they do point towards a value near 2 for M_o. Here therefore the engine design was chosen with a boost Mach number of 2 in mind. The effects of changing M_o, while retaining the same engine, were afterwards investigated.

4.3 Choice of engine design parameters

In choosing the engine design characteristics the aim must be to produce a motor which develops just enough thrust both for level flight at design speed at design altitude, and for climb at lower speeds and altitudes. A preliminary estimate of the requirements for the missiles in this Note suggested that the engine would be well matched to the two requirements if its net thrust coefficient c_T were roughly the same at $M = 2$ and $M = 3$, thus implying that c_T should have a maximum somewhere between $M = 2$ and $M = 3$ while not dropping too far below this value at either $M = 2$ or $M = 3$. On an up-and-along trajectory to 70,000 ft altitude and 100 n.miles range, roughly $\frac{1}{3}$ of the fuel is used in the first half of the climb ($M = 2.0$ to 2.6), $\frac{1}{3}$ in the second half ($M = 2.6$ to 3.0), and $\frac{1}{3}$ in level flight ($M = 3$): so low specific fuel consumption is important throughout the speed range.

In response to a request framed on these rather vague lines, N.G.T.E., Pyestock chose an engine design and provided the curves of net thrust coefficient c_T and specific fuel consumption c plotted in Fig.4. The engine has a simple conical-centrebody intake, with cone angle 60°, and shock-on-lip at $M = 2.6$. After passing through the cylindrical combustion chamber the gas stream enters an exit nozzle with throat/inlet area ratio of 0.85, expanding again to combustion chamber area at the exit plane. On a typical missile trajectory with Mach number 2 at end of boost, 2.6 at the tropopause and 3 at design altitude, the values of c_T at the three corresponding points are 0.81, 1.08 and 0.79 based on combustion chamber cross-sectional area, while the three values of s.f.c. are 3.7, 3.1 and 3.4 lb/hr/lb thrust. These values are for standard atmospheric conditions.

When the missile has to fly level or in shallow climbs at altitudes much lower than its design altitude it is often desirable to reduce thrust to save

fuel and, by keeping down the speed, to minimize aerodynamic heating. Fig.5 shows how the ramjet thrust coefficient and specific fuel consumption in the stratosphere vary when fuel/air ratio is reduced. The most striking feature of Fig.5 is that for this particular engine the s.f.c. is hardly altered when the fuel/air ratio is reduced, i.e. that if the thrust can be cut by a certain percentage the fuel consumption is reduced by roughly the same percentage.

4.4 Choice of climb path

For the purposes of this Note it was decided to approximate to the missile climb path with two straight lines, one from sea level to 36,000 ft altitude, and the other from 36,000 ft to design altitude, usually 70,000 ft. The fuel consumption and the Mach number during flight can then be calculated with adequate accuracy by an analytical method (given in the Appendix), and it has been shown previously (see e.g. ref.9) that such dog-leg paths provide excellent approximations to more realistic paths with finite curvature.

The angles of climb appropriate for the two phases of climb, θ_1 and θ_2, must depend of course on the thrust-drag margins of the individual missiles. Fig.6 shows how the thrust and drag of the standard missile of Fig.1 vary with Mach number and altitude. Two points are worth observing in Fig.6. The first is that induced drag contributes over 40% of the total at 70,000 ft altitude; this means that at low altitudes, where the induced drag is very small, there is a substantial thrust-drag margin. The second point to note is that, with 2g r.m.s. lateral acceleration, the thrust-drag margin at 70,000 ft altitude is extremely small for $2.6 < M < 3$ (and negative for $M < 2.6$). Thus it would be unwise to rely on the missile accelerating to $M = 3$ during level flight at 70,000 ft. A better plan would be to choose a flight path such that the missile could accelerate at lower altitudes where the thrust margin is greater, and arrive at 70,000 ft with its Mach number already up at 3. This was the flight path used here, and the particular pair of values (θ_1, θ_2) chosen for the angles of climb was the one which gave minimum fuel consumption. For the standard missile of Fig.1 $\theta_1 = 75^\circ$ and $\theta_2 = 14.5^\circ$, and on this trajectory the Mach number increases almost linearly with altitude in the troposphere, from $M = 2$ at sea level to $M = 2.7$ at 36,000 ft, then in the stratosphere reaches a maximum of 3.1 near 60,000 ft altitude and drops to $M = 3$ at 70,000 ft (see Fig.31).

This standard flight path gives a reasonable minimum range, for the missile reaches 70,000 ft altitude at a range of 23 n.miles (see Fig.30), a mean climb angle of 27°. Round any missile launching site therefore there would be a circle of 23 miles radius, a 'dead area' within which no targets could be engaged at 70,000 ft altitude. This is not a serious gap in the defences, since targets need to be engaged long before they reach this area. If, however, a substantial reduction in minimum range below 23 miles were required, the missile thrust would have to be greatly augmented, with consequent increases in missile weight.

The standard flight path also gives a reasonably advantageous speed variation. Aerodynamic heating is less than it would be if θ_1 were smaller, and the Mach number greater, during the first half of the climb. If necessary, heating could be further mitigated by making the reduction in angle of climb at the tropopause less abrupt, i.e. replacing the 14.5° climb by two straight lines at angles of, say, 45° and 10°. The speeds during climb are such that the missile performance will not be unduly sensitive to small deficiencies or excesses in thrust. Net thrust (thrust minus drag) exceeds missile weight up to 40,000 ft altitude, and thrust/drag remains above 1.5 up to 55,000 ft altitude, for the standard missile of Fig.1.

When the design parameters depart from their standard values there are changes in the thrust-drag margins and hence in the appropriate angles of climb.

For most of the missiles these changes are small, and "a 75° climb followed by a 10-15° climb" gives a good picture of the climb path. When the missile design altitude is altered however, the net thrust/weight changes greatly and the corresponding pairs of values of (θ_1, θ_2) vary widely, between (75°, 35°) for 80,000 ft design altitude, and (45°, 6°) for 50,000 ft design altitude.

The performance calculations have been made assuming standard I.C.A.N. atmosphere, with 59°F at sea level. On hotter days there would be some loss in thrust and it might be necessary to make the climb path less steep, with consequent reduction in maximum range.

5 Layout

A monoplane twist-and-steer layout has been adopted here, with twin engines mounted on stub wings perpendicular to the main wings. Although it cannot be proved that this is the best layout there is much to commend it. Neither of the two possible single-engine layouts is attractive: if the motor is in a separate pod the missile is grossly asymmetrical, and if the motor is in the main body of the missile a 50% weight increase can be expected (see ref.1), due to difficulties in matching intake area with engine requirements and in the positioning of control surfaces. There are several other possible cartesian-control layouts, all of them having deficiencies: the four-wing missile with two engines indexed at 45° to the wings and the two-wing missile with two engines on small wings in the perpendicular plane both have enough asymmetry to raise doubts about the suitability of cartesian control; the four-wing missile with four wing-tip engines would probably require much thicker wings, and the longer moment arm would increase the lateral destabilizing force due to engine malfunction; the four-wing missile with four engines on stub wings indexed at 45° would suffer because of loss in lift, increase in stub-wing drag, and difficulty in boost arrangement. If twist-and-steer has no other serious penalties as yet unknown, a two-wing twin-engine layout would seem to promise a better performance than any of these others, and, though the advantage may be small, the experience gained in this country with the twin-engine layout - with the JTV test vehicle and the Red Duster missile - tells in favour of this layout.

Fixed wings have been chosen here in preference to moving wings, since the lateral acceleration demands are severe and a fixed-wing layout is much the more efficient in generating lift. A moving-wing layout has also been considered for purposes of comparison, and the pros and cons of the two layouts are discussed more fully in section 7.7.

The layout chosen here has unswept wings, with control surfaces at the rear of the body separated from the wings (Fig.1). An alternative layout with highly swept delta wings and wing-tip control surfaces has recently been considered by the Bristol Aeroplane Co. Changing to such a delta layout would have little effect on missile weight: the delta layout is neater, especially in its boost arrangement, but it is as yet untried in British guided missiles.

A rectangular planform has been assumed for both wings and tail*. The fixed wings have a thickness/chord ratio t/c of 0.03 and a gross aspect ratio of 1.5, and the associated four control surfaces of $t/c = 0.04$, which are indexed at 45° to the wings, have a net aspect ratio of 4. The moving wings of $t/c = 0.04$ act as two separate surfaces each of aspect ratio 1.2. The associated tail surfaces of $t/c = 0.03$, which consist of two panels in line with the wings, have a net aspect ratio of 2.

The missile body consists of an ogival nose of fineness ratio 2.8 and low-drag profile, a cylindrical section at maximum body diameter and an afterbody, in the form of a frustum of a 10° cone, having a base diameter of half

* The planforms have been assumed rectangular to reduce computation; small advantages might be obtained using alternative wing shapes.

the maximum body diameter. A lower limit of 7 has been imposed on the length/diameter ratio of the missile.

A standard missile, for 87 n.miles range, has been sketched in Fig.1 and contains in order from the nose: the radar dish, warhead, guidance equipment, fuel and actuators. This missile has also been sketched in Fig.41 together with the equivalent moving-wing missile for comparison.

To ensure adequate stability of the fixed-wing missile the wings have been placed so that the 1/5 chord line passes through the centre of gravity of the missile at all-burnt. Also the ramjets, which have simple conical-centrebody intakes with 60° cone angles, have been placed with their noses in line with the leading edges of the wings.

For the moving-wing missile, the wings have their 2/5 chord line passing through the centre of gravity of the missile at all-burnt while the ramjets have their noses approximately half-way between the leading edges of the wings and the all-burnt centre of gravity. The fins are at the rear of the missile.

The solid-fuel rocket boosts used would have to be arranged as two units, one behind each wing, in an overlap configuration, as shown in Fig.42.

In making weight estimates for the turbojet missiles an identical layout was assumed; for the rocket missiles too the layout was the same except that the motor was placed in the missile afterbody instead of on stub wings.

6 Method

6.1 General scheme

Since there are eleven parameters (see Table I) involved here in the estimations of missile weight, the complete analysis of every combination would present an impossible task of computation; about a million examples would be necessary.

The method adopted was to choose a standard value for each parameter except range, and estimate the missile weight for this set of values. A sketch of this standard missile for a range of 87 n.miles is shown in Fig.1. Then by considering the parameters design altitude, range, missile diameter, maximum lateral acceleration, mean lateral acceleration, payload weight, payload density, boost Mach number, propulsion and layout, estimates were made of variation in missile weight on up-and-along trajectories when each parameter in turn departed from its standard value. Finally, the performance of the standard missile for four different ranges was investigated for up-and-along, up-along-down-along and beam-riding trajectories for four target altitudes. With this method the number of examples necessary was reduced to about 120.

6.2 Assumptions

The range of values for each parameter together with the standard value is given in Table I (page 6). The payload was assumed to consist of a warhead of density 100 lb/cu ft and guidance equipment of mean density 35 lb/cu ft*. The body structural weight was taken as 3.5 lb/sq ft of body surface area and the weight of the twin ramjets, including stub wings, pumps, etc, as 180S lb, S being the total cross-sectional area of the two ramjets in sq ft. The fuel weight was calculated from the appropriate equation given in the Appendix, the tank weight being taken as 15% of the fuel weight and the volume of fuel and tanks as $m_F/40$ cu ft, where m_F is the weight of fuel in lb. The weight of

* It has been assumed that the payload will fit into the volume allowed, regardless of shape. Payload density has little effect on missile weight (Fig.11).

the fixed wings was taken as 3.5 lb/sq ft of net wing plan area and for the associated controls and actuators as 8.5 lb/sq ft of net control plan area. A lower limit of 5 sq ft was imposed on the net wing plan area. The weight of the moving wings was taken as 6 lb/sq ft of net wing plan area and for the wing actuators, 4 lb/sq ft of net wing plan area. The weight of the associated tail surfaces was assumed to be 1.75% of the missile all-up weight.

These values are based on those in use in British guided missiles now under development, but were increased where appropriate to allow for aerodynamic heating.

To illustrate the superiority of the ramjet over the turbojet under the conditions applicable here, deliberately optimistic assumptions were made for the weight and performance of the turbojet engine.

The weight of the twin turbojets, including stub-wings, pumps, etc. was taken as 1.25T lb, T being the net thrust of the twin turbojets in lb at the design altitude of 70,000 ft at Mach 3. It was assumed that the thrust changed exponentially with altitude from 10T at sea level to T at design altitude. The specific fuel consumption was assumed constant throughout flight at 1.5 lb/lb thrust/hour.

For the rocket missiles the motor was assumed to be liquid-fuel, with a vacuum specific impulse of 250 secs. The motor weight was taken as $27 + 0.028T_{max}$ lb, where T_{max} is the maximum thrust required in lb, and the tank weight as 10% of the propellant weight. An up-and-along trajectory was chosen, the 'up' part being a climb at $M = 2$ at 75° to the horizontal (the best angle for fuel economy) from sea level to a design altitude of 70,000 ft and the 'along' part being divided into an acceleration phase at maximum thrust from $M = 2$ to $M = 3$ and a cruise phase at $M = 3$. The weight of fuel consumed on the climb was calculated by the method of ref.4.

6.3 Drag and lift

Zero-lift drag calculations were made using data in the Aerodynamics Handbook, ref.10. The wave drag of the ogival nose was taken as 80% of the wave drag of the inscribed cone (of 20° apex angle). All wings, including stubs, and control surfaces were assumed to be double-wedge shaped. (Thickness/chord ratios are given in Section 5.) After calculating the total zero-lift drag of the missile, a 10% addition was made to allow for aerials, air intakes to turbopumps, and other irregularities. The external drag of the engines was accounted for in the net thrust coefficient. The induced drag was calculated for a maximum incidence of 25°, assuming lift proportional to incidence. Hence the total drag (zero-lift plus induced) was known and so the ramjet cross-sectional area could be calculated.

For the fixed-wing missile, the maximum wing lift coefficient was taken as $1.76/M$ (M being the design Mach number), based on gross wing area, an allowance having been made for wing-body interference. The body lift coefficient was taken as 3 for an incidence of 25°, based on the body cross-sectional area, a deduction having been made to allow for control surface lift. The net area of the control surfaces was calculated assuming a maximum control lift coefficient of $2.4/M$ and specifying a control time lag of 0.2 sec for the missile to roll through 90° and build up an incidence of 25°.

For the moving-wing missile, the maximum wing lift coefficient was taken as $1.55/M$, based on net wing area.

6.4 Procedure for estimating all-up weight, m_0

The estimation of all-up weight was carried out by an iterative process as follows:

A guess was first made at the values of m_0 and the ramjet cross-sectional area S. This enabled the component weights of the missile to be calculated giving a calculated value of m_0, while the drag calculations enabled the required S to be calculated. If the guessed and calculated values of m_0 and S differed, a better guess was made and the process repeated to give improved calculated values. The guessed and calculated values usually coincided after only two or three such guesses, giving the required values. This procedure differed slightly for the turbojet weight estimates. Here, it was necessary to guess m_0 and T, the thrust at design altitude. This time the drag calculations gave T, the process then being as before.

6.5 Modifications of method for non-standard trajectories

The performance of the standard missile, designed for 70,000 ft altitude, for four different ranges was investigated for up-and-along, up-along-down-along and beam-riding trajectories for target altitudes of 36,000 ft, 50,000 ft, 60,000 ft, 70,000 ft and in one case 80,000 ft.

6.5.1 Up-and-along trajectory

Here, the missile climbed at constant angle θ_1 from sea level to 36,000 ft and then at a smaller angle θ_2 from 36,000 ft to target altitude, which was then maintained.

For target altitudes up to 60,000 ft it was assumed that the Mach number at target altitude could be reduced to 2.6 (since adequate lateral acceleration was still available) with the result that thrust and hence fuel consumption could be considerably reduced at these altitudes.

At 80,000 ft target altitude, however, the standard missile has inadequate thrust if the r.m.s. lateral acceleration remains at 2g and the missile could only fly at this altitude if the r.m.s. demand were reduced to 1.24g. The fuel weight was then calculated as for the standard missile but with appropriate reductions for altitudes of 60,000 ft and below.

6.5.2 Up-along-down-along trajectory

Here, the climb path was taken as that of the previous up-and-along trajectory to maximum altitude at which the missile cruised before diving to target altitude, assumed less than 70,000 ft. During the dive the thrust was reduced to keep the Mach number constant; during the flight at target altitude (for either 10 or 20 n.miles) a reduced fuel consumption was assumed as in 6.5.1.

6.5.3 Beam-riding trajectory

Actual beam-riding trajectories were drawn in Fig.27 for 36,000 and 70,000 ft target altitudes and 80 and 200 n.miles range at launch*. The target was assumed to have a constant speed of 0.32 n.miles/sec (Mach 2) and the missile to have constant speeds of 0.4 n.miles/sec from launch to 36,000 ft and 0.45 n.miles/sec from 36,000 ft to target altitude.

---

* In drawing these trajectories the earth was assumed flat and the beam straight. With curved earth and beam the 113-mile trajectory in Fig.27(d) would be on the average 1500 ft below the trajectory drawn, implying an increase in fuel consumption of not more than 5%. At shorter ranges the difference would be less. Since the missile might anyway be constrained to ride 3-5000 ft above the beam this correction is not very important.

Each trajectory was then replaced by two straight lines, shown broken in Fig.39 - one from sea level to 36,000 ft and one from 36,000 ft to target altitude - to approximate as closely as possible to the actual trajectory. Approximations for other target altitudes and ranges at launch were obtained by interpolation.

For climbs to 36,000 ft target altitude, the thrust was reduced to save fuel and, by preventing the speed from becoming too high, reduce aerodynamic heating. The fuel consumption was then calculated as for the standard trajectory.

6.6 Boost weight

Solid-fuel rocket boosts were assumed to accelerate the missile to Mach 2 at sea level. The specific impulse was taken as 200 secs and the boost charge/weight ratio as 0.65. The boost weight was calculated using the methods of refs.11 and 12 and for the standard missile these assumptions gave the boost weight as being equal to the missile weight.

6.7 Accuracy check of assumptions

To check the accuracy of the assumptions made in the Appendix, a step-by-step numerical integration was performed for the standard missile. The accuracy of the approximations was found to be excellent and is discussed in more detail in the Appendix.

7 Discussion of results

The results presented in Figs.8-29 and Table II (page 28) are arranged to show the effects of each design parameter in turn and are discussed in sections 7.1-7.10.

7.1 Design altitude and range

Fig.8 shows that increasing the range from 100 to 200 n.miles increases the missile all-up weight by about 35% for 50,000 ft altitude and about 16% for altitudes above 70,000 ft. Fig.9 shows that for any given range up to 200 n.miles, all-up weight varies little with design altitude up to 70,000 ft but increases by about 65% from 70,000 to 80,000 ft design altitude due to the rapid increase in size, and so weight, of the wings and control surfaces required to maintain a specified lateral acceleration and a specified control time lag.

7.2 Missile body diameter, d

This parameter shows the effect of varying the dish diameter.

The missile length/diameter ratio, L/d, must not be so small as to aggravate the control problems. Here, L/d has been fixed at 7.89, the standard missile value, for body diameters between 2 and 3 ft. For smaller diameters, between 1.6 and 2.2 ft, however, the missile length was adjusted to maintain sufficient volume for the contents.

For constant L/d, increasing the missile diameter from 2 to 3 ft resulted in an increase of 45% in all-up weight due to large increases in body structural weight and body wave drag, both of which increase as the square of the diameter. This is shown in Fig.10. For the fixed-volume missile, variation in diameter over the range considered (1.6-2.2 ft) has only a small effect on the missile all-up weight.

7.3 Payload weight and density

Missile weight increases linearly with payload (warhead plus guidance) weight as shown in Fig.11; each 1 lb increase in payload causes an increase of about 2.2 lb in missile weight.

Payload density has only a small effect on missile weight; reducing the density from 100 to 50 lb/cu ft increases the missile weight by only about 7%. Increasing σ, the guidance/warhead weight ratio, by an order of magnitude over the range 0.1-10 increases the missile weight by only about 5%.

7.4 Maximum and r.m.s. lateral accelerations, Ng and ng

The mean acceleration has the greater influence on missile weight as can be seen in Figs.12 and 13. For the lower values of n, less than the standard value of 2, variation of N has only a small effect on missile weight. For higher values of n, however, missile weight increases rapidly as N decreases due to high induced drag. When the induced drag is high, excess thrust is available in climb and so can be reduced to give a saving in fuel. This saving partly offsets the large increase in weight.

The dotted line in Fig.14 shows the value of maximum lateral acceleration which gives minimum missile weight for a given mean lateral acceleration; for $n = 2$, a maximum acceleration of 7.3g gives minimum weight (about 1860 lb) whereas for $n = 3$, $N = 11$ gives minimum weight (about 2260 lb).

7.5 Mach number at the end of boost

Although an increase in boost Mach number above the standard value of 2 would give a smaller missile all-up weight, as shown in Fig.15, a much larger increase in total missile weight at launch (including boosts) would result. A reduction in boost Mach number below 2 could give a small saving in weight at launch but in this case the initial thrust-drag margin would be critical; a slight deficiency in boost impulse might lead to drag exceeding thrust.

7.6 Propulsion

Although optimistic assumptions were made for the weight and fuel consumption of turbojet and rocket engines, Fig.16 shows that the ramjet missile is considerably lighter than either turbojet or rocket at all relevant ranges. For any given range the turbojet missile is about 2000 lb heavier than the ramjet missile; the weight increase with range is about 9% per 100 n.miles for the turbojet and about 16% for the ramjet. For 100 n.miles range the all-up weight of the rocket missile is double that of the ramjet although its empty weight is only 19% greater. The liquid-fuel rocket assumed here has an overall vacuum specific impulse, $\frac{\text{(total impulse in vacuo)}}{\text{(total weight of motor, propellant and tanks)}}$, of 206 sec; changing to solid-fuel motor having an overall vacuum specific impulse of less than 180 sec, would of course further increase missile weight.

7.7 Fixed and moving wings

The variation of missile all-up weight with range for the fixed-wing and moving-wing missiles is given in Fig.17, for the standard conditions of 8g maximum lateral acceleration, 2g r.m.s. lateral acceleration and 70,000 ft design altitude. For all ranges between 30 and 200 miles the moving-wing missile is about 30% heavier. If the lateral acceleration demands are reduced to 5g (maximum) and 1.5g (r.m.s.) the difference between the two is narrowed to about 7%.

In Fig.41 sketches are given of the standard fixed-wing missile of Fig.1 and the comparable moving-wing missile. The vast difference in wing size - the two spans are 5.6 ft and 13.4 ft - is at once apparent. For the moving-wing missile the weight of wings, actuators and tail fins is 580 lb, while for the fixed-wing missile the comparable weight, of wings, control surfaces and actuators, is 250 lb. The wing drag is greater by a factor of 5 on the moving-wing missile.

The fundamental reason for the moving-wing missile's poor showing is its relative inefficiency as a lift-generator. For the fixed-wing missile the wing lift, taking into account wing-body interference, is roughly the same as the lift due to the gross wing, i.e. the wing carried through the area occupied by the body; for the moving-wing missile, it is virtually only the net area of the two moving panels which is effective in producing lift*. For the fixed-wing missile the lift from the body and ramjets at $M = 3$, even with a generous deduction for possible negative control-surface trim-force, makes a substantial addition (about 70% for the standard missile) to the wing lift; for the moving-wing missile the body should be at zero incidence and so give no lift.

As a result of these differences lift at maximum incidence is reduced by quite a large factor in changing a missile of given geometry from fixed to moving wings. Fig.40 shows how this factor varies with the ratio $\frac{\text{(body diameter)}}{\text{(wing span)}}$, d/b. For a typical value of d/b, 0.3, the fixed-wing layout gives a lift about 3.5 times greater than the moving-wing at 25° incidence. In calculating this factor the methods of ref.10 were used, with maximum body lift coefficient 3 at $M = 3$.

If the lateral acceleration demands on the missile are not too severe, the poor lift-producing properties of the moving-wing layout are not so important, since there is little weight difference between a missile with small wings and one with hardly any wings at all.

If, as in the standard conditions defined here, the lateral acceleration demands are exacting, strong subsidiary reasons are needed to resuscitate the moving-wing layout. Some possible reasons are touched on below.

(1) The moving-wing layout automatically provides a rapid roll response when the wings are moved differentially, whereas large control surfaces are needed to give the fixed-wing missile an adequate roll acceleration: that is why the moving-wing missile in Fig.41 is not more than 30% heavier than the fixed-wing.

(2) Moving wings should ease radome aberration problems by keeping the body near zero incidence. This is probably the main potential disadvantage of the fixed-wing layout, but its importance cannot be evaluated until much more work has been done to determine, first, the maximum aberration that can be tolerated without seriously degrading the homing performance and, second, the likely aberration characteristics of future practical radomes.

(3) A fund of experience on the moving-wing layout has been built up in this country, with Red Duster. The problems of the fixed-wing twist-and-steer missile have not yet been fully explored.

---

* Using data in an as yet unpublished chapter of ref.10, the increase in lift of the moving-wing missile of Fig.41 due to wing-body interference is found to be only 3%.

(4) With the moving-wing layout the angle of incidence of the ramjets should remain small throughout flight. With the fixed-wing layout incidence would have to be limited during mid-course flight to the maximum at which stable combustion could be maintained, perhaps 10-12°. This would mean that, after reaching design altitude the missile lateral acceleration would effectively be limited to about 3-4g until the final homing phase - a limitation which should not seriously degrade mid-course accuracy. In the final phase the ramjets are not likely to remain alight if the maximum incidence of 25° is demanded. This is not as bad as might at first appear (a) because deceleration would be rapid even if the ramjets gave full thrust (for the missile of Fig.1 thrust is 830 lb and drag at 25° incidence is 6300 lb, at M = 3 at 70,000 ft), and (b) because deceleration has very little effect on the maximum tolerable mid-course errors for collision-course interceptions near head-on. Ref.7 indicates that for a homing lock-on range of 10 n.miles and a Mach 2 target, deceleration would reduce the maximum tolerable mid-course error for the standard missile from its constant-speed value of 3.35 n.miles to 3.24 n.miles (if the ramjets remained alight) or 3.22 n.miles (if the ramjets went out).

7.8 Missile fuel/weight ratio on standard trajectory

Fig.18 shows that the missile fuel/weight ratio decreases with increase in design altitude for a given range, although the ratio is not affected much by altitude variation at short ranges. For a design altitude of 70,000 ft, the fuel/weight ratio does not exceed 0.25 up to 200 n.miles range and so is small enough not to dominate the missile design.

7.9 Dimensions

Ramjet combustion chamber diameter has been plotted against several design parameters in Fig.19 where it can be seen that the diameter is affected most by design altitude and r.m.s. lateral acceleration. The values of ramjet diameter are within the limits of what is practically attainable, lying between 10 and 20 inches except for the highest values of r.m.s. lateral acceleration. Range has little effect on ramjet diameter.

Missile length has similarly been plotted against several design parameters in Fig.20. The length varies linearly with payload weight, increasing by about 1 ft for every 100 lb of payload. The length is also nearly linear with range, increasing by about 10% for an increase in range from 100 to 200 n.miles. For design altitudes below 70,000 ft the variation in length is small, although an increase from 70,000 to 80,000 ft altitude increases the length by about 20%.

7.10 Trajectory

For the up-and-along trajectory, all-up weight, range and target altitude have been plotted in pairs as shown in Figs.21-23 for a design altitude of 70,000 ft. The broken line in Fig.21 shows what the variation of missile weight with range would be, for a target altitude of 80,000 ft, if the induced drag could be reduced to prevent drag exceeding thrust. (If the r.m.s. lateral acceleration had to be kept at 2g, drag would greatly exceed thrust at 80,000 ft.) Fig.22 shows that for a given range, there is little change in all-up weight for missiles climbing to target altitudes between 55,000 and 70,000 ft, mainly due to the thrust being reduced for altitudes below 70,000 ft. For lower altitudes, however, there is a rapid increase in weight, particularly at the longer ranges. Fig.23 shows that for a given missile weight there is little change in range above 60,000 ft.

For the up-along-down-along trajectory Figs.24 and 25 show that for a given range up to 100 n.miles the all-up weight does not change by more than 7% for all cruising and target altitudes. This percentage change remains just as small for ranges up to 400 n.miles except for missiles cruising at altitudes

below 60,000 ft and descending to target altitudes below 40,000 ft. Fig.26 shows that the optimum cruising altitude (broken line), which gives the maximum range for a given all-up weight, varies little with range and is nearly independent of target altitude. Between 100 and 400 n.miles range the optimum cruising altitude increases from 63,000 to 67,000 ft.

For the beam-riding trajectory, Fig.28 shows a rapid rise in missile weight at relatively short ranges; increasing range from 50 to 100 n.miles increases missile weight by about 35%.

Fig.29 shows the relative performance of the standard missile on the three trajectories. There is little difference in weight between missiles on up-and-along and optimum up-along-down-along trajectories above 60,000 ft target altitude. For 100 n.miles range and 60,000 ft target altitude the respective missile weights for the beam-riding, up-and-along and optimum up-along-down-along trajectories are 2880, 1885 and 1880 lb.

8 Summary of results, and conclusions

The missile sketched in Fig.1, which weighs 1870 lb at launch without boosts, has standard values of the eleven design parameters - payload 700 lb, maximum lateral acceleration 8g at the design altitude of 70,000 ft and the design Mach number of 3, range 87 n.miles, r.m.s. lateral acceleration 2g, ramjet propulsion, up-and-along trajectory, etc.

Table II (page 28) and Figs.8-29 show how the weight changes when each parameter in turn departs from its standard value. Increasing design altitude from 70,000 ft to 80,000 ft increases weight by 65%; reducing design altitude has much less effect. 100 miles extra range at 70,000 ft altitude puts up the weight by 16%. Changing missile diameter from 24" to 30" results in a 20% weight increase. Every extra lb of payload leads to 2.2 lb extra all-up weight. Reducing payload density from 52 to 35 lb/cu ft adds 6% to the weight. Increasing maximum lateral acceleration from 8g to 12g (while retaining 2g r.m.s. lateral acceleration) puts up the weight by only 7%, but increasing r.m.s. lateral acceleration from 2g to 3g (while retaining 8g maximum lateral acceleration) leads to a 30% weight increment. A small saving in weight at launch with boosts (4%) might be made by reducing the boost Mach number from 2 to 1.8. Changing from ramjet to turbojet or rocket propulsion doubles the all-up weight for 100 n.miles range. Using moving wings instead of fixed increases missile weight by 30%. There is little to be gained by changing from the standard up-and-along trajectory to an up-along-down-along type unless the target altitude is below 50,000 ft: when the target altitude is 36,000 ft the saving in weight is 12% for 100 n.miles range. Changing from up-and-along to beam-riding trajectory leads to a weight increase of about 50% at 100 miles range.

These results suggest the following broad conclusions:

(1) If 8g lateral acceleration is to be developed by aerodynamic lifting surfaces without incurring unreasonable increases in missile weight, the upper limit for design altitude can be taken as about 75,000 ft.

(2) Range can be increased from 100 to 250 miles without an excessive increase in weight (for up-and-along trajectory to 70,000 ft altitude).

(3) To avoid undue increase in weight, missile diameter should be kept down to about 24".

(4) The r.m.s. lateral acceleration in mid-course flight can have an important effect on missile performance, and should not be allowed to rise above 2g (including the inevitable 1g for counteracting gravity).

(5) The ramjet appears to be the most suitable form of propulsion. An appropriate climb programme for a ramjet missile is: boost to $M \simeq 2$; increase M steadily during steep climb ($\simeq 75^\circ$) to 36,000 ft altitude, followed by shallower climb ($\simeq 15^\circ$) to design altitude of 70,000 ft; maintain $M \simeq 3$ at design altitude.

(6) Provided its radome aberration characteristics are acceptable, a fixed-wing layout will be preferable to a moving-wing, since the weight at launch is 30% lower, for 8g lateral acceleration at 70,000 ft altitude. If the lateral acceleration demand were reduced to 5g, the weight difference would be much less, 7%.

(7) If a beam-riding trajectory had to be used instead of up-and-along or up-along-down-along, the increase in missile weight for a given range would be large and ranges much greater than 100 n.miles would be impossible.

Acknowledgement

The authors wish to thank Mr. P. Philpot of NGTE, Pyestock, who provided the thrust and fuel consumption estimates for the ramjet motor.

REFERENCES

No.	Author	Title, etc
1	D.G. King-Hele K.J. Bowcock I.W. Peattie	Effects of radome shape on the weights of surface-to-air guided missiles. RAE Tech Note GW 220 (1952). ARC 15,885.
2	A. Wall D.L. Martlew C.A.M. Taylor	A comparison of turbojet and ramjet engines when applied to surface-to-air guided missiles having a design Mach number of 2. NGTE Memo M.208. (1954).
3	D.G. King-Hele G.V. Groves	The estimated performance of surface-to-air guided missiles powered by expendable jet engines. RAE Tech Note GW 117. (1951).
4	D.G. King-Hele H. Hiller	Effect of design altitude on the weight and size of short-range rocket and ramjet surface-to-air guided missiles. RAE Tech Note GW 371. (1955).
5	D.G. King-Hele F.A. Varker K.J. Bowcock	Surface-to-air ramjet missiles at Mach numbers of 2, 2.5 and 3. RAE Tech Note GW 204. (1952). ARC 15,710.
6	J.A. Bird	A preliminary study of the relations between homing radar characteristics and weapon effectiveness for a long-range surface-to-air guided weapon. RAE Tech Note GW (to be issued).
7	D.C.M. Leslie	An elementary theory of correctable errors in missile homing. Part II: Effect of aerodynamic deceleration. RAE Tech Note GW (to be issued).

REFERENCES (Contd)

No.	Author	Title, etc
8	J. Poole D.G. King-Hele E.F. Lawlor	An assessment of supersonic missiles and aeroplanes for long-range bombardment. RAE Tech Note Aero 2231, GW 243. (1953).
9	P.R. Owen C.L. Barham J.E.P. Dunning	Report of the RAE Project Group on medium-range anti-aircraft guided missiles. Appendix I. RAE Report GW6 Part III. (1949).
10	-	Handbook of supersonic aerodynamic data applicable to guided weapon design. RAE GW Dept GW/Handbook/1.
11	J.D. Burgess	Drag coefficient of RTV2 with wrap-round boosts, derived from flight measurements. RAE Tech Note GW 224. (1952).
12	R.G. Thorne R.W. Bain	An analytical solution for the performance of a single-stage boost burning fuel at a constant rate. RAE Tech Note GW 98. (1950). ARC 13,823.

Attached:- Appendix
Table II
Drawings GW/P/6704 to 6745
Detachable Abstract Cards

Advance Distribution

Ministry of Supply

Chief Scientist	
CGWL	
PDSR(G)	
DG/GW	70
AD Eng RD6	
DG of A	
TPA3/TIB	90

R.A.E.

Director	
DD(E)	
DD(A)	
RPD	3
Aero	3
Arm	
IAP	
NGTE	
Patents	
NAE Library	
Library	

APPENDIX

Method of calculating missile speed and fuel consumption during climb

A.1 Missile speed

A.1.1 Climb from sea level to the tropopause (36,090 ft altitude)

The missile is assumed to be boosted to Mach number M_o at sea level, and, as stated in section 4.4, to follow a straight-line path during its climb from sea level to 36,000 ft. The altitude in thousands of feet is denoted by y. The suffix $*$ denotes values at the tropopause, where $y = y^* = 36.09$. The suffix o denotes values at sea level.

To simplify the analysis it is assumed that the variation of net thrust (T-D) with altitude can adequately be represented by expressing the quantity $\frac{T-D}{mp}$ as a linear function of y (m being the mass of the missile and p the atmospheric pressure at altitude y). Thus we have

$$\frac{T-D}{mg} = F, \quad \text{say} \tag{1}$$

$$= \left\{F_o + \frac{y}{y^*}\left(F^* \frac{p_o}{p^*} - F_o\right)\right\} \frac{p}{p_o} . \tag{2}$$

The actual variation of $\frac{(T-D)}{mp}$ with y for the missile of Fig.1 is plotted in Fig.31, from which it appears that the approximation is a good one: the maximum error is 6%, and the mean error 1.5% - the approximation erring on the optimistic side.

Now if v is the velocity of the missile (in thousands of feet per second) the equation of motion is

$$T - D - mg \sin \theta_1 = 1000\, m\dot{v}$$

$$= 1000\, mv \sin \theta_1 \frac{dv}{dy} \tag{3}$$

where θ_1 is the angle of climb. Equation (3) may be rewritten

$$\left(1 + \frac{1000v}{g}\frac{dv}{dy}\right) \sin \theta_1 = \frac{T-D}{mg} , \tag{4}$$

or, by (2),

$$\left\{1 + \frac{500}{g}\frac{d(v^2)}{dy}\right\} \sin \theta_1 = \left\{F_o + \frac{y}{y^*}\left(F^* \frac{p_o}{p^*} - F_o\right)\right\} \frac{p}{p_o} . \tag{5}$$

Integrating (5) we have

$$\left(y + \frac{v^2 - v_o^2}{0.0644}\right) \sin \theta_1 = F_o H + \left(F^* \frac{p_o}{p^*} - F_o\right) I , \qquad (6)$$

where

$$\left.\begin{aligned} H &= H(y) = \int_o^y \frac{p(u)}{p_o} du \\ I &= I(y) = \int_o^y \frac{up(u)}{y^* p_o} du , \end{aligned}\right\} \qquad (7)$$

$p(u)$ being the atmospheric pressure at altitude u. Using the I.C.A.N. formulae for the standard atmosphere, it can easily be shown that, for $y < 36.09$,

$$\left.\begin{aligned} H(y) &= 23.24 \left(1 - \frac{p}{p_o} \frac{\tau}{\tau_o}\right) \\ I(y) &= 12.90 \left\{1 - \frac{p}{p_o} \left(\frac{\tau}{\tau_o}\right)^2\right\} - 0.6439\, y \frac{p}{p_o} \frac{\tau}{\tau_o} , \end{aligned}\right\} \qquad (8)$$

where $\frac{\tau}{\tau_o}$ is the relative temperature (^oK) at altitude y; and, for $y > 36.09$,

$$\left.\begin{aligned} H(y) - H(y^*) &= H - H^* = 4.640 - 20.79 \frac{p}{p_o} \\ I(y) - I(y^*) &= I - I^* = 7.312 - (11.98 + 0.5761y) \frac{p}{p_o} . \end{aligned}\right\} \qquad (9)$$

Values of the functions H and I are given in Table III and plotted in Fig.32.

Table III

The 'atmosphere functions' H and I defined by equations (7)

Altitude y thousand ft	H	H-H*	I	I-I*
0	0		0	
10	8.36		1.086	
20	14.03		3.392	
30	17.77		5.941	
36.09 = y*	19.34		7.379	
40	20.14	0.80	8.235	0.856
50	21.60	2.26	10.03	2.65
60	22.51	3.17	11.40	4.02
70	23.07	3.73	12.40	5.02
80	23.42	4.08	13.12	5.74

Equation (6), written in the form

$$v^2 = v_o^2 + 0.0644\ [\{F_oH + (4.481F^* - F_o)I\}\ \text{cosec}\ \theta_1 - y] \qquad (10)$$

can be used to find the velocity and hence the Mach number at any altitude during the climb, if v_o, θ_1, F_o and F^* are known. The variation of Mach number with altitude, for $M_o = 2$ and various typical values of θ_1, F_o and F^*, is shown in Fig.33, which suggests that for most practical purposes the variation of M with y can be taken as linear.

M^* can be found by inserting in (10) the appropriate numerical values, $y = 36.09$, $H = 19.34$, $I = 7.379$ and $v = 0.9685M^*$. This gives

$$M^{*2} = 1.330\ M_o^2 - 2.478 + (0.821\ F_o + 2.270\ F^*)\ \text{cosec}\ \theta_1 . \qquad (11)$$

M^* is plotted against F^* for various values of F_o and θ_1, and for $M_o = 2$, in Fig.34.

A.1.2 Climb in the stratosphere

It is assumed that in the stratosphere the missile climbs at a constant angle θ_2, chosen so that at design altitude y_1 it is flying at design Mach number M_1 (= 3). Again, taking $\frac{(T-D)}{mp}$ as a linear function of altitude, we have, since by definition $T = D$ at $M = M_1$ and $y = y_1$,

$$\frac{T-D}{mg} = \frac{T^*-D^*}{m^*g} \cdot \frac{y_1-y}{y_1-y^*} \cdot \frac{p}{p^*} \qquad (12)$$

$$= F^* \frac{y_1-y}{y_1-y^*} \cdot \frac{p}{p^*} . \qquad (13)$$

The actual variation of $\frac{(T-D)}{mp}$ is shown in Fig.31, from which it appears that the linear approximation is pessimistic. The discrepancy is largest where the value of $(T-D)$ is least, so that the largest error in net thrust/weight ratio, $\frac{(T-D)}{mg}$, is less than 0.06, i.e. the largest error in estimating acceleration is 0.06g.

Using equation (13) the equation of motion (4) becomes

$$1 + \frac{500}{g}\frac{d}{dy}\left\{(0.9685M)^2\right\} = \frac{F^*\ \text{cosec}\ \theta_2\ p_o\ y^*}{p^*(y_1-y^*)} \qquad (14)$$

or, integrating,

$$y-y^* + 14.57\ (M^2-M^{*2}) = \frac{p_o}{p^*} \cdot \frac{y^*}{y_1-y^*}\left\{\frac{y_1}{y^*}(H-H^*) - (I-I^*)\right\} F^*\ \text{cosec}\ \theta_2 . \qquad (15)$$

Since $M = M_1$ at $y = y_1$, θ_2 is given by

$$\sin\theta_2 = \frac{\frac{p_o}{p^*}\cdot\frac{y^*}{y_1-y^*}\left\{\frac{y_1}{y^*}(H_1-H^*)-(I_1-I^*)\right\}F^*}{y_1-y^*+14.57\,(M_1^{\,2}-M^{*2})}, \qquad (16)$$

θ_2 is plotted against F^* for $M_1 = 3$ and various y_1 and M^* in Fig.36. Re-arranging (15) we have for the Mach number M at intermediate altitudes,

$$M^2 = M^{*2} + 0.0686\left[\frac{p_o}{p^*}\cdot\frac{y^*}{y_1-y^*}\left\{\frac{y_1}{y^*}(H-H^*)-(I-I^*)\right\}F^*\operatorname{cosec}\theta_2-(y-y^*)\right], \qquad (17)$$

$F^*\operatorname{cosec}\theta_2$ being found from (16). M is plotted against y for various M^* and y_1 and $M_1 = 3$ in Fig.37.

A.1.3 Accuracy of the approximate method

The values of Mach number during climb found by numerical integration are compared with those given by the approximate method outlined in this Appendix in Fig.31. The maximum error in Mach number incurred by using the approximate method is 0.03.

A.2 Fuel consumption

A.2.1 Method

If a_T is the ramjet thrust coefficient, S the total cross-sectional area of the motors (i.e. $S = 2A_3$ if there are two motors each of cross-sectional area A_3), and c the specific fuel consumption (lb/hr/lb thrust), the rate of fuel flow is

$$\dot{m}_F = \frac{Sa_T\,1481\,M^2\,c}{3600}\cdot\frac{p}{p_o} \quad \text{lb/sec}. \qquad (18)$$

The fuel burnt in a climb from sea level to altitude y_1 is therefore,

$$m_{F_o} = \int_0^{t_1}\dot{m}_F\,dt = 0.4114S\int_0^{y_1} a_T M^2 c\,\frac{p}{p_o}\,\frac{\operatorname{cosec}\theta\,dy}{v}. \qquad (19)$$

For climb at angle θ_1 to altitude y^* followed by climb at angle θ_2 to altitude y_1, (19) becomes

$$m_{F_o} = 0.4114S\operatorname{cosec}\theta_1\int_0^{y^*} a_T c\,\frac{M}{a}\,\frac{p}{p_o}\,dy + 0.4248S\operatorname{cosec}\theta_2\int_{y^*}^{y_1} a_T Mc\,\frac{p}{p_o}\,dy, \qquad (20)$$

a being the speed of sound. The values of a_T and c given in Fig.4 have been used to plot $a_T c$ and $a_T Mc$ as a function of M and y in Fig.38. From Fig.38(a) it appears that $a_T c$ increases from 2.97 at sea level to 3.37 at the stratosphere on a typical climb, and since this variation is small a

suitably weighted mean value, 3.09, has been taken here. Then, assuming $\frac{M}{a}$ is a linear function of y (with $M^* = 2.6$), the first term in (20) reduces to

$$S \operatorname{cosec} \theta_1 \; 1.271 \; (1.791H^* + 0.894I^*)$$

$$= 52.4S \operatorname{cosec} \theta_1 \; . \qquad (21)$$

Fig.38(b) shows that $\sigma_T Mc$ does not vary greatly with Mach number in the stratosphere. A mean value $\bar{B}$ was therefore taken for the quantity $0.4248 \; \sigma_T Mc$ in the stratosphere, the value of $\bar{B}$ being altered to suit the particular missile's Mach number. The second term in (20) then reduces to

$$S \operatorname{cosec} \theta_2 \; \bar{B} \; (H_1 - H^*) \; . \qquad (22)$$

Adding (21) and (22) gives as the fuel burnt during climb

$$m_{F_c} = S \; \{52.4 \operatorname{cosec} \theta_1 + \bar{B}(H_1 - H^*) \operatorname{cosec} \theta_2\} \text{ lb} \qquad (23)$$

where, in the standard case, $\bar{B}$ has the value 3.6, and, for $y_1 = 70$, $H_1 - H^* = 3.73$, from Table III.

When the missile reaches its design altitude it has covered a horizontal distance

$$y^* \cot \theta_1 + (y_1 - y^*) \cot \theta_2 \quad \text{thousand feet,}$$

and if x is its total horizontal range in nautical miles the distance flown at design altitude on an up-and-along trajectory is

$$6.08x - y^* \cot \theta_1 - (y_1 - y^*) \cot \theta_2 \quad \text{thousand feet.}$$

The rate of burning fuel during level flight is by (18)

$$\frac{S\sigma_T \; 1481 \; M^2 c}{3600v} \cdot \frac{p_1}{p_o} \quad \text{lb/th. ft.}$$

$$= 3.40 \frac{p_1}{p_o} S \quad \text{lb/th. ft. at } M = 3.$$

Hence the total fuel load required for range x is

$$m_F = S \Big[52.4 \operatorname{cosec} \theta_1 + \bar{B}(H_1 - H^*) \operatorname{cosec} \theta_2 + 3.40 \frac{p_1}{p_o} \{6.08x - y^* \cot \theta_1 - (y_1 - y^*) \cot \theta_2\} \Big] . \qquad (24)$$

A.2.2 <u>Accuracy of method</u>

The method is of course exact during the level-flight period. For the missile of Fig.1 on its standard climb path step-by-step numerical integration gives 192 lb for the fuel used on the climb, while equation (24) gives 194 lb. The approximate method thus overestimates the total fuel load for the standard missile by 2 lb in 300 lb.

TABLE II

Variation of missile weight when each of the eleven design parameters departs from its standard value

PARAMETER												WEIGHTS						
Design Altitude ft	Range n.miles	Missile Diameter ft	Payload Weight lb	Payload Density lb/cu ft	Maximum Lat. Accn. g	R.M.S. Lat. Accn. g	Boost Mach Number	Motor	Layout	Trajectory	Target Altitude ft	Payload	Motors* and Stub Wings	Fuel	Wings	Body Structure (Incl. Tanks)	Controls and Actuators	All-up Weight lb
y_1	x	d	P		N	n	M_0				y_T							m_0
70,000	87	2	700	52	8	2	2	RJ	FW	A	70,000	700	320	300	50	300	200	1870
60,000	"	"	"	"	"	"	"	"	"	"	60,000	700	250	300	20	280	110	1660
80,000	"	"	"	"	"	"	"	"	"	"	80,000	700	720	490	240	350	540	3080
70,000	30	"	"	"	"	"	"	"	"	"	70,000	700	340	200	40	270	200	1720
"	200	"	"	"	"	"	"	"	"	"	"	700	340	540	60	370	220	2230
"	87	1.6	"	"	"	"	"	"	"	"	"	700	300	270	70	300	210	1850
"	"	3	"	"	"	"	"	"	"	"	"	700	540	490	20	650	250	2690
"	"	2	1200	"	"	"	"	"	"	"	"	1200	470	430	110	420	330	2980
"	"	"	700	25	"	"	"	"	"	"	"	700	360	330	60	410	230	2090
"	"	"	"	52	12	"	"	"	"	"	"	700	360	310	120	300	220	2010
"	"	"	"	"	8	3.5	"	"	"	"	"	700	870	620	100	410	320	3020
"	"	"	"	"	"	2	1.8	"	"	"	"	700	330	330	50	310	210	1930
"	"	"	"	"	"	"	2	TJ	"	"	"	700	1960	220	200	300	470	3850
"	"	"	"	"	"	"	"	RK	"	"	"	700	210	1770	60	610	230	3580
"	"	"	"	"	"	"	"	RJ	MW	"	"	700	410	370	320	340	260†	2400
"	"	"	"	"	"	"	"	"	FW	B	"	700	360	680	60	430	230	2460
"	"	"	"	"	"	"	"	"	"	A	36,000	700	340	470	50	360	220	2140
"	"	"	"	"	"	"	"	"	"	B	"	700	380	890	70	490	250	2780
"	"	"	"	"	"	"	"	"	"	C	"	700	330	340	50	310	210	1940

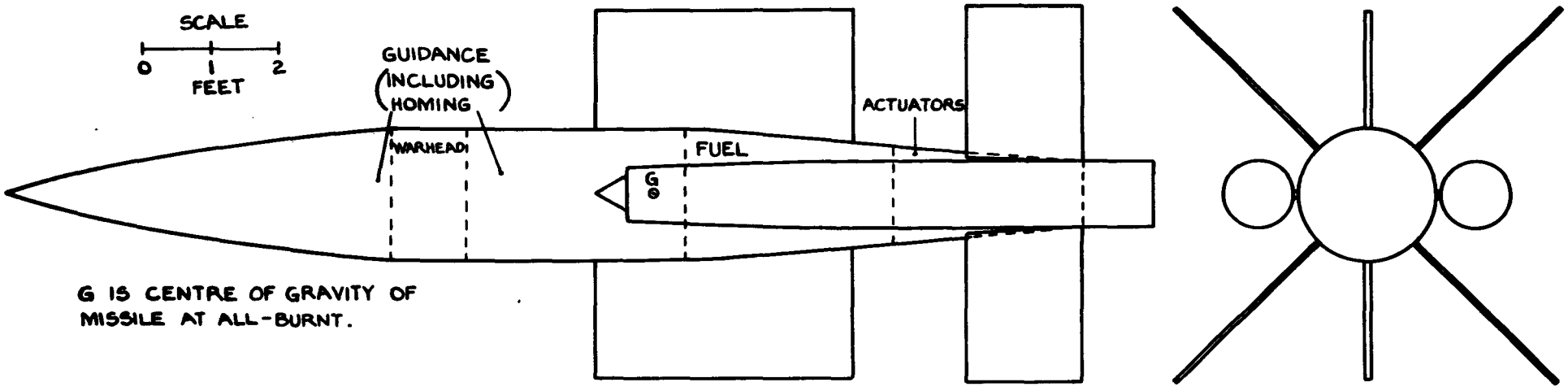
* Motors include pumps, turbines, etc.

† This value is weight of tail + wing-actuators.

A = Up-and-along; B = Beam-Riding; C = Optimum up-along-down-along.

RJ = Ramjet; TJ = Turbojet; RK = Liquid-fuel rocket.

MW = Moving-wing; FW = Fixed-wing.



G IS CENTRE OF GRAVITY OF MISSILE AT ALL-BURNT.

COMPONENT	WEIGHT-LB
WARHEAD	350
GUIDANCE	350
BODY STRUCTURE	260
WINGS	50
CONTROLS & ACTUATORS	200
RAMJETS, STUBS, ETC	320
FUEL	300
TANKS	40
ALL-UP WEIGHT	1870
BOOSTS (NOT SHOWN)	1870

DIMENSIONS, ETC.	
LENGTH (FEET)	15·8
DIAMETER (FEET)	2·0
WING SPAN (FEET)	5·6
CONTROL SPAN (FEET)	8·1
RAMJET DIAM. (INCHES)	12·8
DESIGN MACH Nº	3
BOOST MACH Nº	2
MAX. LATERAL ACCN.	8g
R.M.S. LATERAL ACCN.	2g
DESIGN ALTITUDE (FT.)	70,000
RANGE (N. MILES)	87
TRAJECTORY :- UP-AND-ALONG	

DRAG AT M=3 AT 70,000 FT. ALTITUDE

DRAG COMPONENT	%	C_D
NOSE WAVE	27	0·070
AFTERBODY WAVE	10	0·026
BASE WAVE	10	0·025
BODY SKIN FRICTION	14	0·036
STUB WINGS	2	0·006
WING WAVE	2	0·005
WING SKIN FRICTION	6	0·017
CONTROL WAVE	6	0·017
CONTROL SKIN FRICTION	14	0·038
ADDITION	9	0·025
TOTAL	100	0·265
ZERO-LIFT DRAG	485 LB.	
INDUCED DRAG	343 LB.	
TOTAL DRAG	828 LB.	

ZERO-LIFT DRAG COEFFICIENT, C_D, BASED ON BODY FRONTAL AREA.

FIG. 1. STANDARD MISSILE OF 87 N. MILES RANGE.

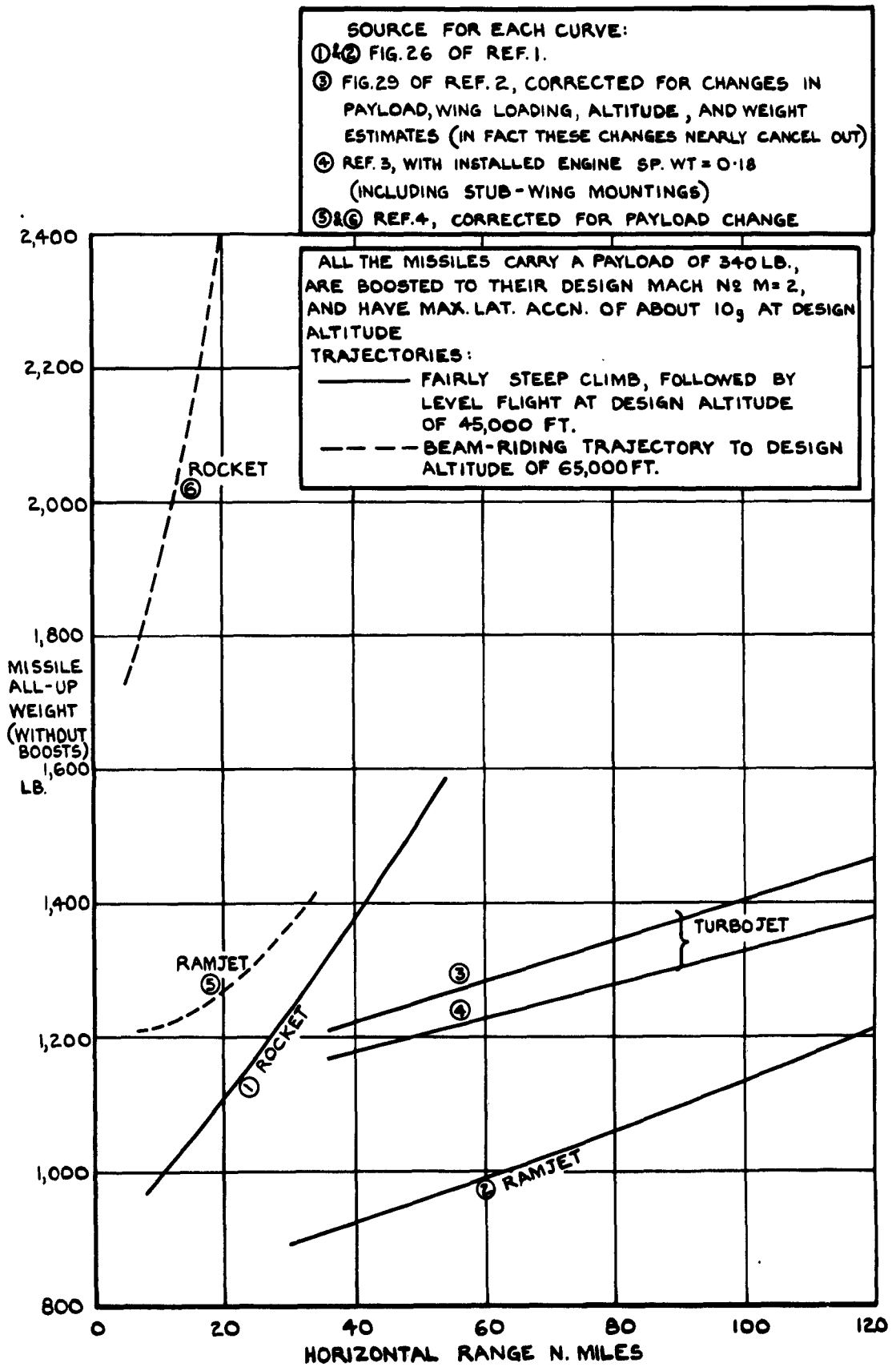


FIG. 2. VARIATION OF WEIGHT WITH RANGE FOR ROCKET, RAMJET AND TURBOJET SURFACE-TO-AIR MISSILES.

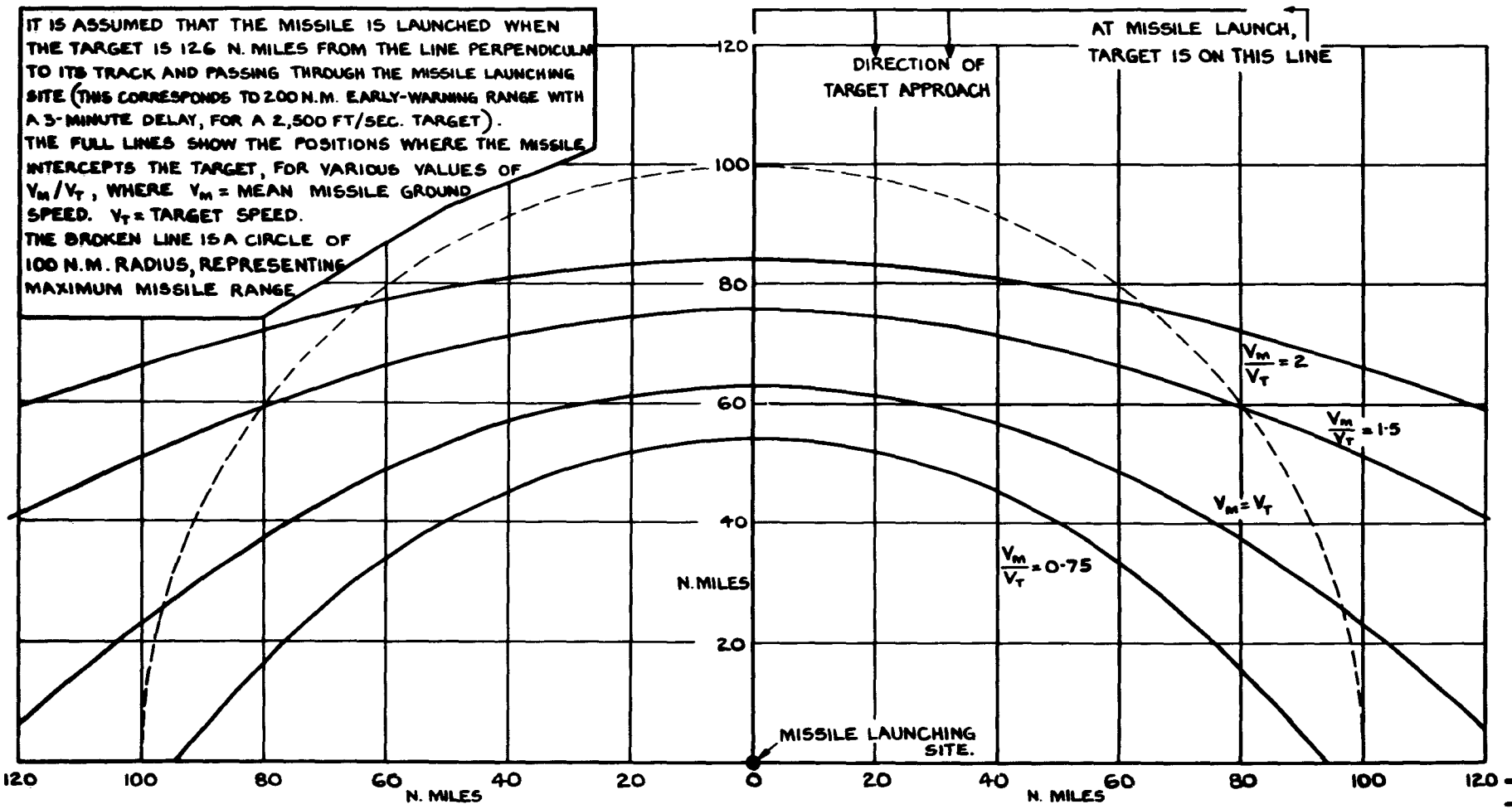


FIG. 3. COVER DIAGRAM, SHOWING THE EFFECT OF MISSILE/TARGET SPEED RATIO ON THE AREA WITHIN WHICH TARGETS CAN BE INTERCEPTED.

FIG. 4. (a & b)

$$C_T = \frac{T_N}{0{\cdot}7 p M^2 A_3}$$

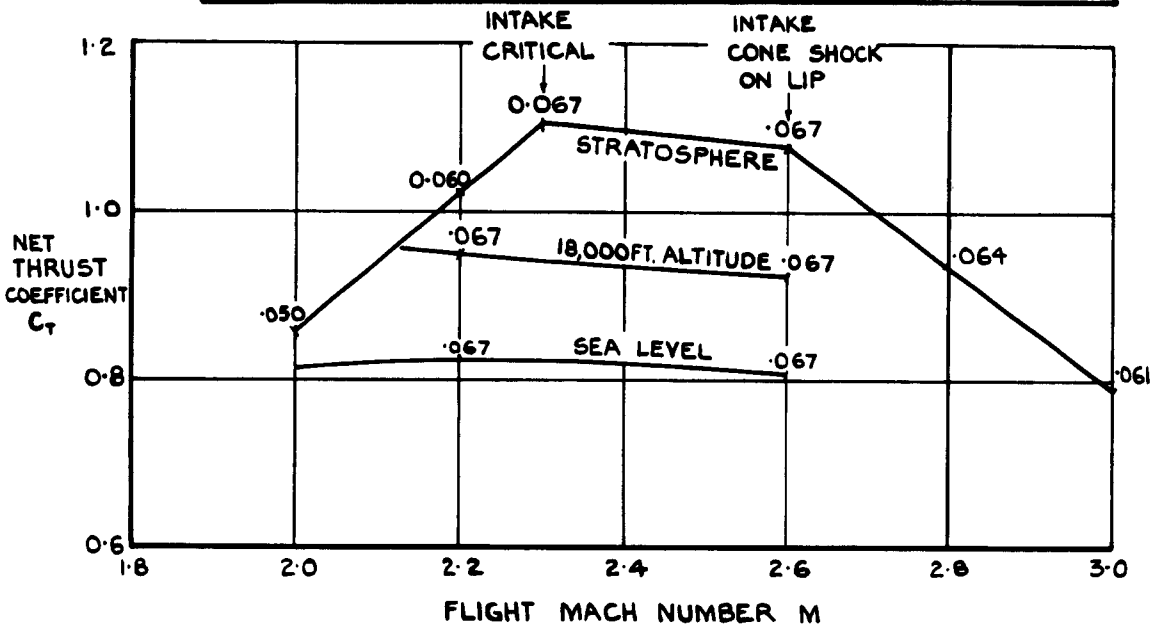
WHERE T_N = NET THRUST (ALLOWING FOR EXTERNAL DRAG OF RAMJET DUCT)

p = AMBIENT ATMOSPHERIC PRESSURE

A_3 = MOTOR MAX. CROSS-SECTIONAL AREA

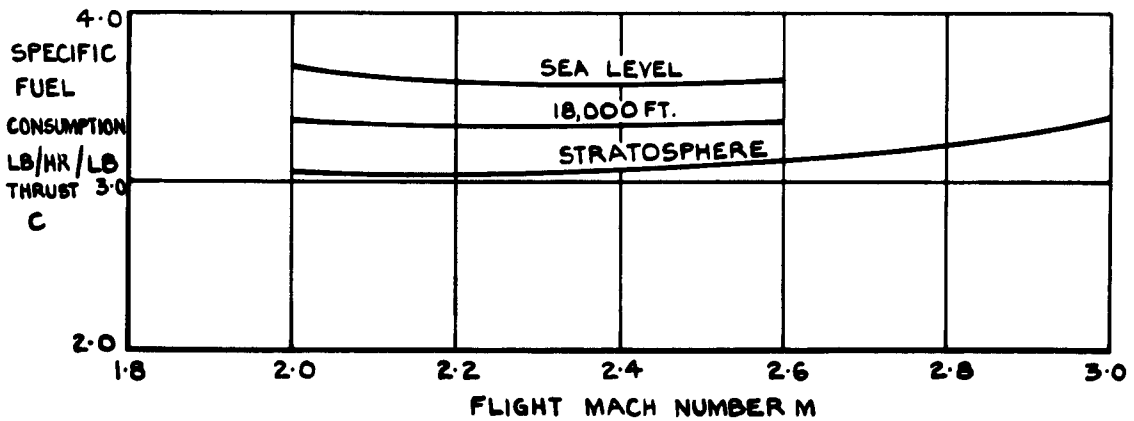
FOR MOTOR SHAPE, SEE FIG. 1.

NUMBERS ON CURVES INDICATE FUEL/AIR RATIO.



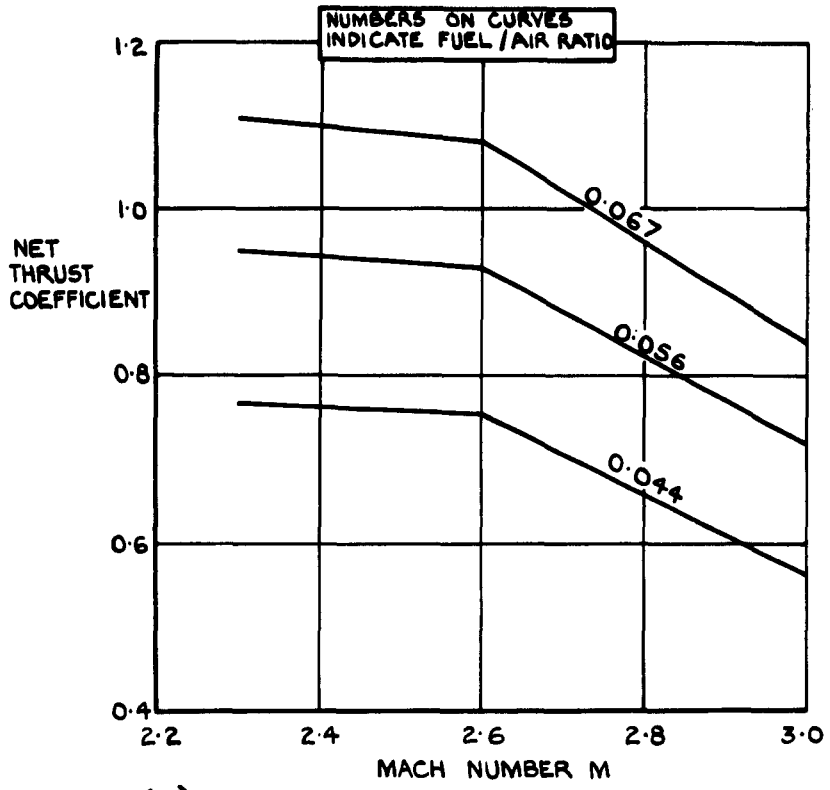
(a) THRUST COEFFICIENT.

S.F.C. = (FUEL CONSUMPTION IN LB/HOUR) ÷ (NET THRUST IN LB.)

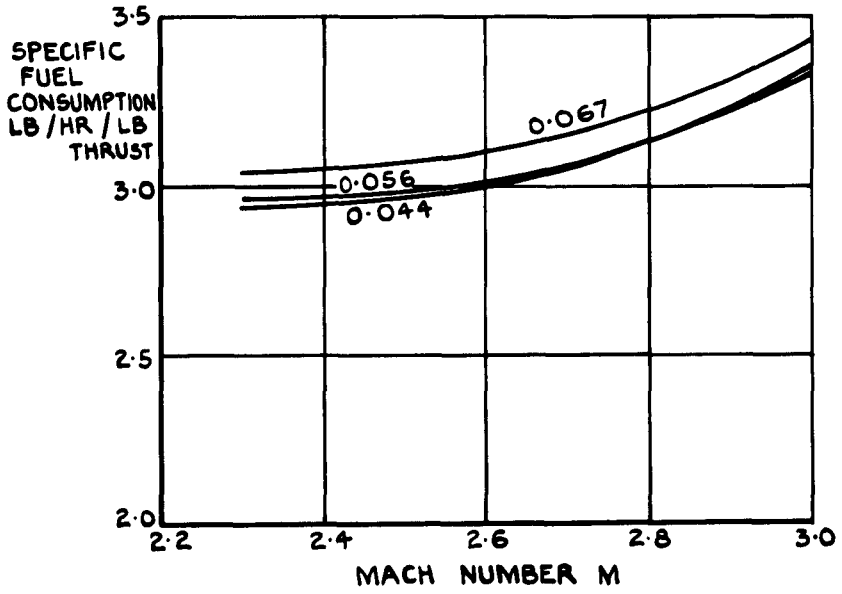


(b) SPECIFIC FUEL CONSUMPTION.

FIG. 4. (a & b) THRUST COEFFICIENT AND FUEL CONSUMPTION OF RAMJET MOTOR AS A FUNCTION OF MACH NUMBER AND ALTITUDE.



(a) THRUST COEFFICIENT.



(b) SPECIFIC FUEL CONSUMPTION.

FIG. 5 (a&b) VARIATION OF RAMJET THRUST AND FUEL CONSUMPTION IN THE STRATOSPHERE WITH MACH NUMBER AND FUEL/AIR RATIO.

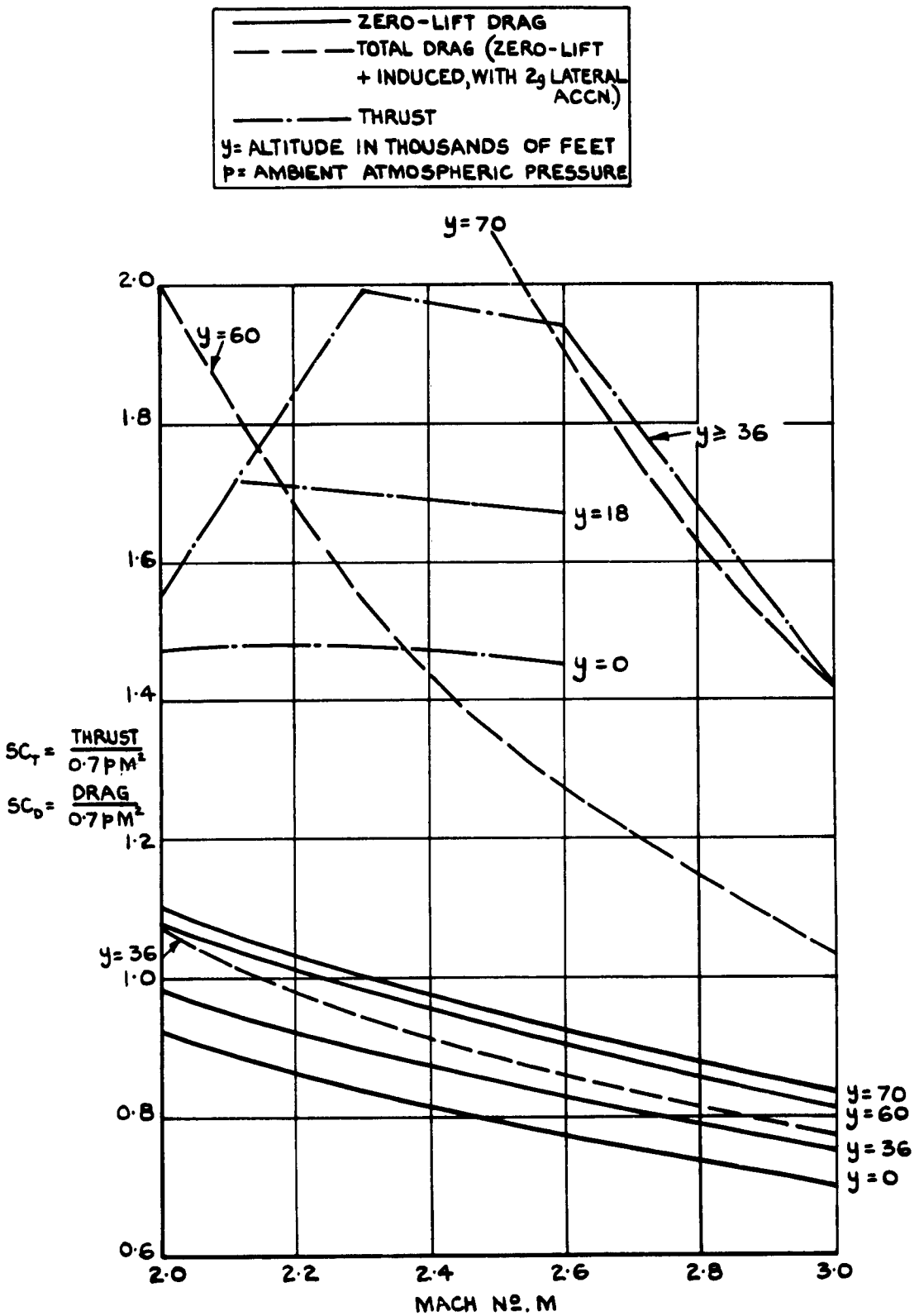


FIG. 6. VARIATION OF THRUST AND DRAG OF THE MISSILE OF FIG. 1. WITH MACH NUMBER AND ALTITUDE.

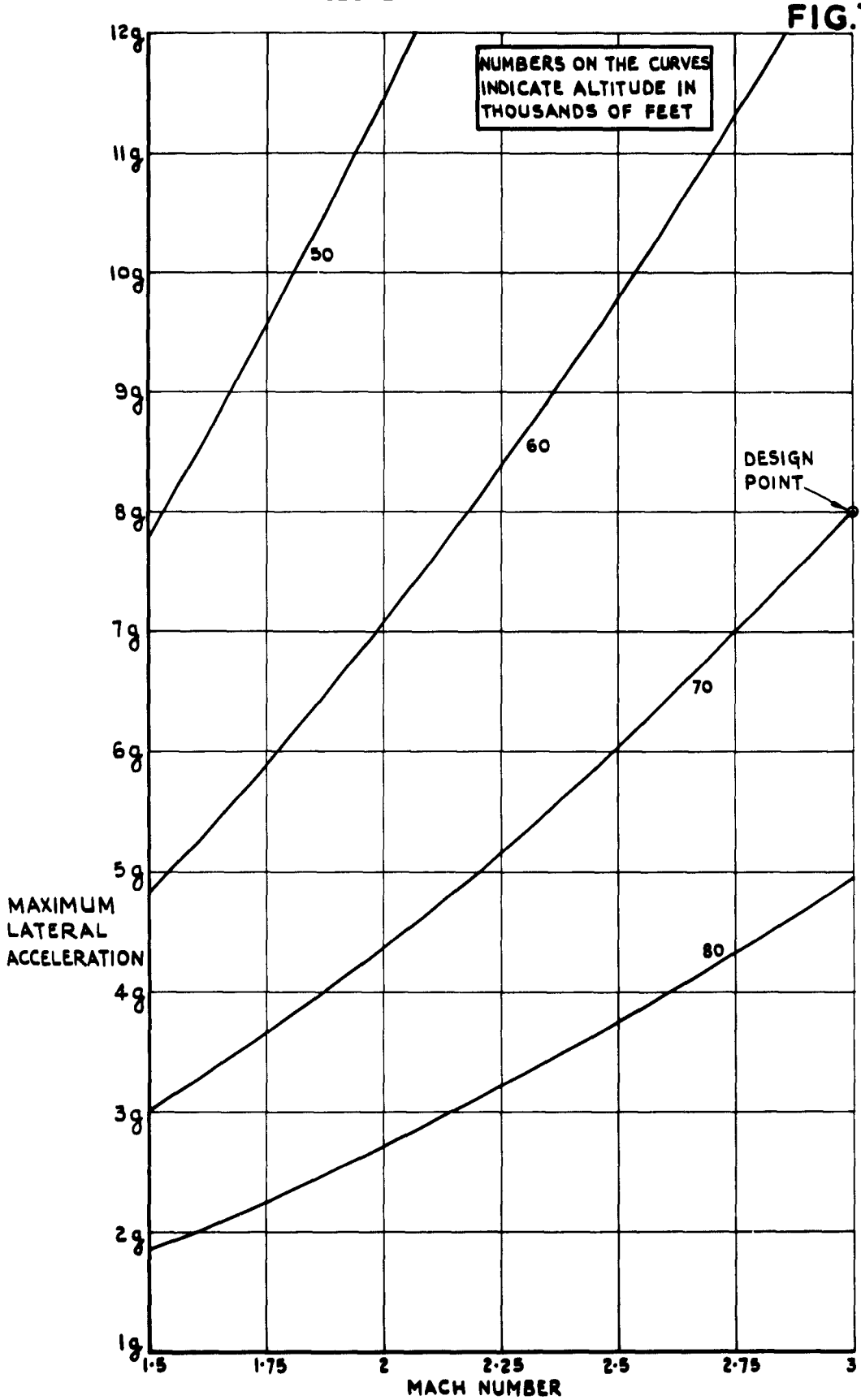


FIG.7. VARIATION OF MAX. LATERAL ACCELERATION OF THE MISSILE OF FIG.1 WITH MACH NUMBER AND ALTITUDE.

UP-AND-ALONG TRAJECTORY: STEEP CLIMB TO 36,000 FT. ALTITUDE, FOLLOWED BY CLIMB AT LOWER ANGLE (5-35°) TO DESIGN ALTITUDE, AND LEVEL FLIGHT AT DESIGN ALTITUDE y_1. SPEED M = 2 AT SEA LEVEL, M = 3 AT ALTITUDE y_1.
ALL-UP WEIGHT = WEIGHT AT LAUNCH WITHOUT BOOSTS

MISSILE BODY DIAMETER = 2 FEET
PAYLOAD WEIGHT = 700 LB.
PAYLOAD DENSITY = 51·8 LB/CU.FT.
MAX. LATERAL ACCN. = 8g (N = 8)
R.M.S. LATERAL ACCN. = 2g (π = 2)
MACH Nº AT END OF BOOST M_0 = 2
y_1 IS DESIGN ALTITUDE IN THOUSANDS OF FEET

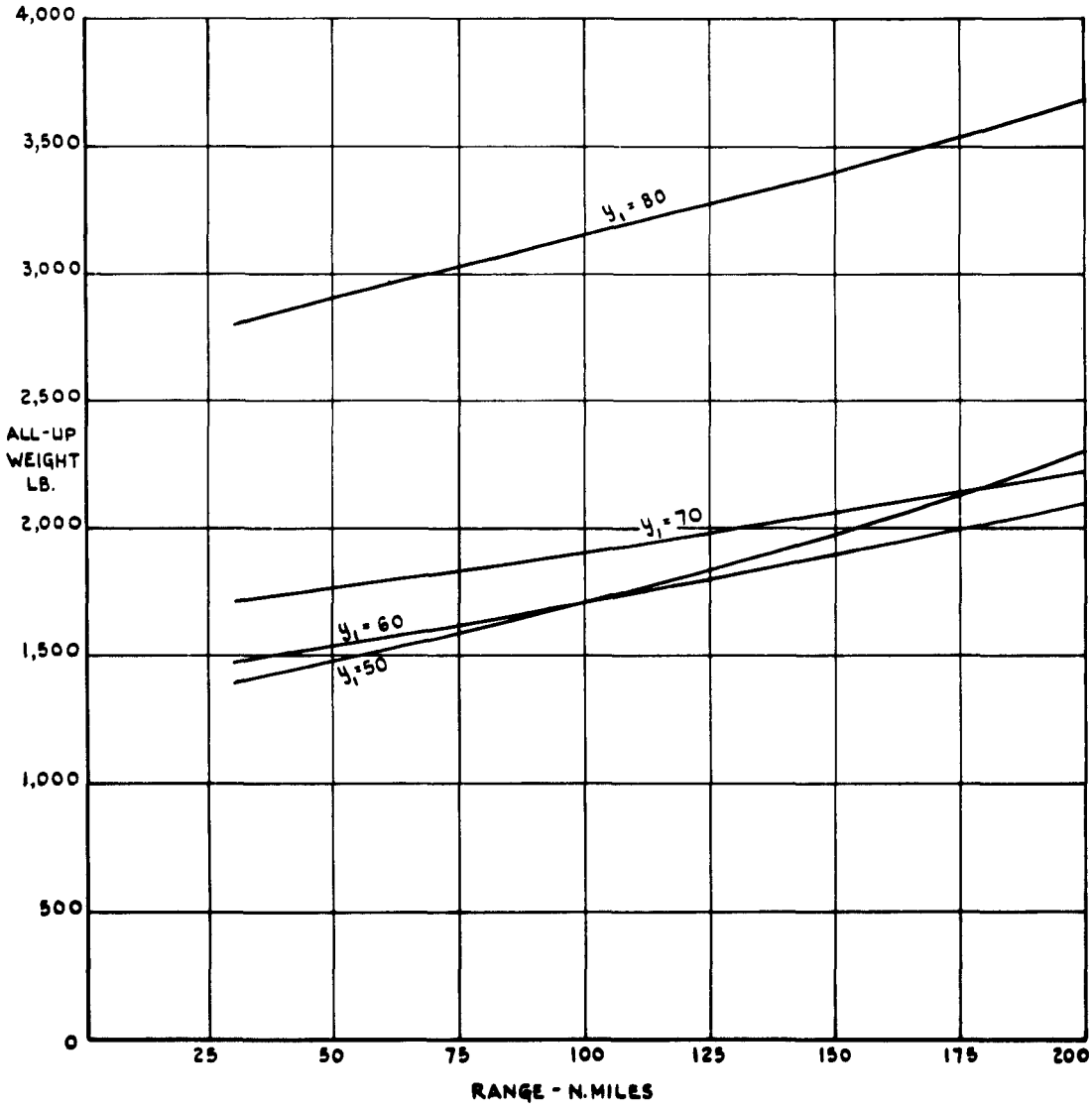


FIG. 8. VARIATION OF MISSILE WEIGHT WITH RANGE, FOR A SERIES OF DESIGN ALTITUDES.

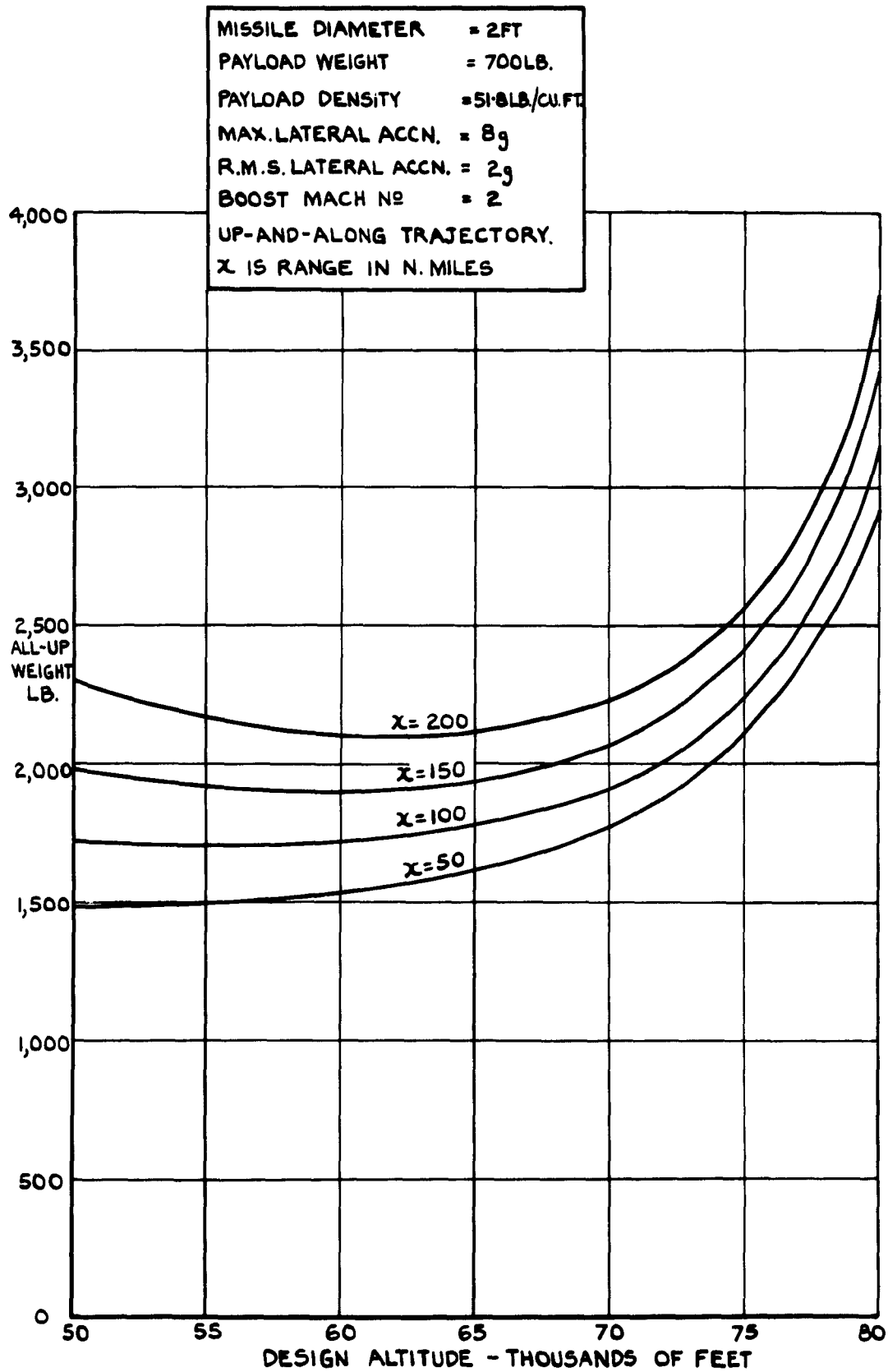


FIG. 9. VARIATION OF MISSILE WEIGHT WITH DESIGN ALTITUDE FOR A SERIES OF RANGES.

DESIGN ALTITUDE	= 70,000 FT.
RANGE	= 87 N. MILES
PAYLOAD WEIGHT	= 700 LB.
PAYLOAD DENSITY	= 51·8 LB/CU.FT.
MAX. LAT. ACCN.	= 8g
R.M.S. LAT. ACCN.	= 2g
BOOST MACH Nº	= 2
UP-AND-ALONG TRAJECTORY	

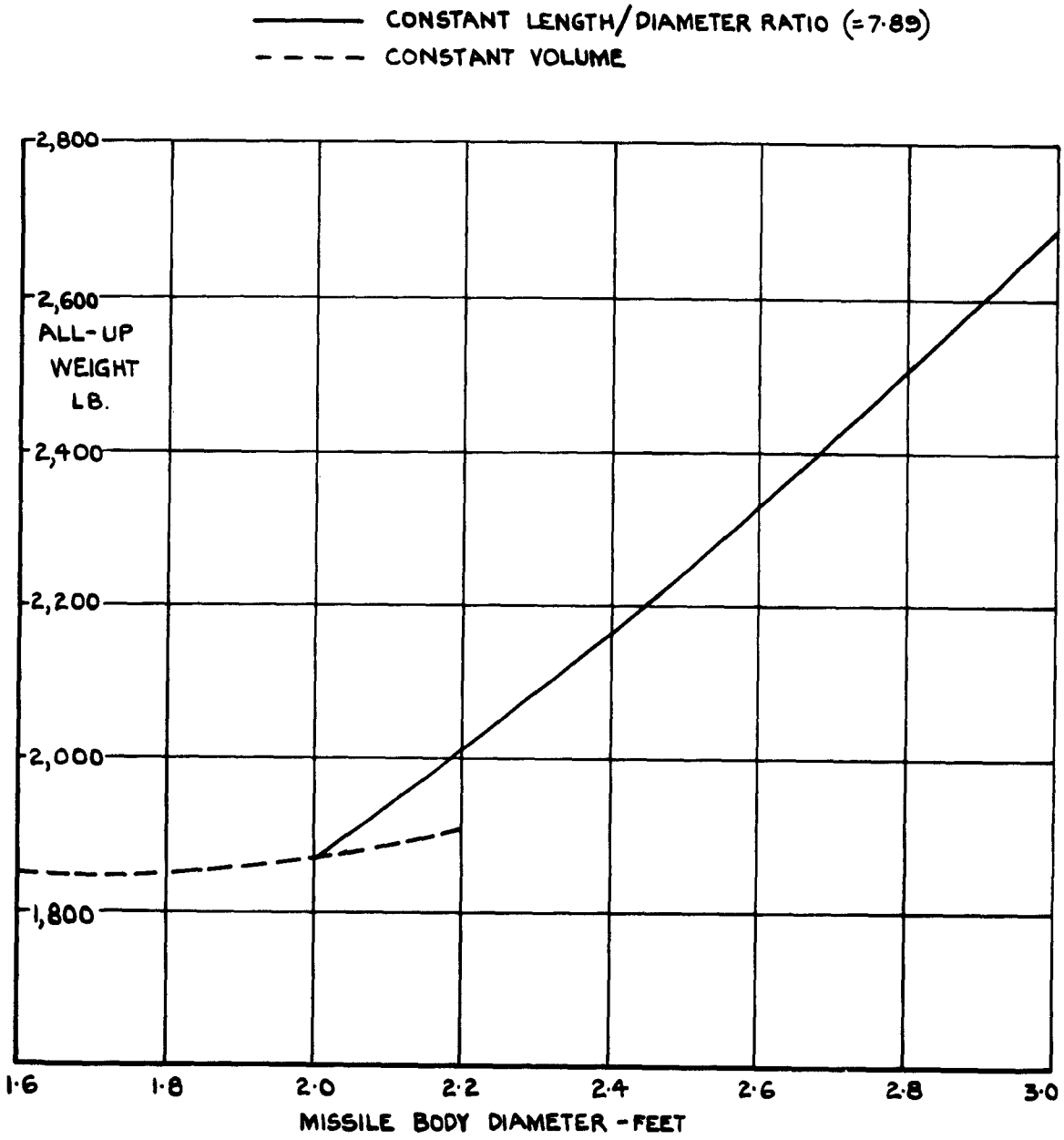


FIG. 10. VARIATION OF MISSILE WEIGHT WITH DIAMETER.

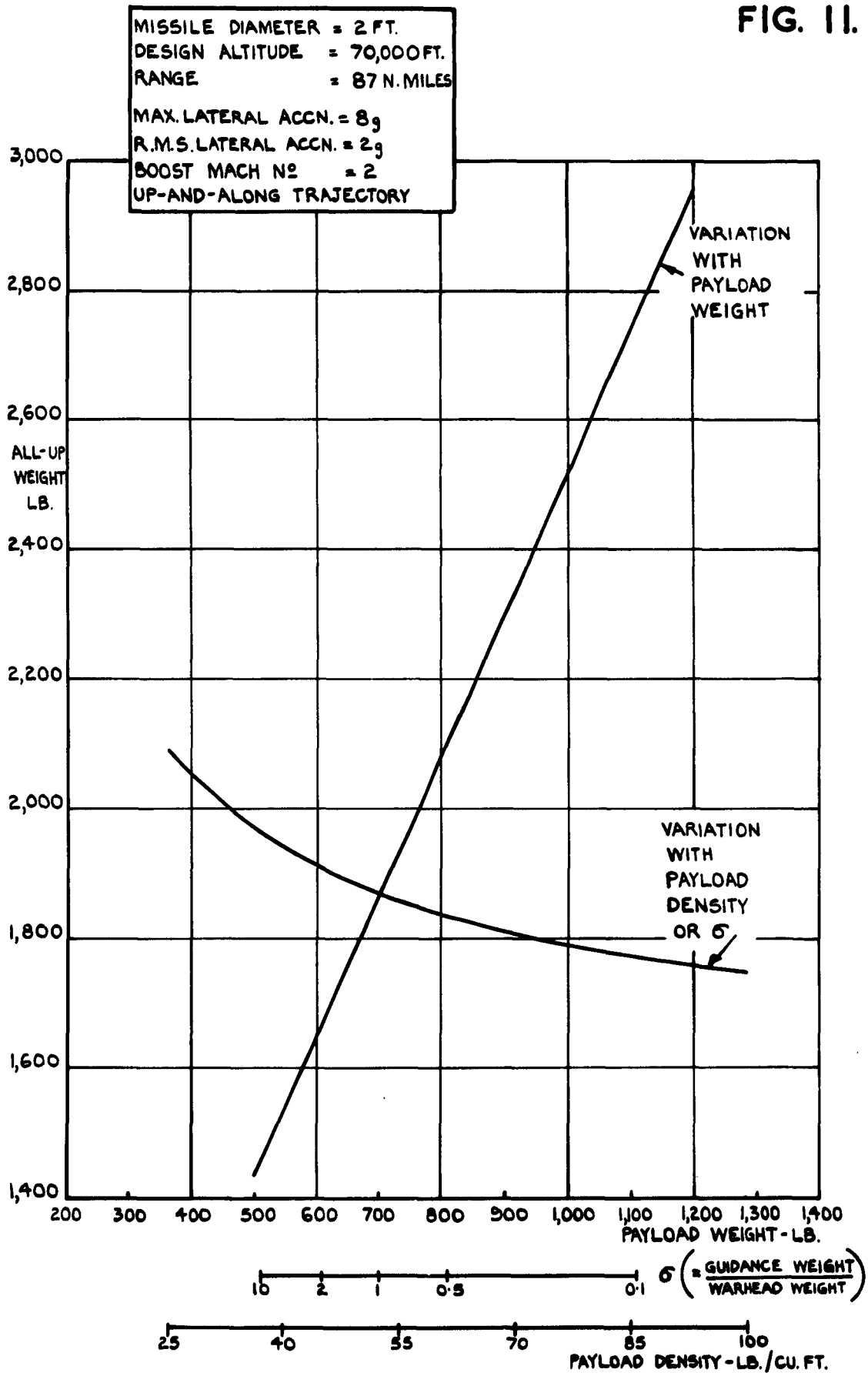


FIG. 11. VARIATION OF MISSILE WEIGHT WITH PAYLOAD WEIGHT AND DENSITY (PAYLOAD = WARHEAD PLUS GUIDANCE.)

FIG. 12.

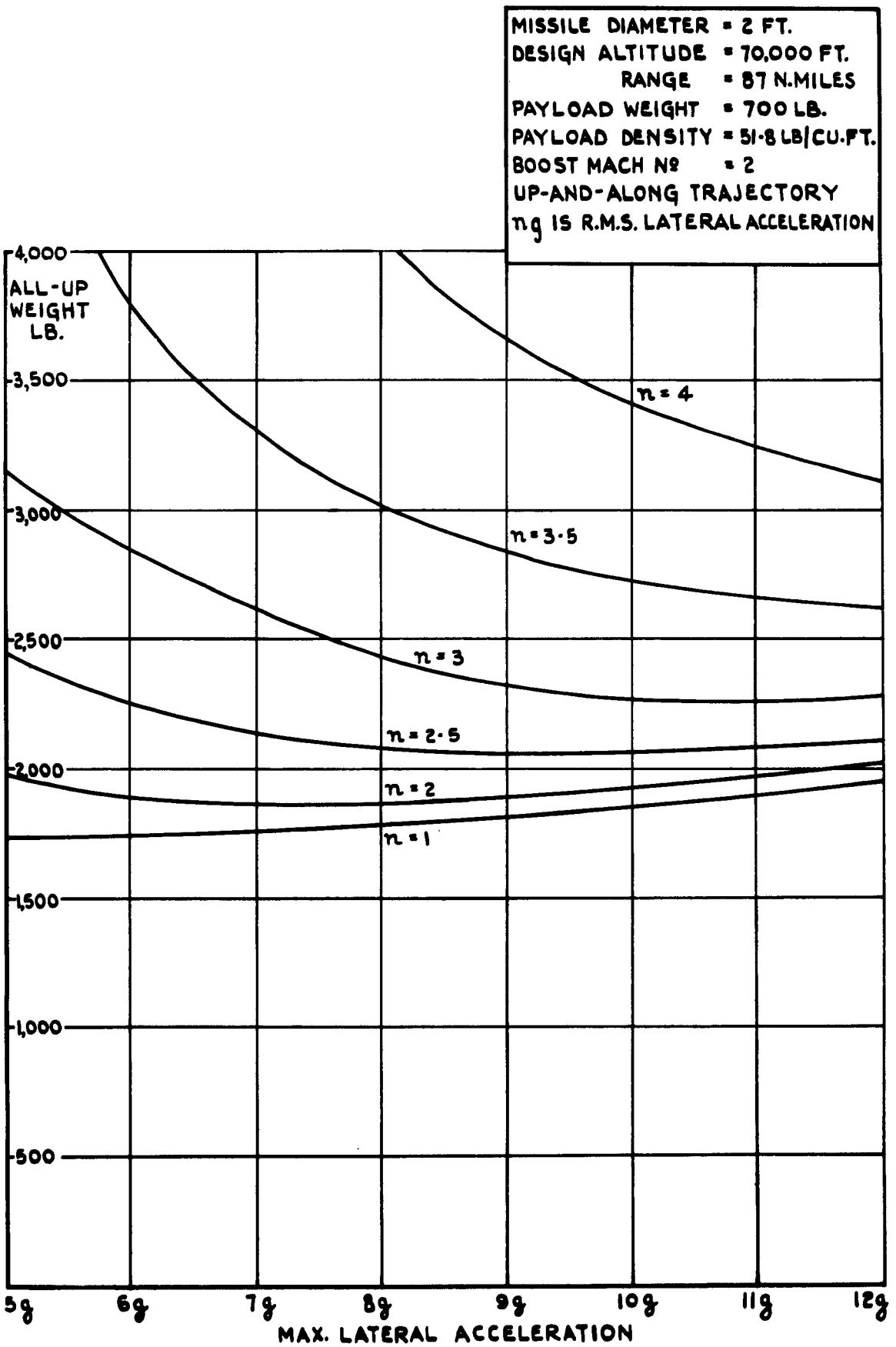


FIG. 12. VARIATION OF MISSILE WEIGHT WITH MAX. LATERAL ACCELERATION FOR A SERIES OF R.M.S. LATERAL ACCELERATIONS.

SECRET.

FIG. 13.

MISSILE DIAMETER	= 2 FT.
DESIGN ALTITUDE	= 70,000 FT.
RANGE	= 87 N. MILES
PAYLOAD WEIGHT	= 700 LB.
PAYLOAD DENSITY	= 51·8 LB./CU. FT.
BOOST MACH Nº	= 2
UP-AND-ALONG TRAJECTORY	
N_g IS MAX. LATERAL ACCELERATION	

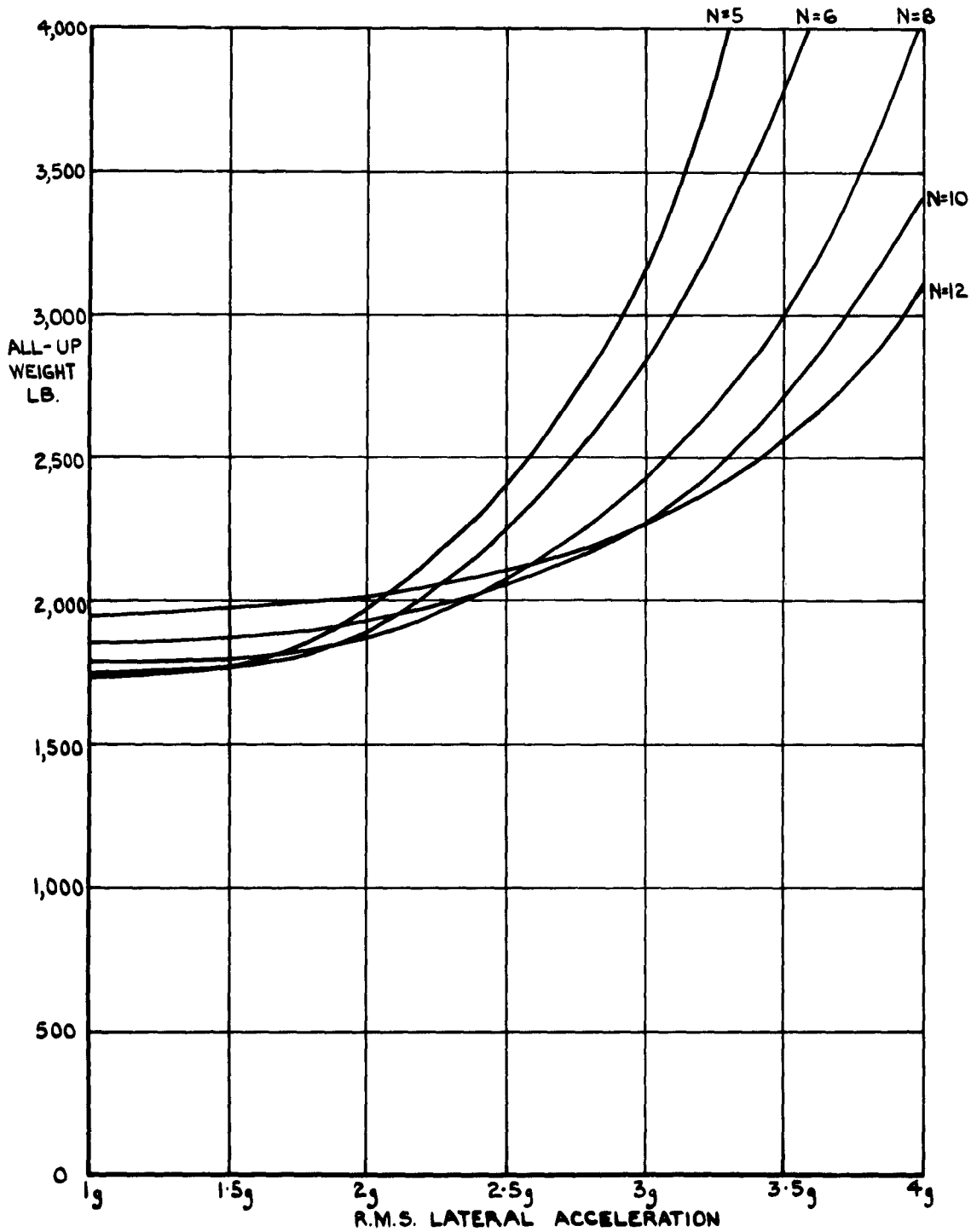


FIG. 13. VARIATION OF MISSILE WEIGHT WITH R.M.S. LATERAL ACCELERATION FOR A SERIES OF MAX. LATERAL ACCELERATIONS.

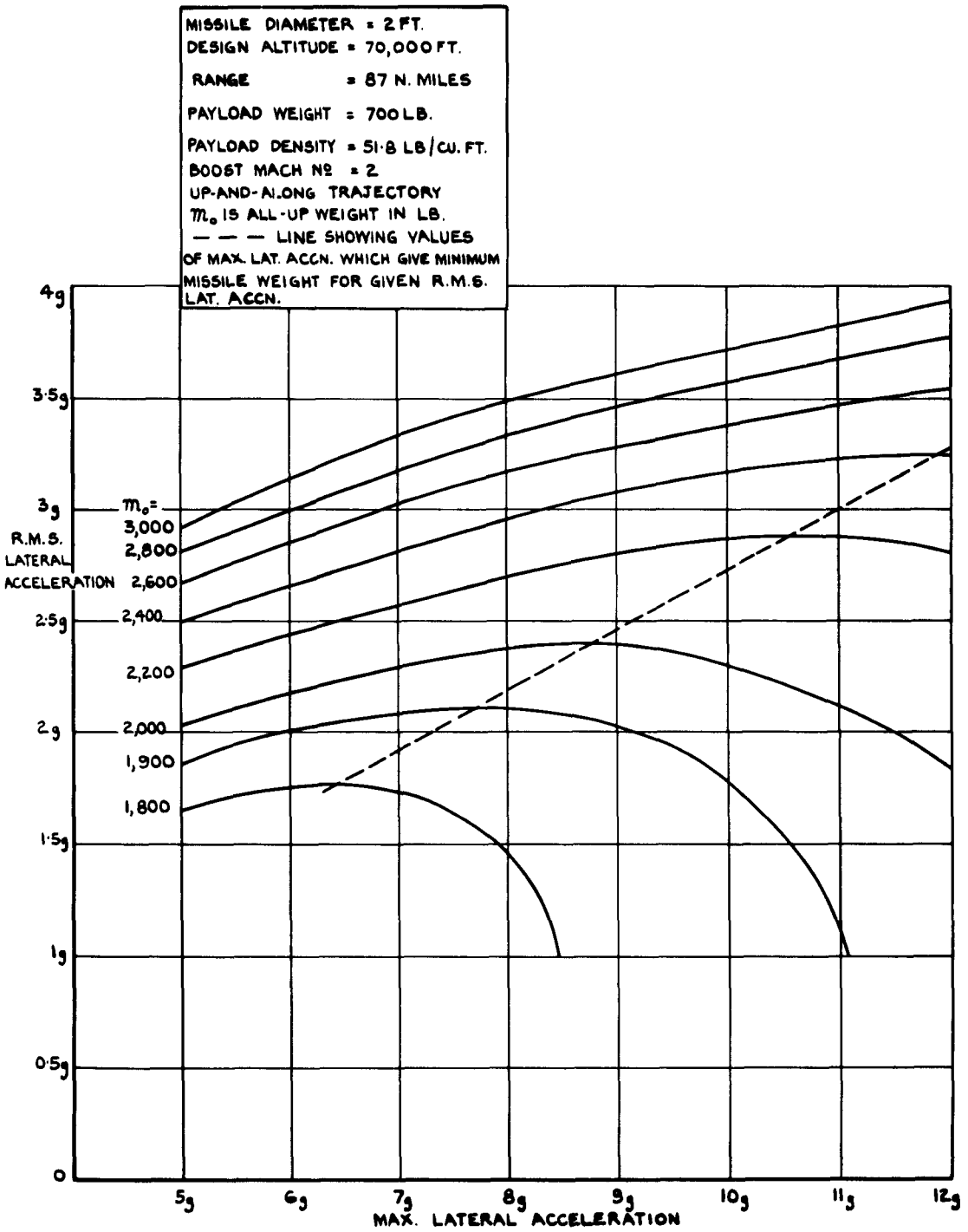


FIG. 14. VARIATION OF R.M.S. LATERAL ACCELERATION WITH MAX. LATERAL ACCELERATION FOR A SERIES OF MISSILE WEIGHTS.

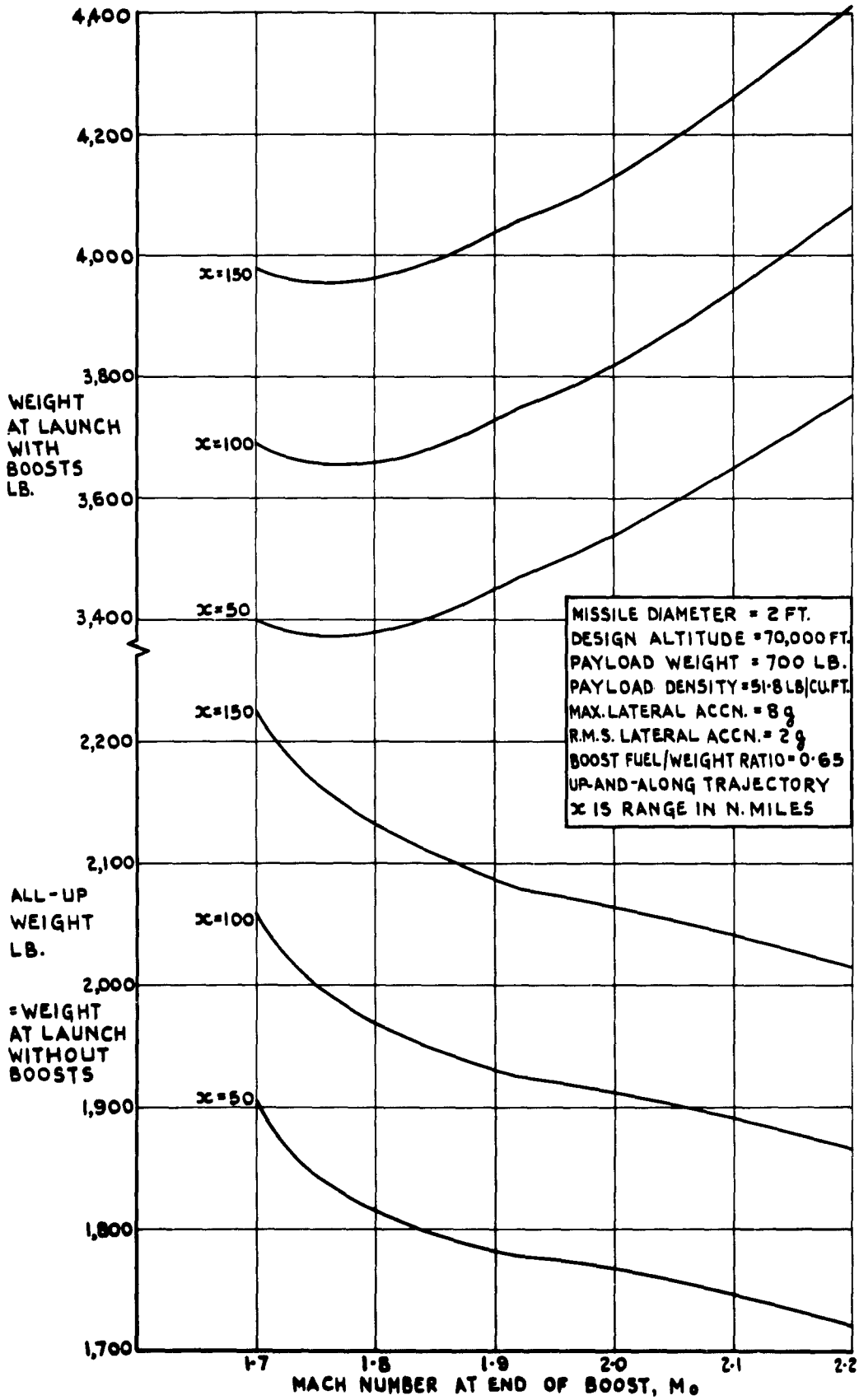


FIG.15. WEIGHT AT LAUNCH, WITH AND WITHOUT BOOSTS, AS A FUNCTION OF MACH NUMBER AT END OF BOOST FOR A SERIES OF RANGES.

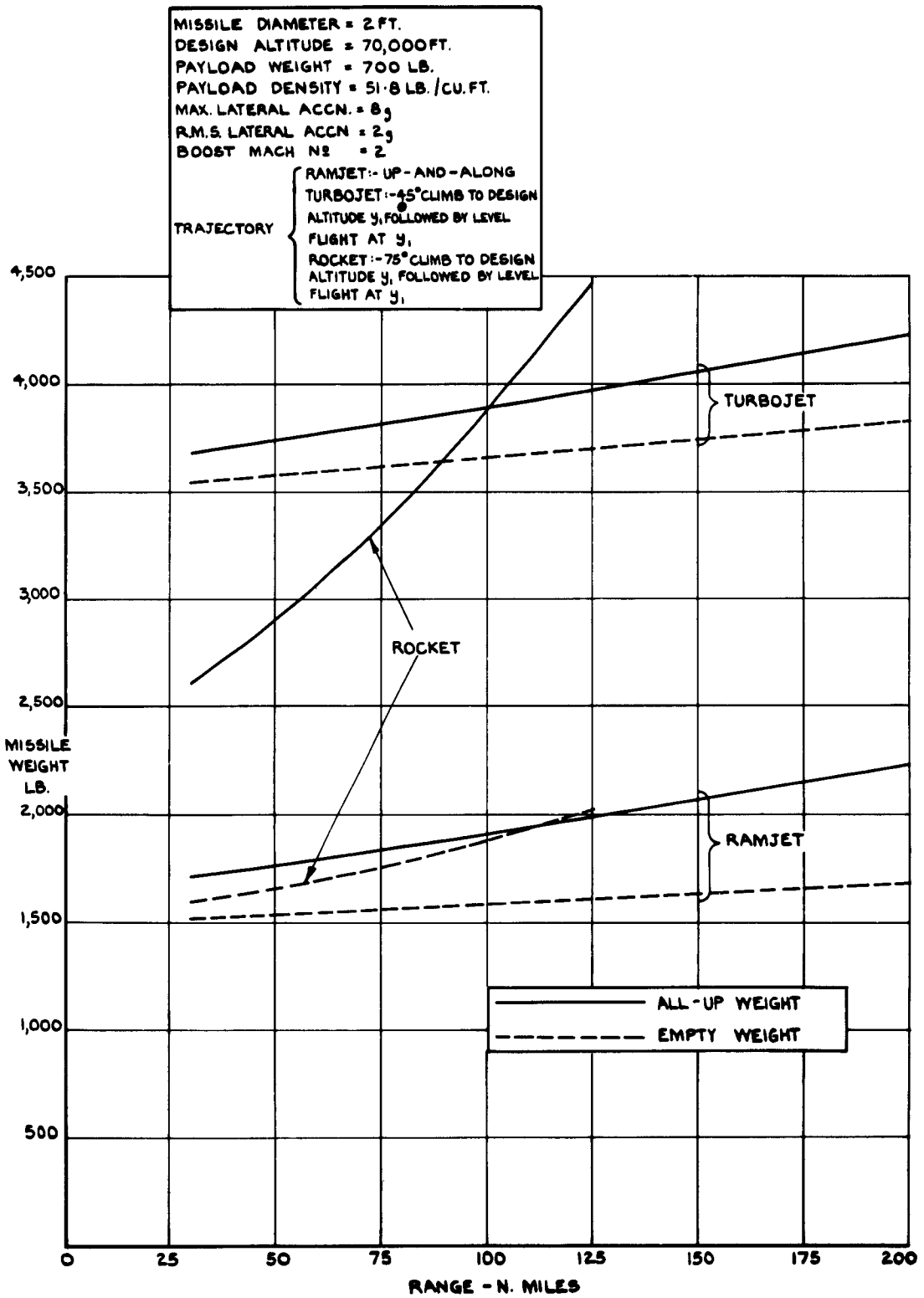


FIG. 16. VARIATION OF WEIGHT WITH RANGE FOR TURBOJET, ROCKET AND RAMJET MISSILES.

FIG. 17.

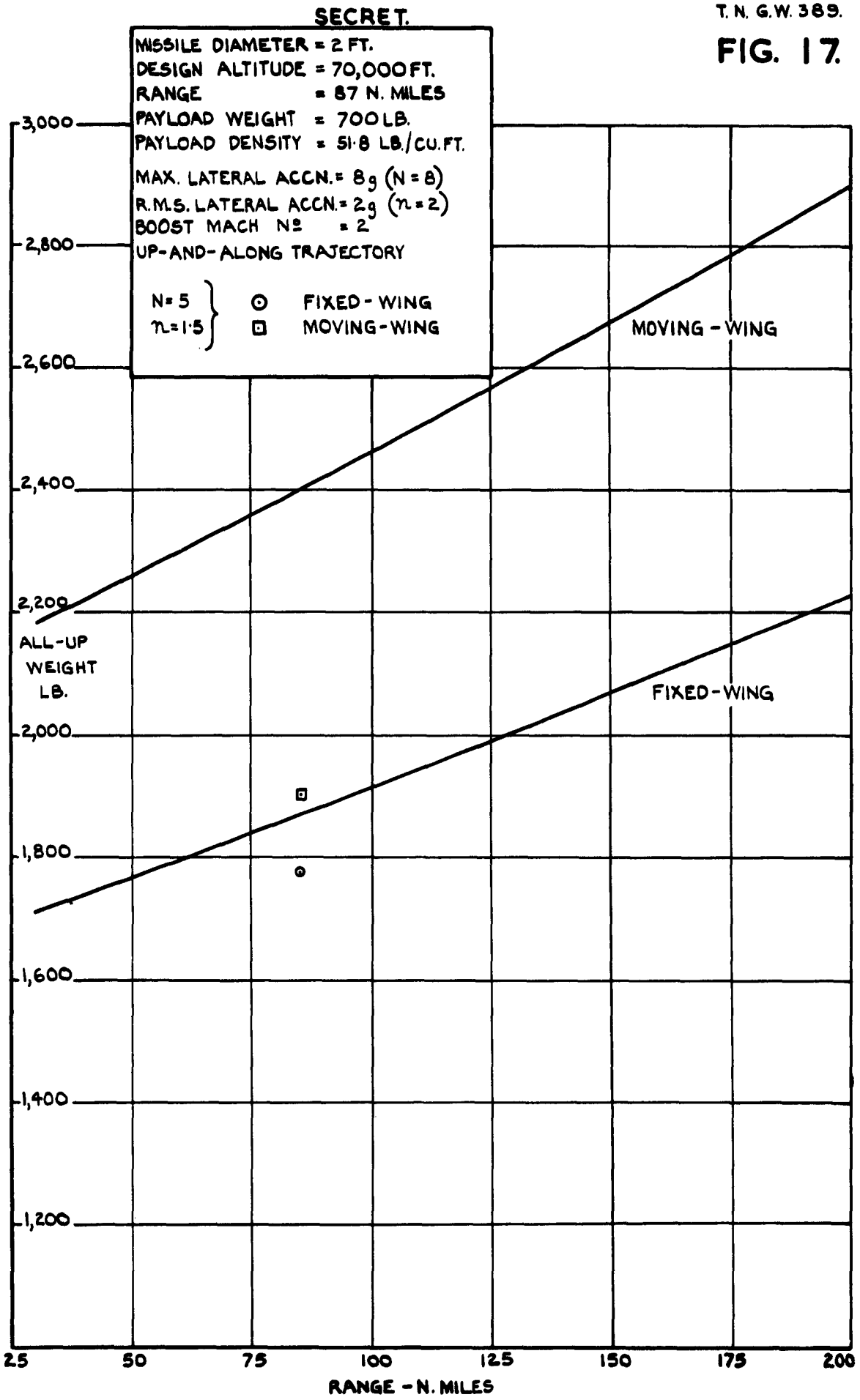


FIG. 17 VARIATION OF WEIGHT WITH RANGE FOR MOVING-WING AND FIXED-WING MISSILES.

MISSILE DIAMETER = 2FT.
PAYLOAD WEIGHT = 700LB
PAYLOAD DENSITY = 51·8 LB/CUFT
MAX. LATERAL ACCN. = 8g
R.M.S. LATERAL ACCN. = 2g
BOOST MACH Nº = 2
UP-AND-ALONG TRAJECTORY.

y_1 IS MISSILE DESIGN ALTITUDE IN THOUSANDS OF FEET.

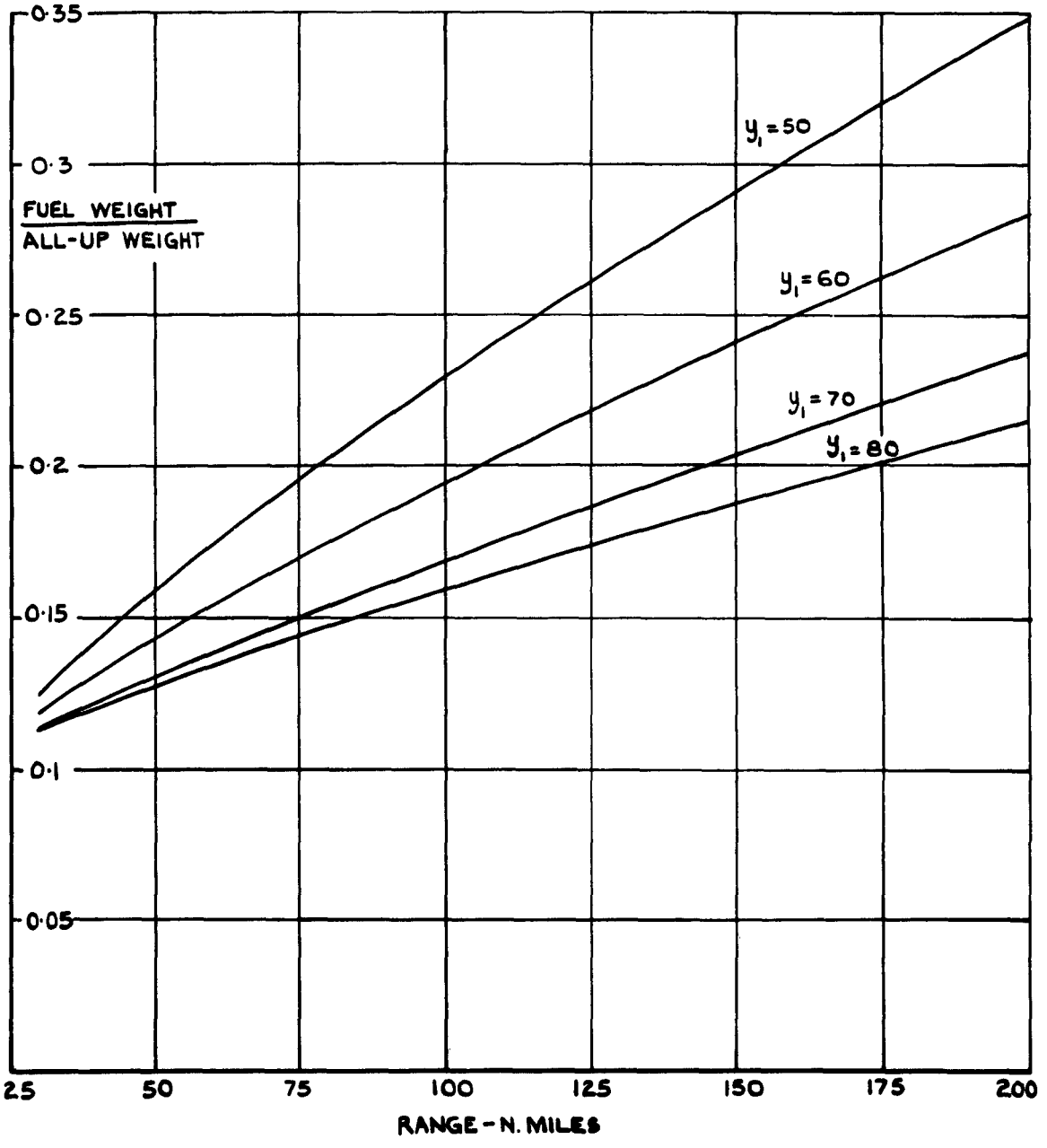


FIG. 18. VARIATION OF FUEL/ALL-UP WEIGHT RATIO WITH RANGE ON UP-AND-ALONG TRAJECTORIES FOR A SERIES OF MISSILE DESIGN ALTITUDES.

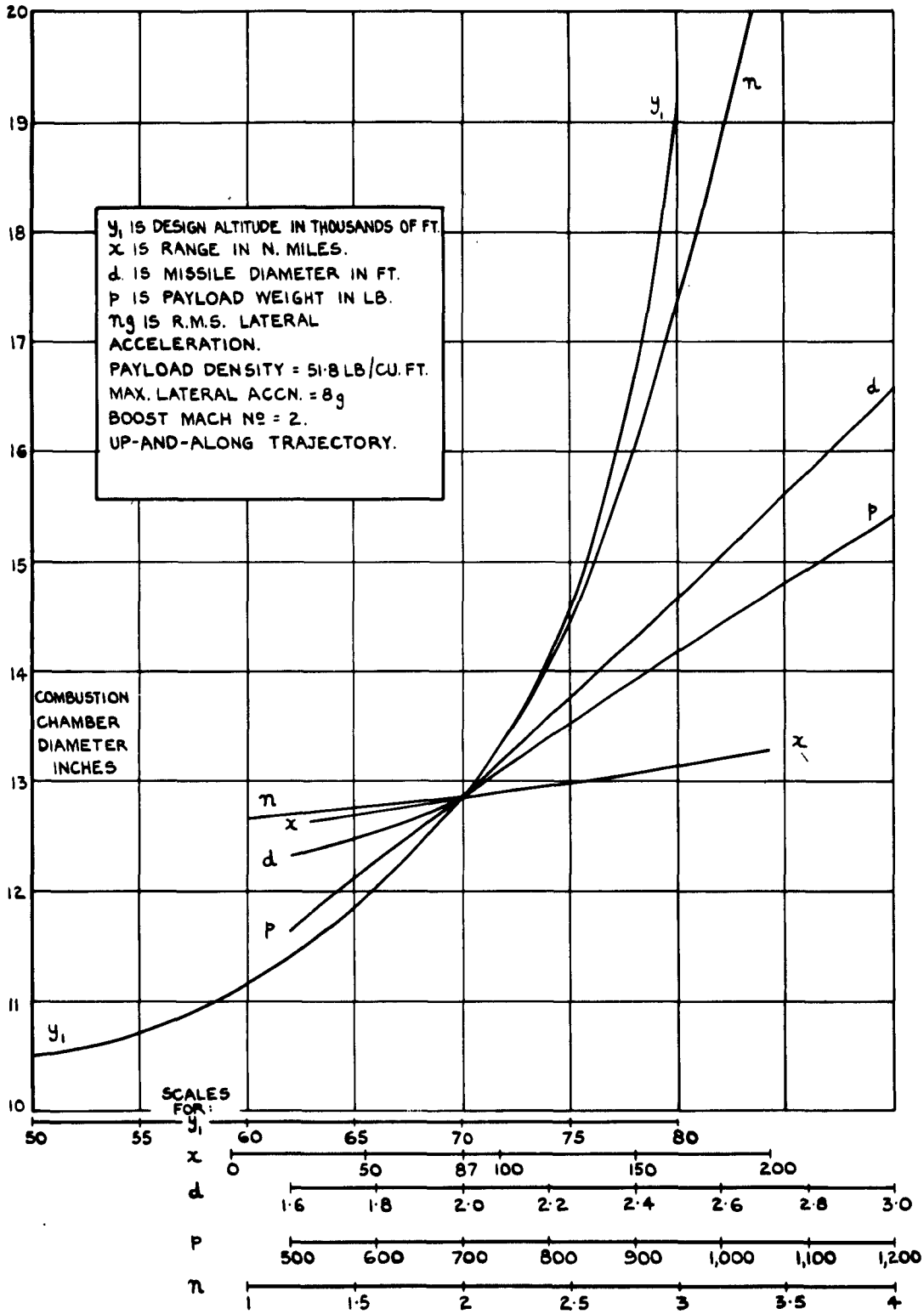


FIG. 19. VARIATION OF RAMJET COMBUSTION CHAMBER DIAMETER WHEN EACH OF THE DESIGN PARAMETERS SHOWN DEPARTS IN TURN FROM ITS STANDARD VALUE.

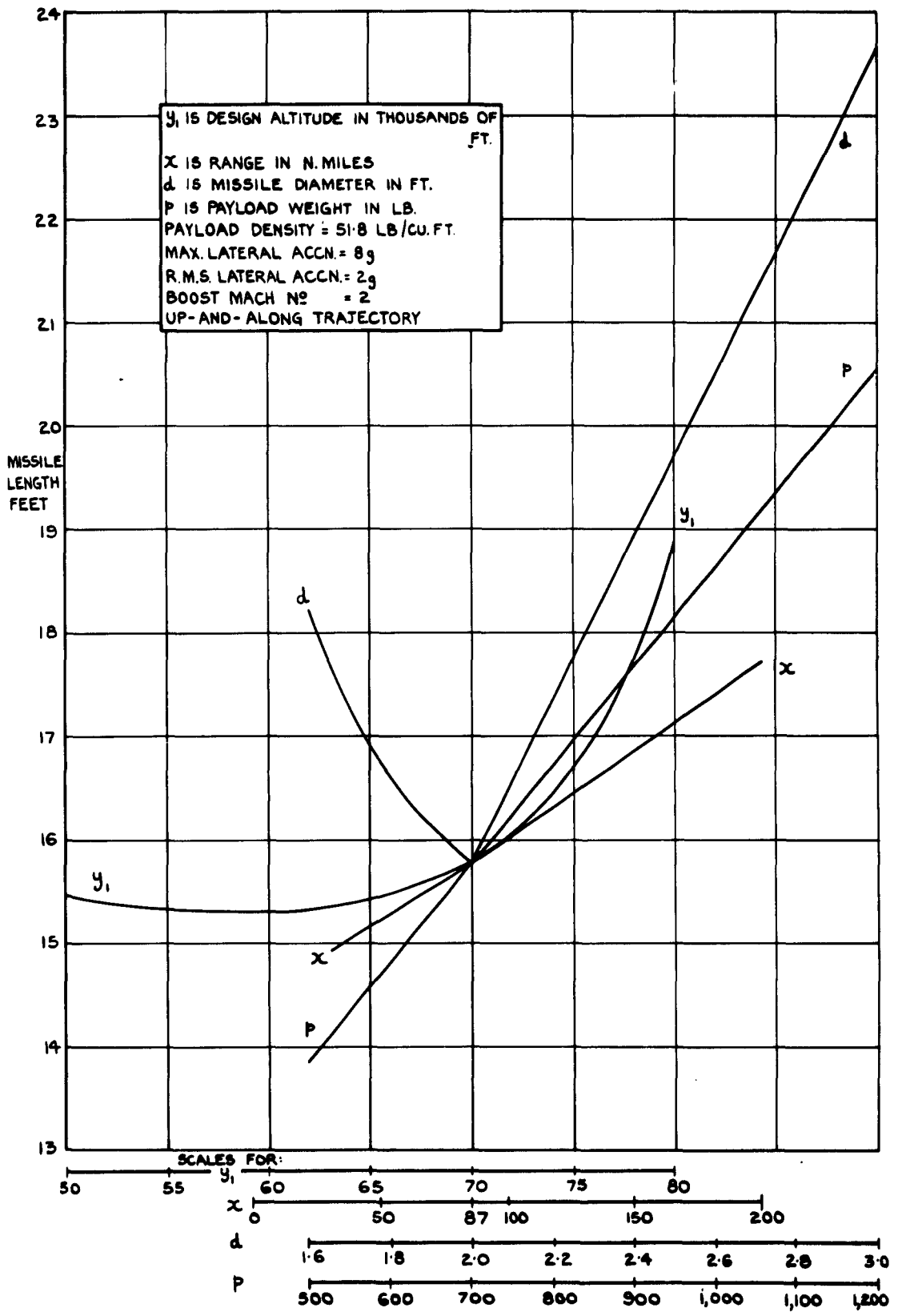


FIG. 20. VARIATION OF MISSILE LENGTH WHEN EACH OF THE DESIGN PARAMETERS SHOWN DEPARTS IN TURN FROM ITS STANDARD VALUE.

MISSILE DIAMETER = 2 FT.
DESIGN ALTITUDE y_1 = 70,000 FT.
PAYLOAD WEIGHT = 700 LB
PAYLOAD DENSITY = 51·8 LB/CU.FT.
MAX. LATERAL ACCN. = 8g
R.M.S. LATERAL ACCN. = 2g
BOOST MACH Nº = 2
UP-AND-ALONG TRAJECTORY
y_T IS TARGET ALTITUDE IN THOUSANDS OF FEET.

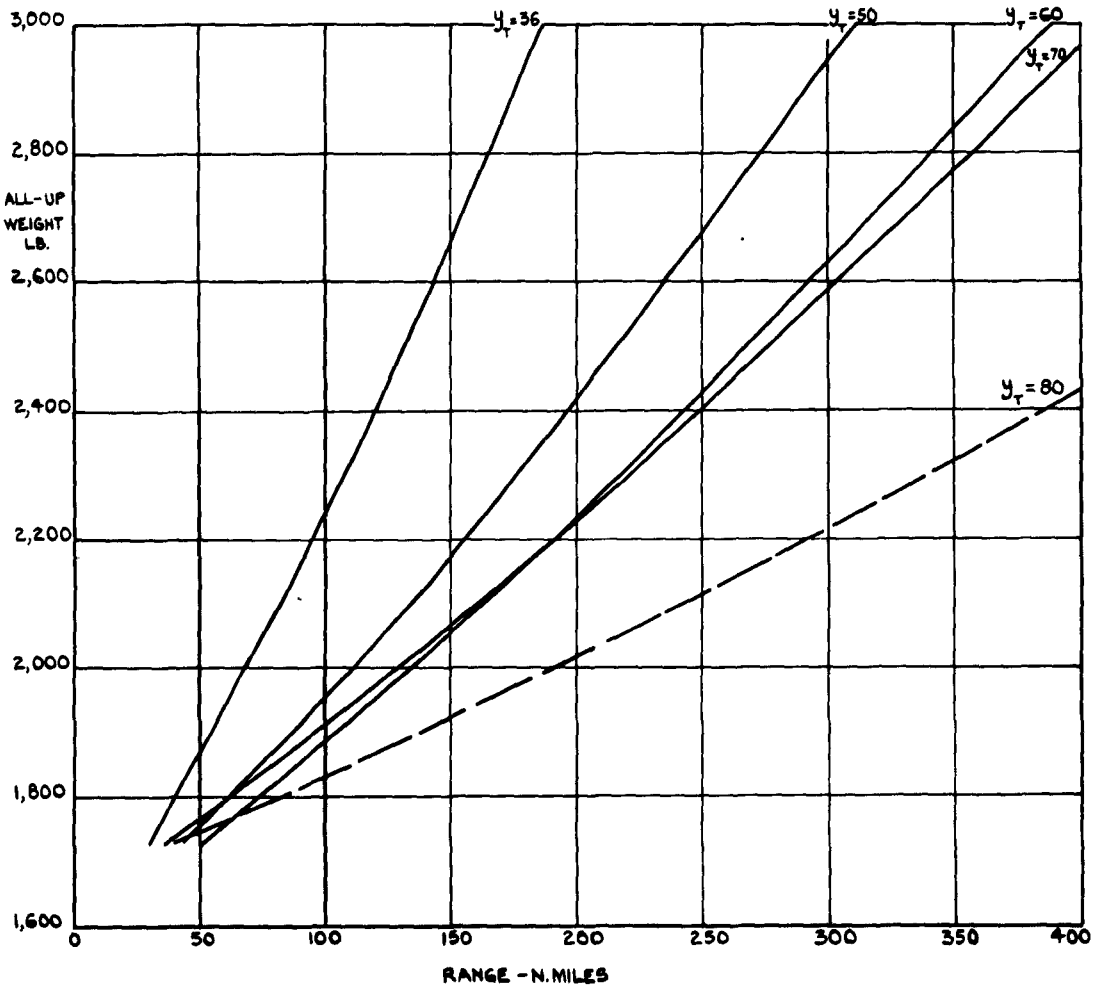


FIG. 21. VARIATION OF WEIGHT WITH RANGE FOR STANDARD MISSILES (DESIGNED FOR 70,000 FT. ALTITUDE) ON UP-AND-ALONG TRAJECTORIES TO VARIOUS TARGET ALTITUDES.

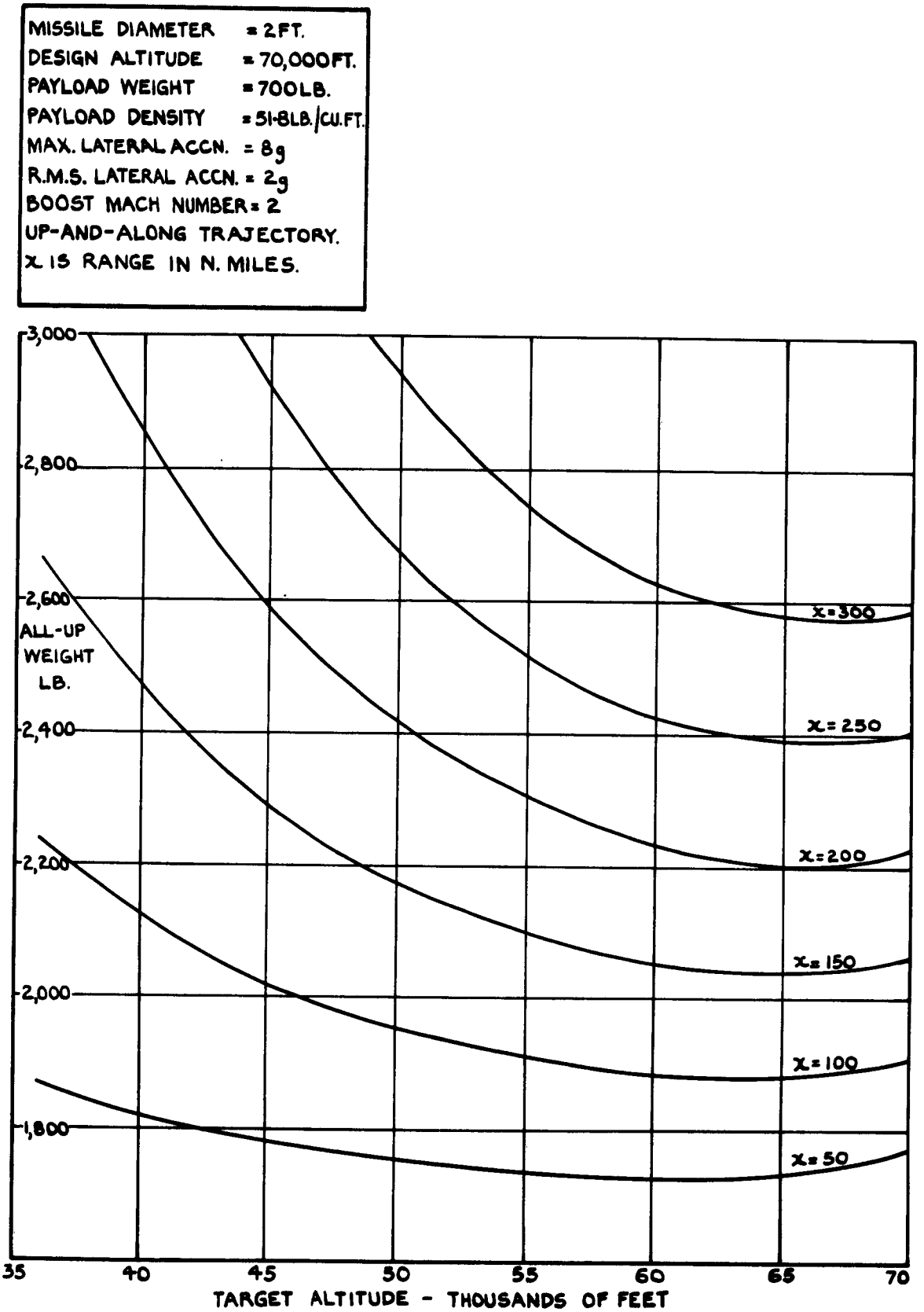


FIG. 22. VARIATION OF MISSILE WEIGHT WITH TARGET ALTITUDE FOR STANDARD MISSILES (DESIGNED FOR 70,000 FT. ALTITUDE) ON UP-AND-ALONG TRAJECTORIES, FOR A SERIES OF RANGES.

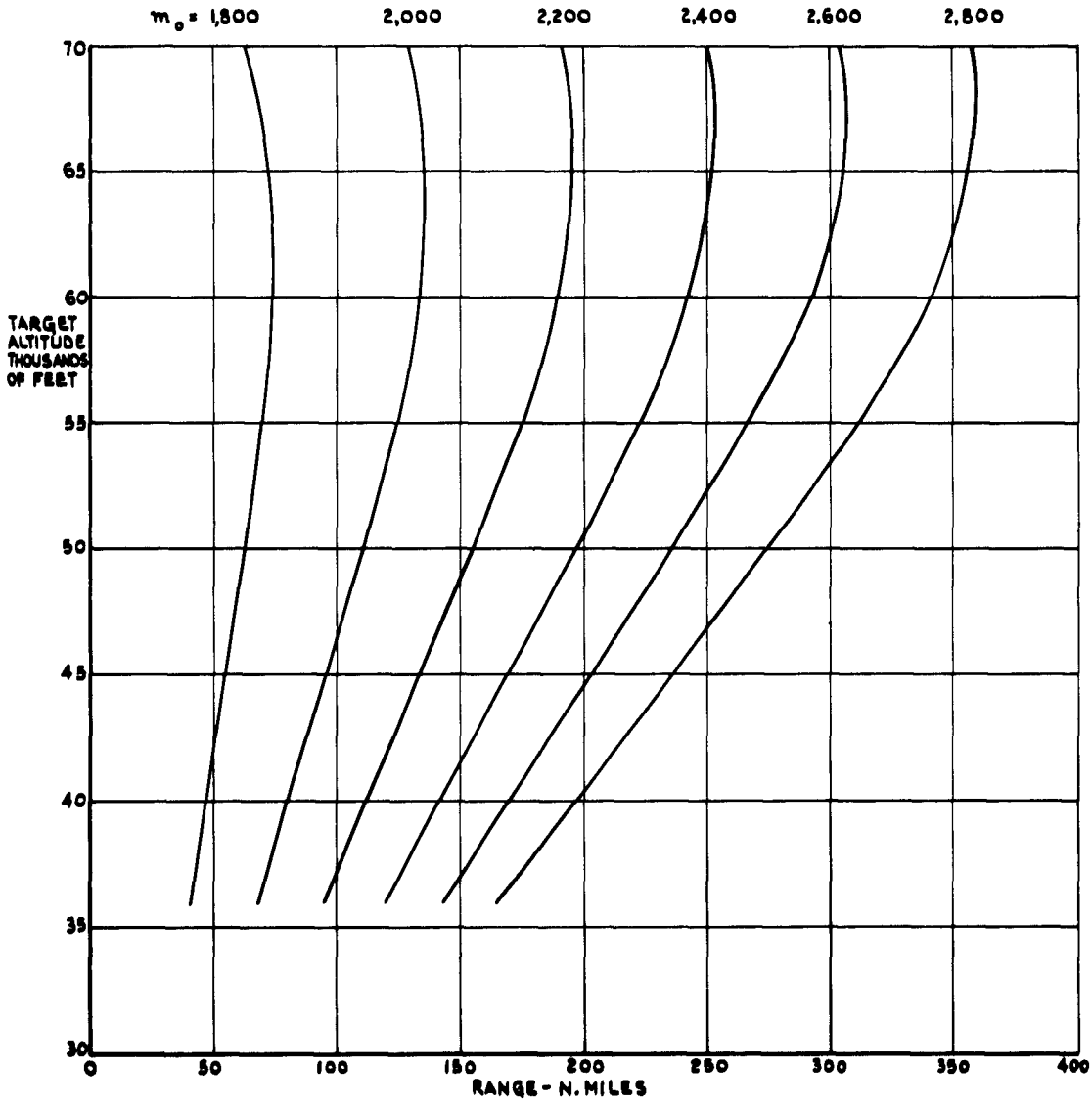


FIG.23. VARIATION OF RANGE WITH TARGET ALTITUDE FOR STANDARD MISSILES (DESIGNED FOR 70,000 FT. ALTITUDE) ON UP-AND-ALONG TRAJECTORIES, FOR A SERIES OF MISSILE WEIGHTS.

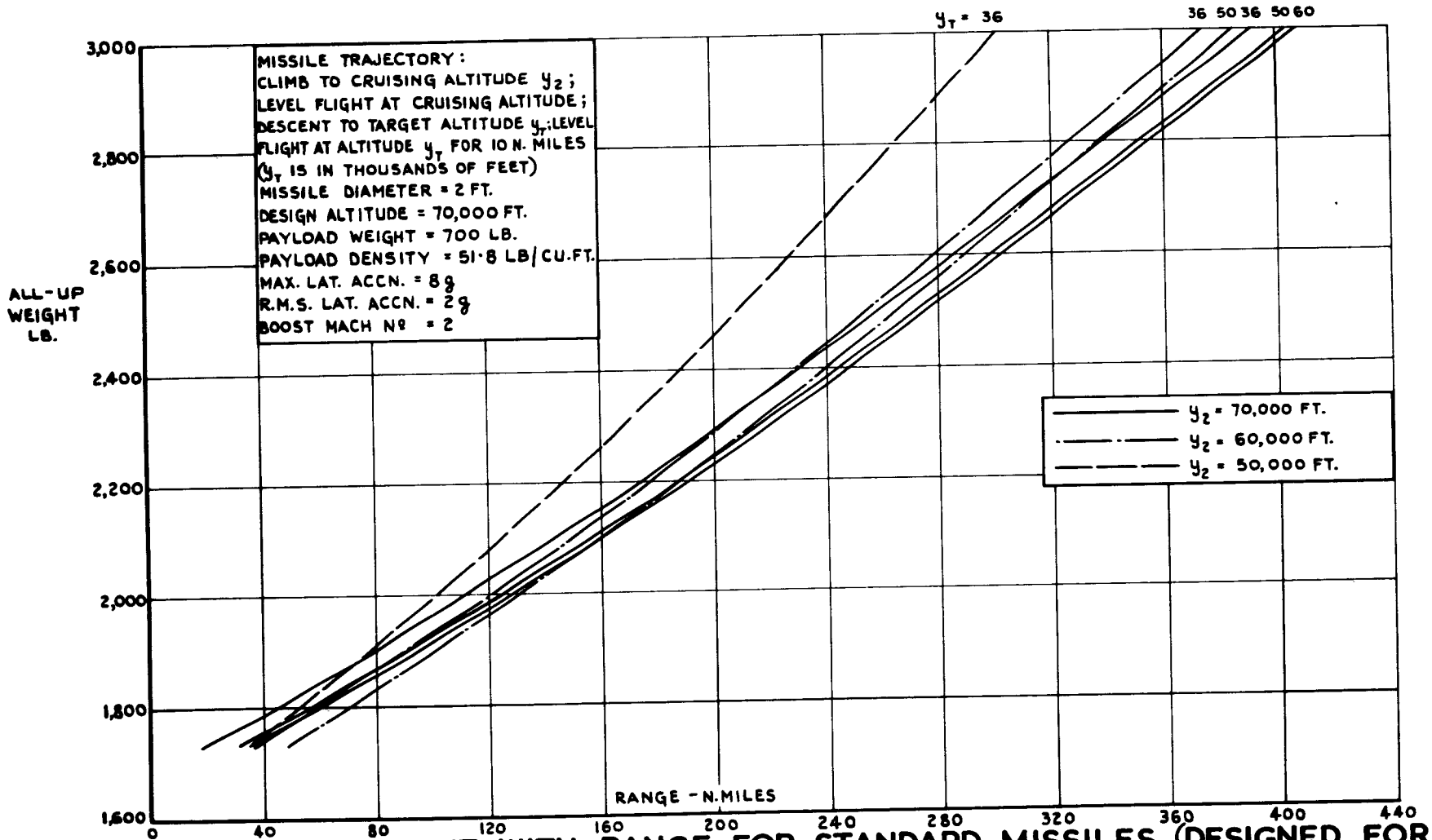


FIG.24. VARIATION OF WEIGHT WITH RANGE FOR STANDARD MISSILES (DESIGNED FOR 70,000 FT. ALTITUDE) ON UP-ALONG-DOWN-ALONG TRAJECTORIES, WITH 10N. MILES FLIGHT AT TARGET ALTITUDE.

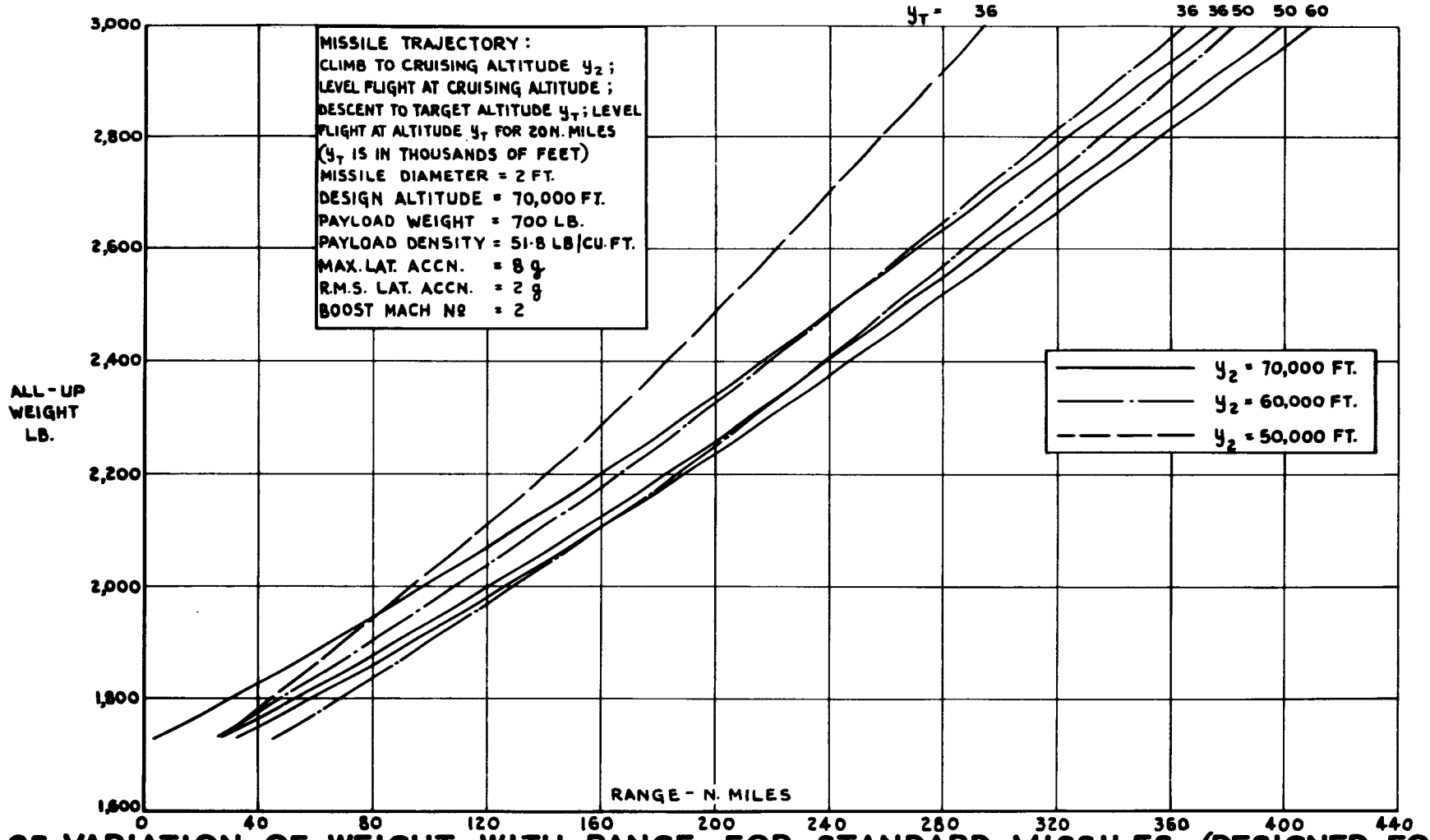


FIG.25. VARIATION OF WEIGHT WITH RANGE FOR STANDARD MISSILES (DESIGNED FOR 70,000 FT. ALTITUDE) ON UP-ALONG-DOWN-ALONG TRAJECTORIES, WITH 20 N. MILES FLIGHT AT TARGET ALTITUDE.

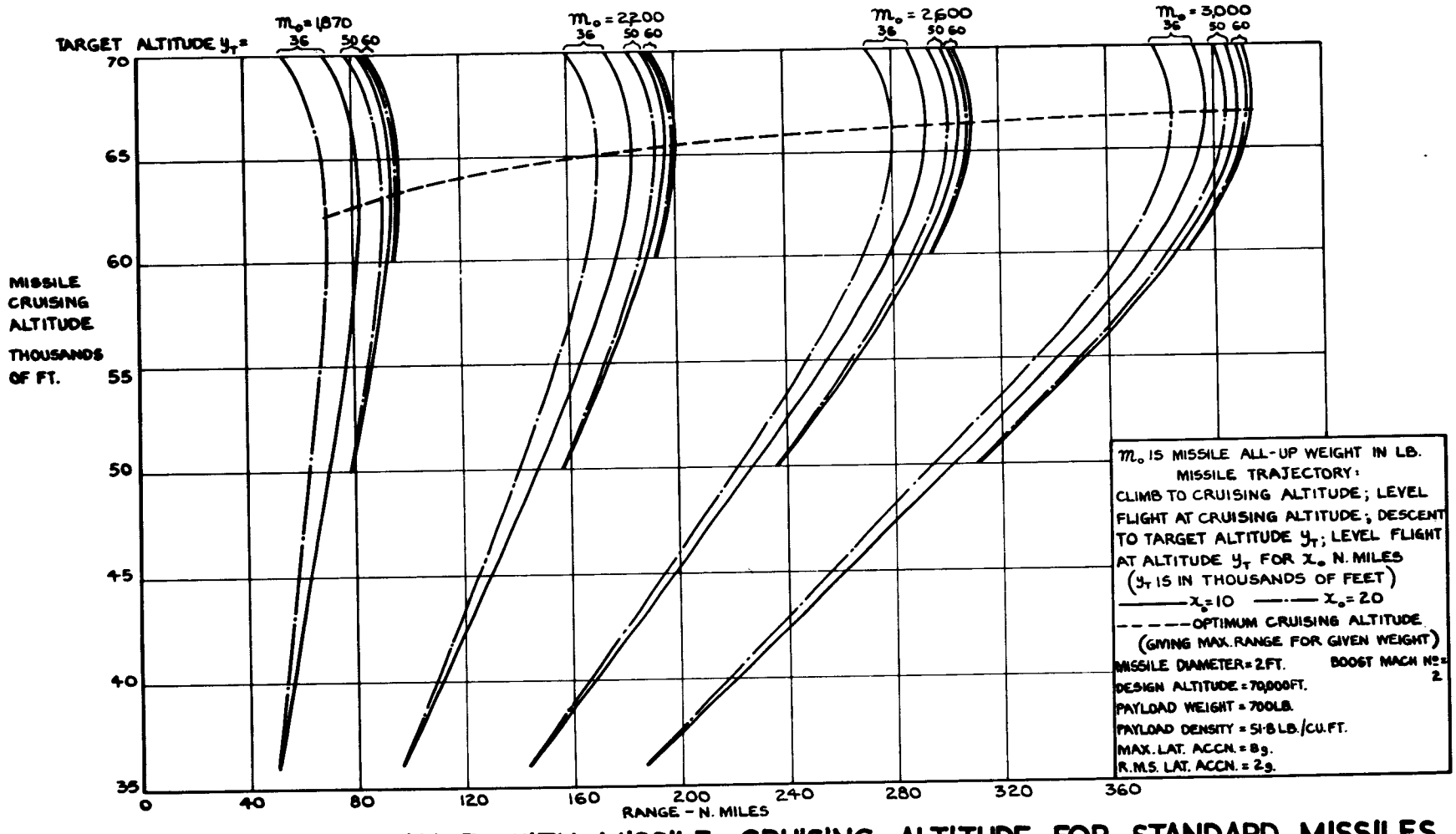
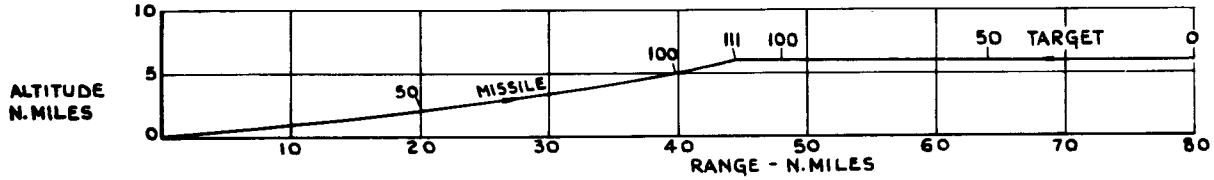
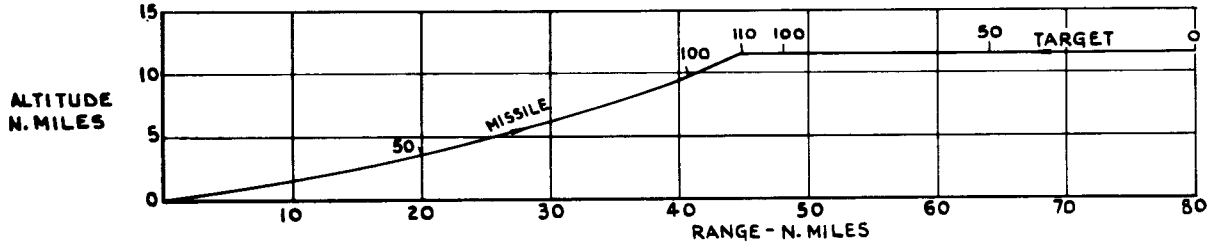


FIG. 26. VARIATION OF RANGE WITH MISSILE CRUISING ALTITUDE FOR STANDARD MISSILES (DESIGNED FOR 70,000 FT. ALTITUDE) ON UP-ALONG-DOWN-ALONG TRAJECTORIES, FOR A SERIES OF MISSILE WEIGHTS AND TARGET ALTITUDES.

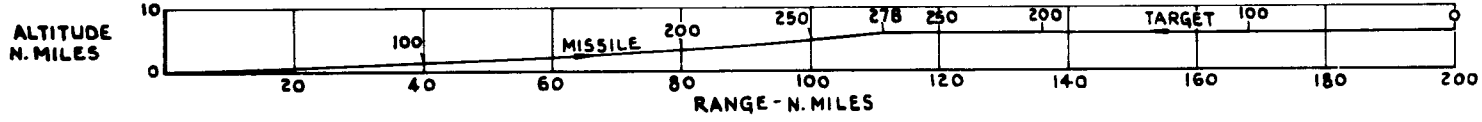


NUMBERS ON TRAJECTORIES REPRESENT TIME FROM MISSILE LAUNCH IN SECS.

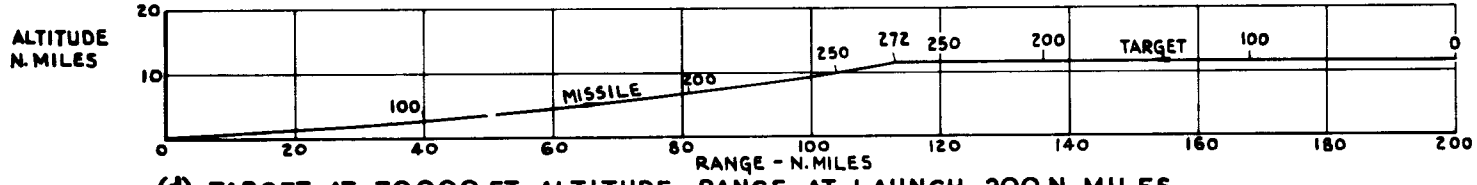
(a). TARGET AT 36,000 FT. ALTITUDE. RANGE AT LAUNCH 80 N. MILES.



(b). TARGET AT 70,000 FT. ALTITUDE. RANGE AT LAUNCH 80 N. MILES.



(c). TARGET AT 36,000 FT. ALTITUDE. RANGE AT LAUNCH 200 N. MILES.



(d). TARGET AT 70,000 FT. ALTITUDE. RANGE AT LAUNCH 200 N. MILES.

FIG.27(a-d). BEAM-RIDING TRAJECTORIES OVER FLAT EARTH.

TARGET SPEED 0·32 N.M/SEC. ($M \simeq 2$). MISSILE SPEED 0·4 N.M./SEC. FOR ALTITUDE $y < 6$ N.M., 0·45 N.M./SEC. FOR $y \geq 6$ N.M.

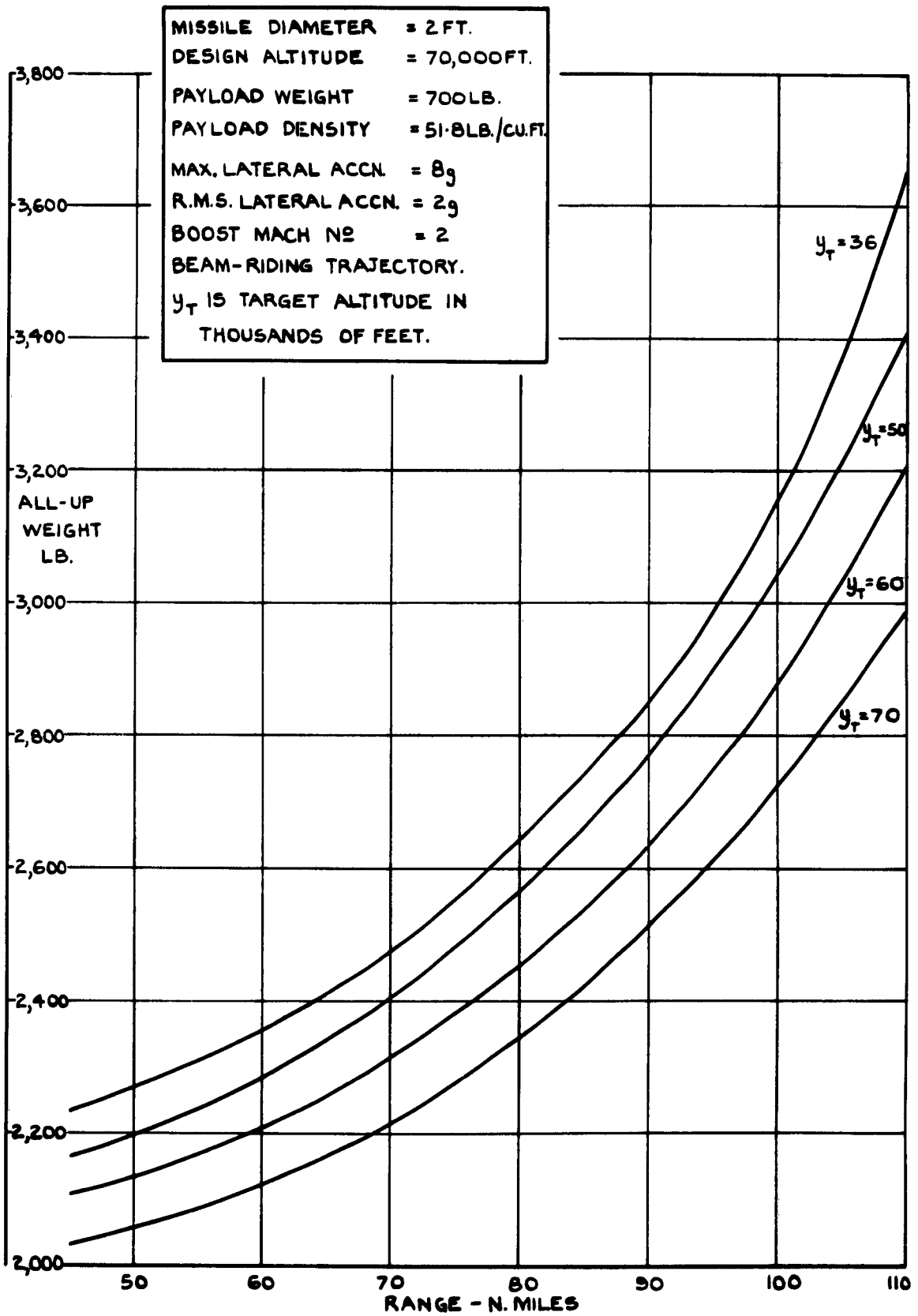


FIG. 28. VARIATION OF WEIGHT WITH RANGE FOR STANDARD MISSILES (DESIGNED FOR 70,000 FT. ALTITUDE) ON BEAM-RIDING TRAJECTORIES TO VARIOUS TARGET ALTITUDES.

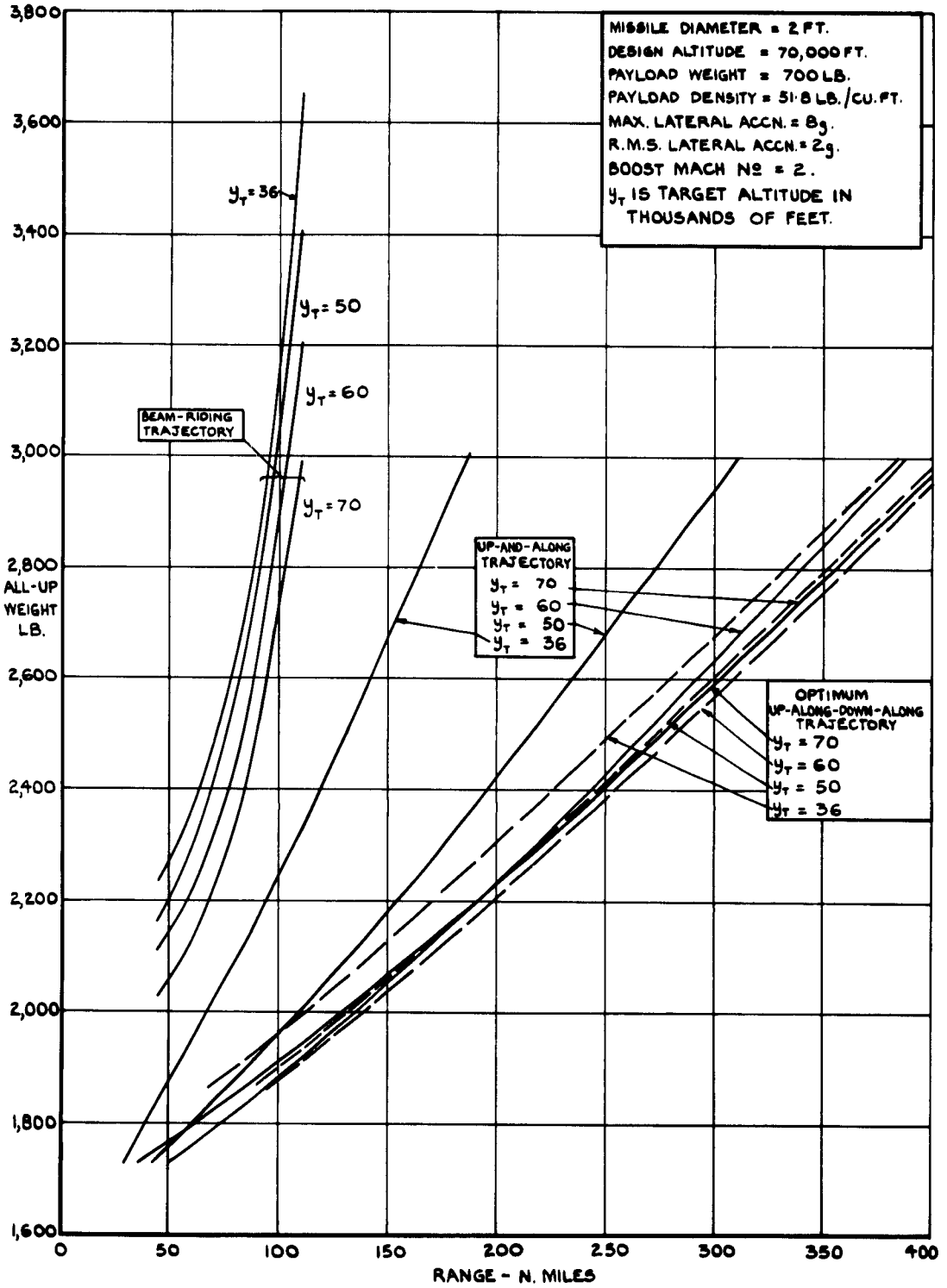


FIG. 29. VARIATION OF WEIGHT WITH RANGE FOR STANDARD MISSILES (DESIGNED FOR 70,000 FT. ALTITUDE) ON VARIOUS TRAJECTORIES TO VARIOUS TARGET ALTITUDES y_T.

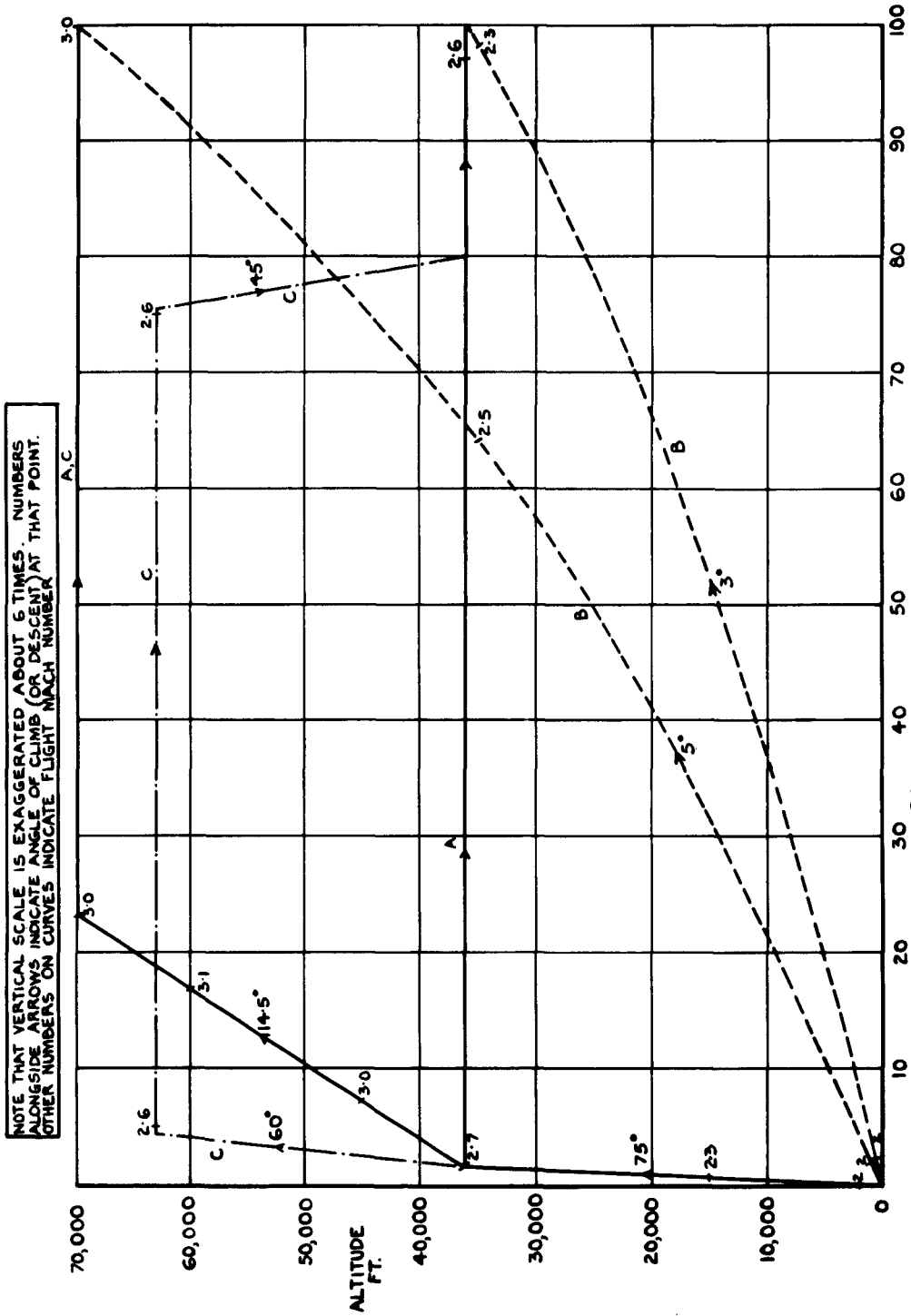


FIG. 30. DIAGRAM OF (A) UP-AND-ALONG, (B) BEAM-RIDING AND (C) UP-ALONG-DOWN-ALONG TRAJECTORIES TO TARGET ALTITUDES OF 36,000 AND 70,000 FT.

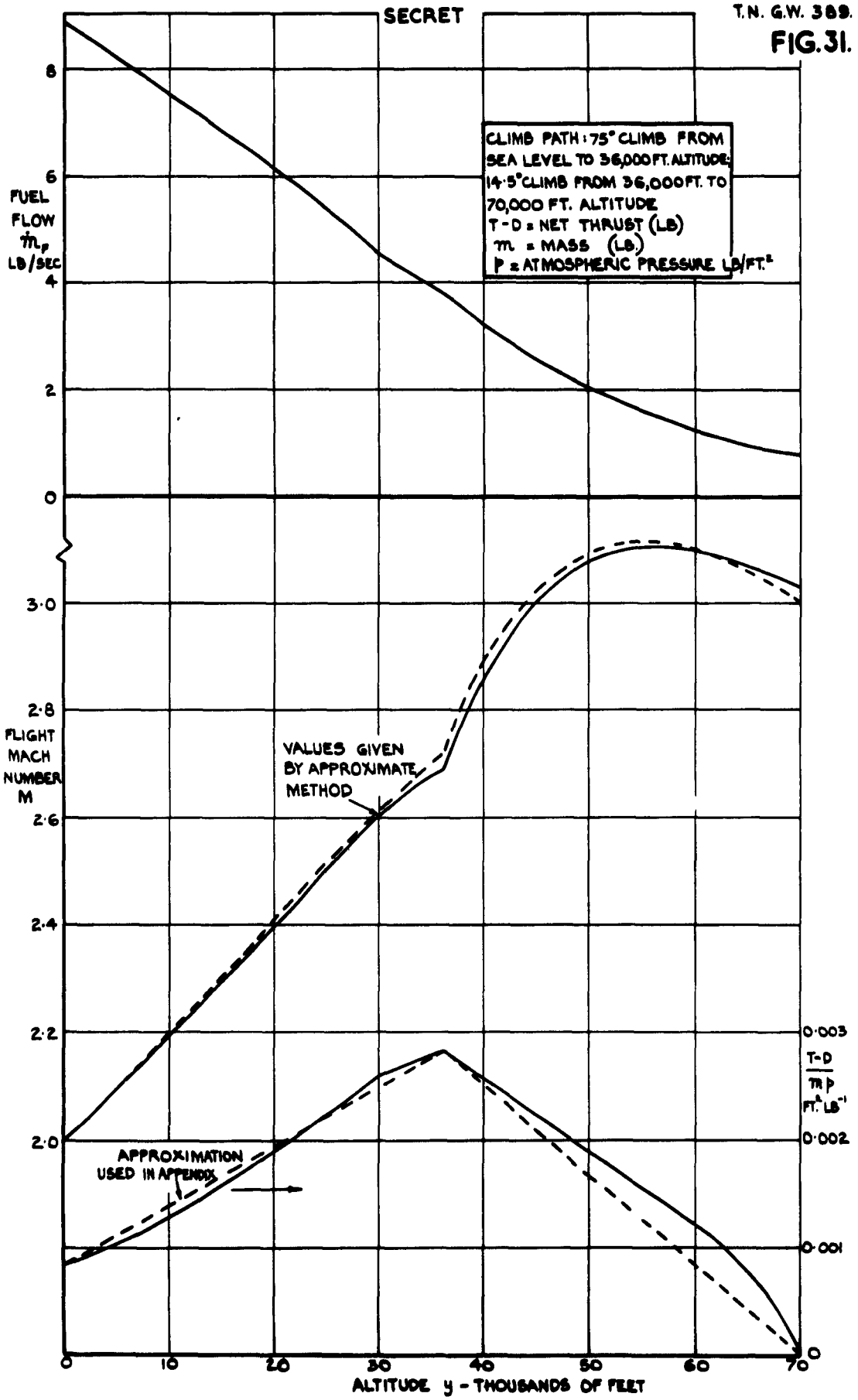


FIG. 31. VARIATION OF MACH NUMBER, NET THRUST PARAMETER, AND FUEL FLOW WITH ALTITUDE FOR THE MISSILE OF FIG. 1 ON ITS STANDARD CLIMB PATH.

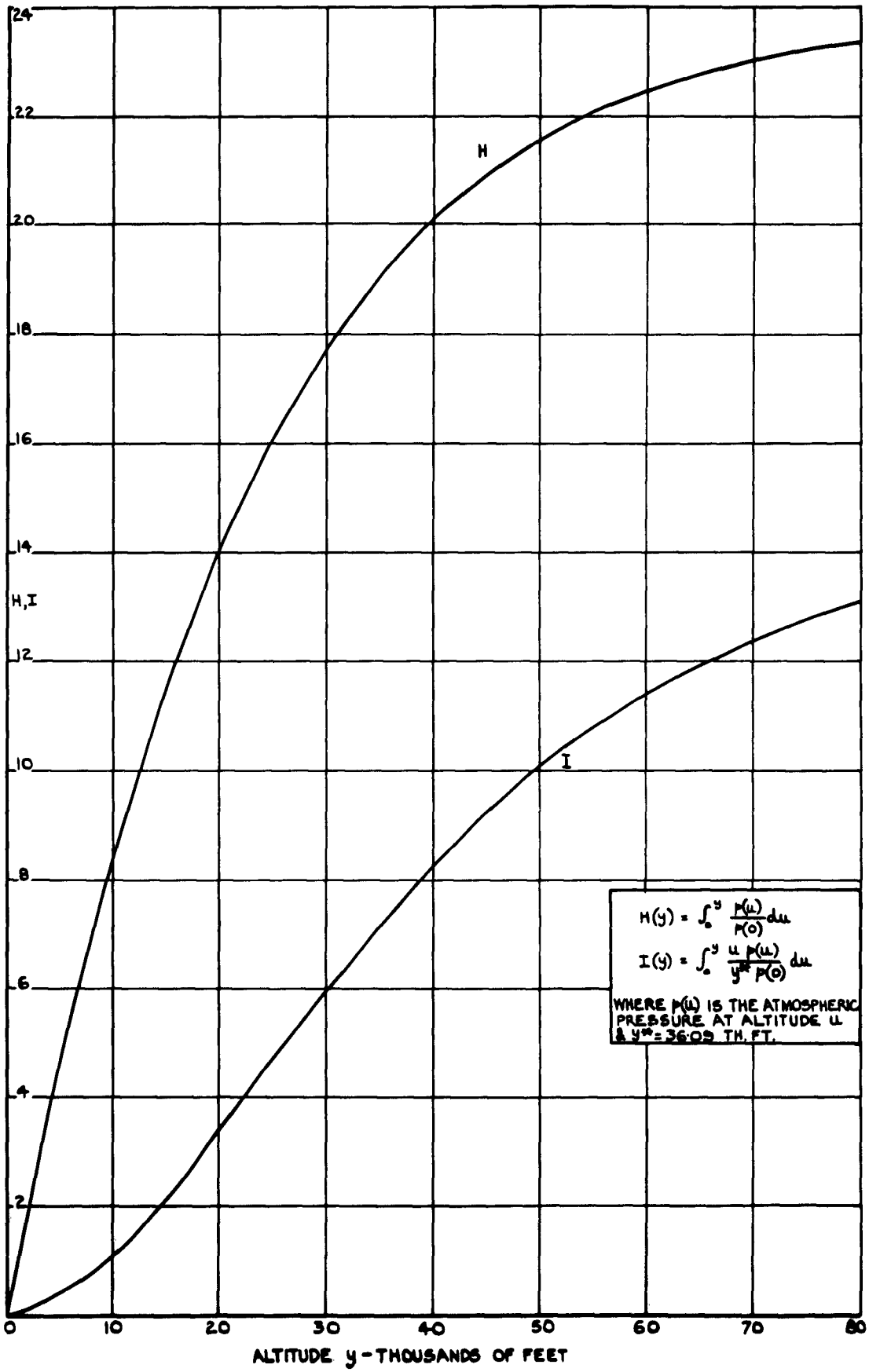


FIG. 32. VARIATION OF THE 'ATMOSPHERE FUNCTIONS' H AND I WITH ALTITUDE y.

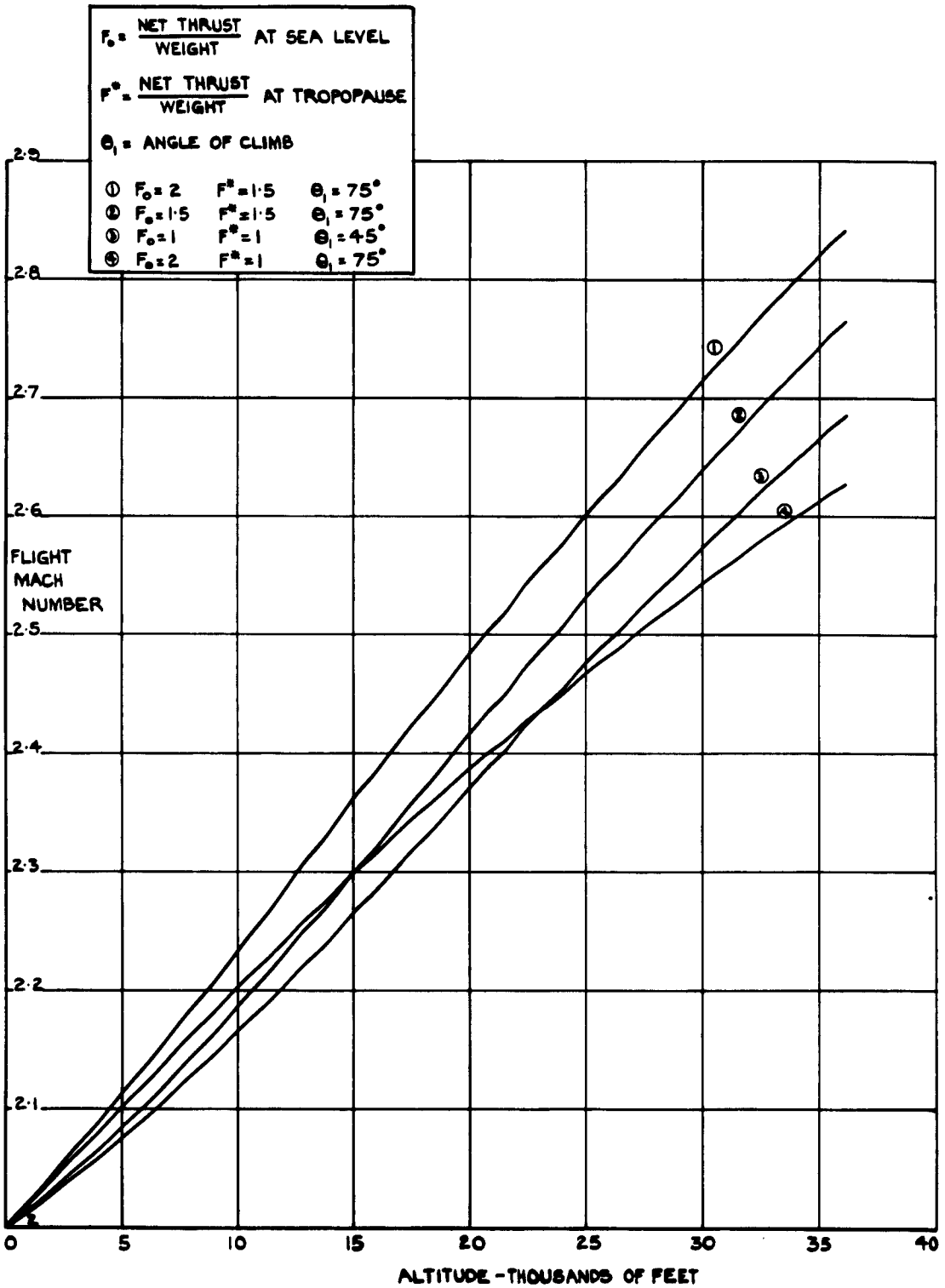
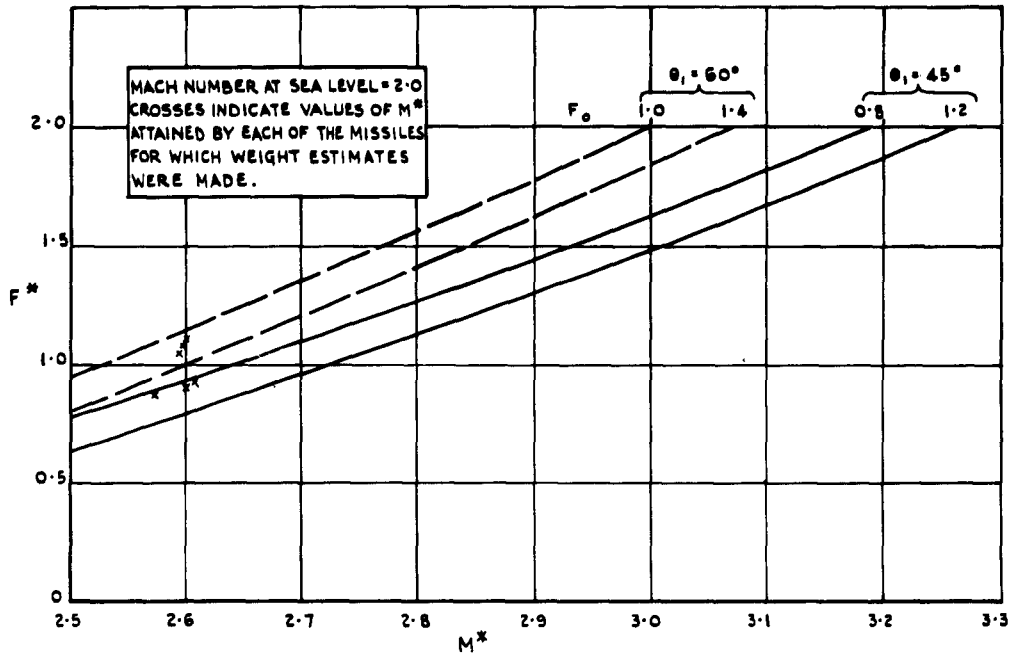
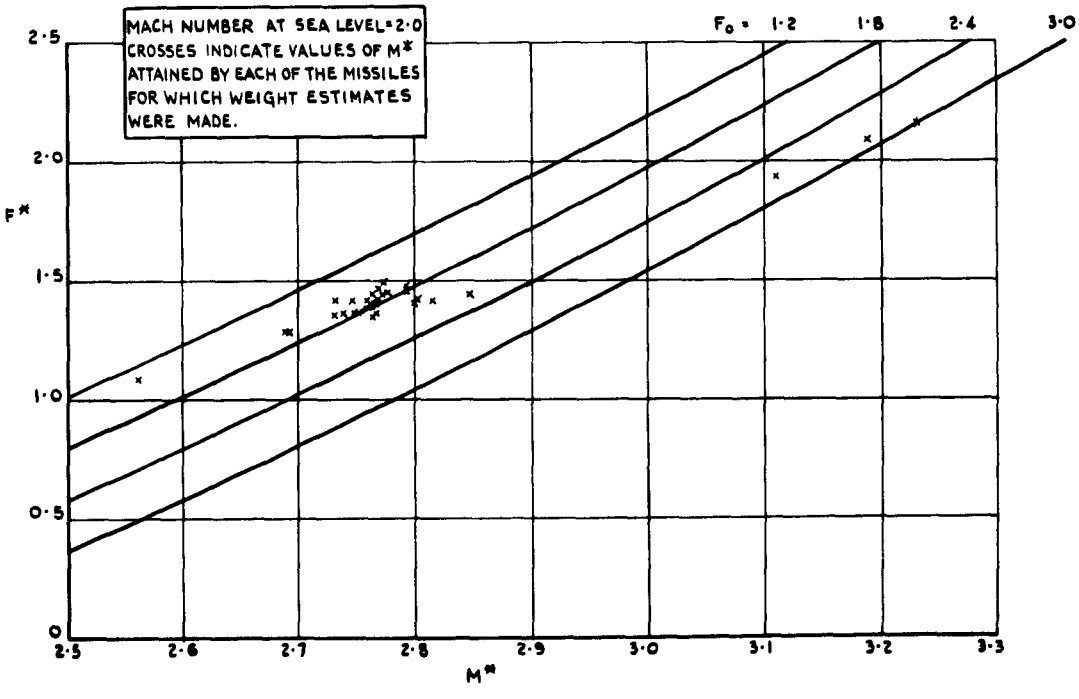


FIG. 33. VARIATION OF MISSILE FLIGHT MACH NUMBER WITH ALTITUDE FOR STRAIGHT-LINE CLIMBS FROM SEA LEVEL TO TROPOPAUSE.

FIG. 34. (a & b)



(a) FOR ANGLES OF CLIMB $\theta_1 = 60°$ AND $45°$



(b) FOR 75° ANGLE OF CLIMB.

FIG. 34. (a & b) MACH NUMBER AT THE TROPOPAUSE, M*, AS A FUNCTION OF NET THRUST/WEIGHT RATIOS AT SEA LEVEL (F_o) AND TROPOPAUSE (F^*), FOR STRAIGHT — LINE CLIMBS.

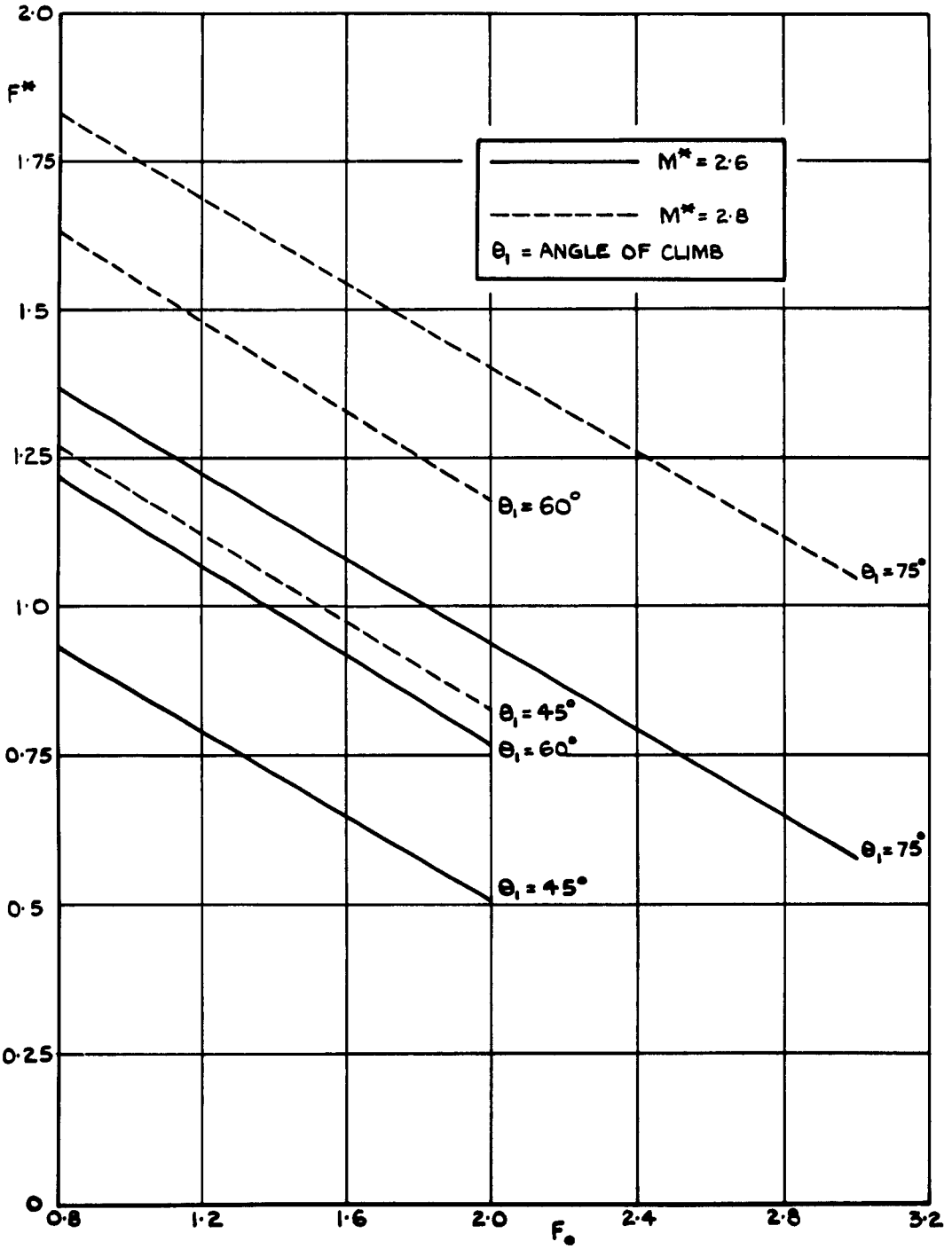


FIG. 35. VARIATION OF NET THRUST / WEIGHT RATIO AT TROPOPAUSE (F^*) WITH THAT AT SEA LEVEL (F_0) FOR STRAIGHT-LINE CLIMBS, SHOWING EFFECT OF MACH NUMBER AT THE TROPOPAUSE (M^*).

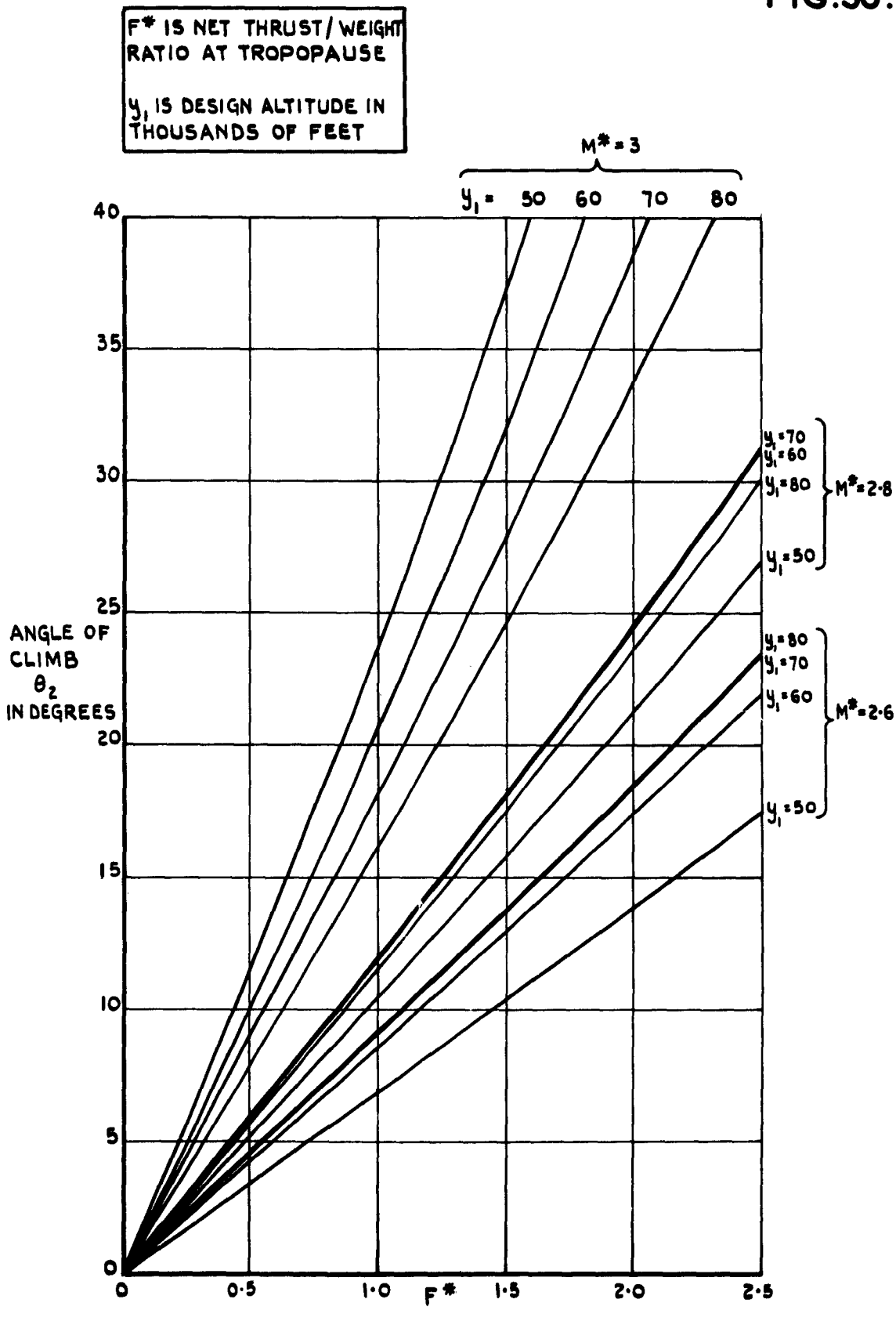


FIG.36. VARIATION OF ANGLE OF CLIMB WITH F* FOR STRAIGHT-LINE CLIMBS IN THE STRATOSPHERE. MACH NUMBER M* AT THE TROPOPAUSE. M = 3 AT DESIGN ALTITUDE y_1.

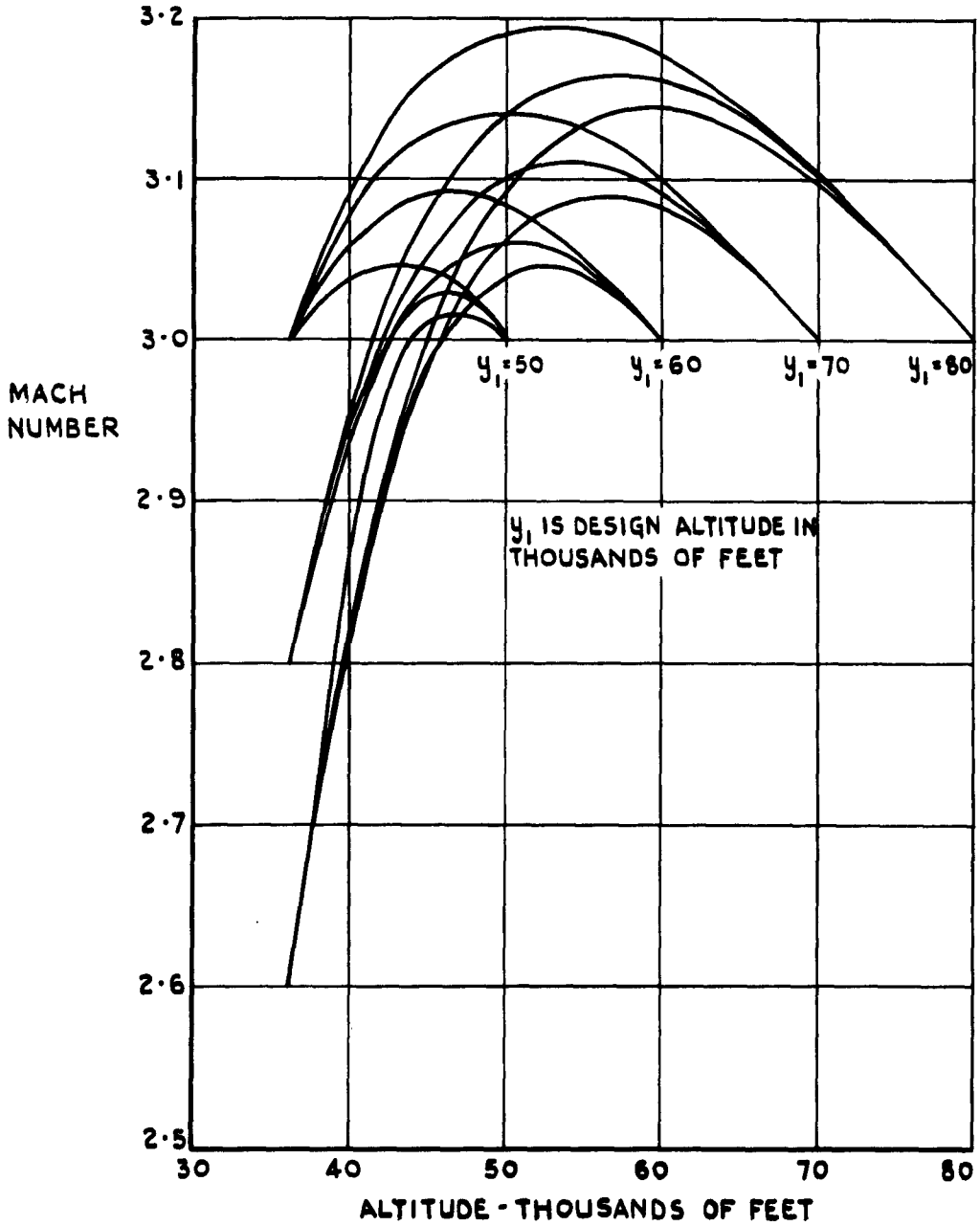
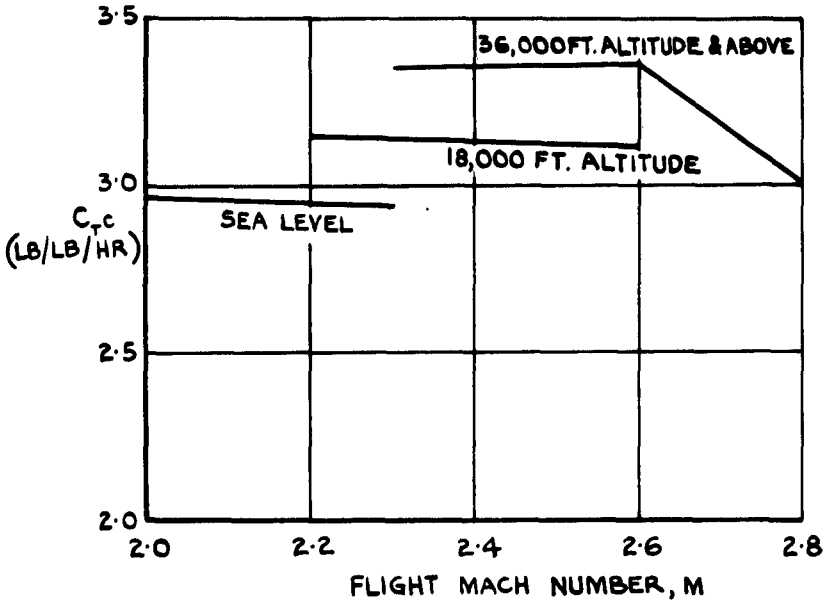
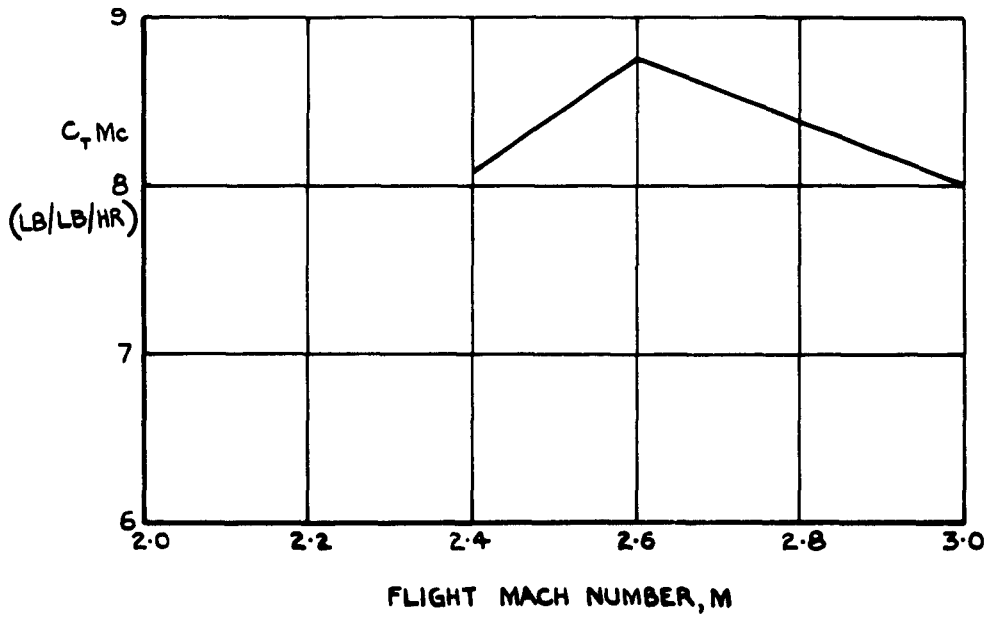


FIG.37. VARIATION OF MACH NUMBER WITH ALTITUDE ON STRAIGHT-LINE CLIMBS IN THE STRATOSPHERE FROM THE TROPOPAUSE TO DESIGN ALTITUDE y_1. M = 3 AT DESIGN ALTITUDE.



(a). (THRUST COEFFICIENT) × (SPECIFIC FUEL CONSUMPTION), $C_T c$.



(b) $c_T Mc$, IN THE STRATOSPHERE.

FIG. 38. (a&b) RAMJET FUEL CONSUMPTION PARAMETERS AS FUNCTIONS OF MACH NUMBER AND ALTITUDE.

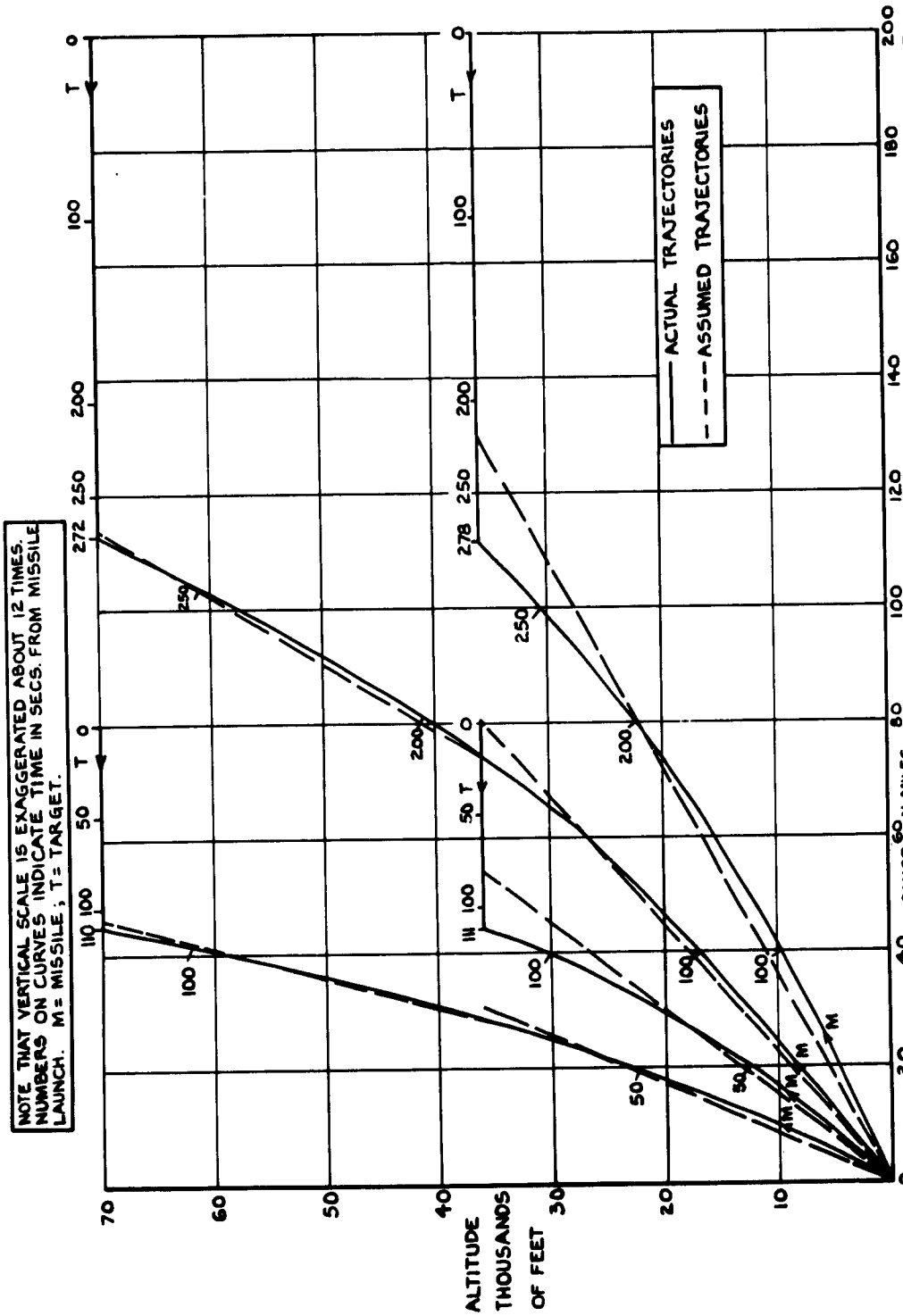


FIG. 39. BEAM-RIDING TRAJECTORIES FOR RANGES AT LAUNCH OF 80 AND 200 N. MILES AND TARGET ALTITUDES OF 36,000 AND 70,000 FT.

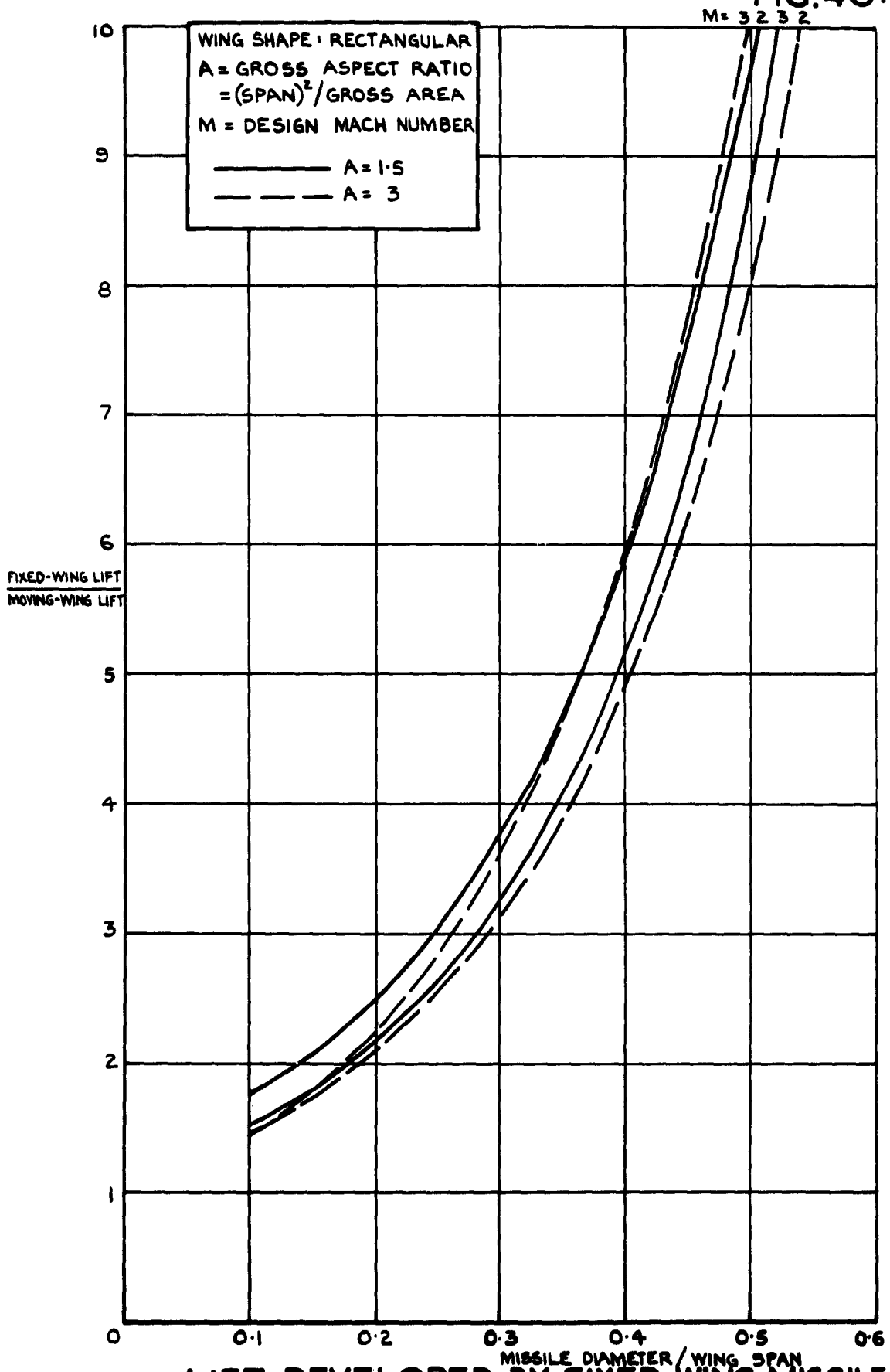
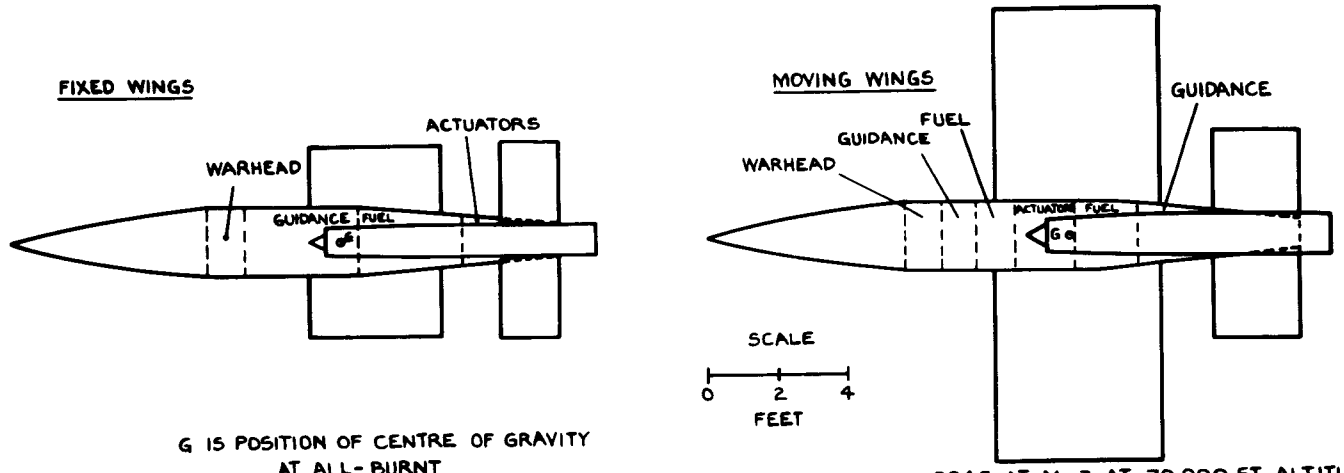


FIG. 40. LIFT DEVELOPED BY FIXED-WING MISSILE / LIFT DEVELOPED BY MOVING-WING MISSILE FOR GIVEN MISSILE CONFIGURATION FOR 25° INCIDENCE.



G IS POSITION OF CENTRE OF GRAVITY AT ALL-BURNT

MISSILE COMPONENT	WEIGHT - LB	
	FIXED WING	MOVING WING
WARHEAD	350	350
GUIDANCE	350	350
BODY STRUCTURE	260	280
WINGS	50	320
CONTROLS OR FINS	120	40
ACTUATORS	80	220
RAMJETS, STUBS ETC.	320	410
FUEL	300	370
TANKS	40	60
ALL-UP WEIGHT	1870	2400

DIMENSIONS, ETC.	FIXED WING	MOVING WING
BODY LENGTH (FEET)	15·8	17·1
DIAMETER (FEET)	2·0	2·0
WING SPAN (FEET)	5·6	13·4
WING NET PLAN AREA (SQ.FT)	13·4	53·9
CONTROL OR FIN SPAN (FEET)	8·1	6·2
RAMJET DIAM. (INCHES)	12·8	14·4
DESIGN MACH NUMBER	3	
MAX. LATERAL ACCN.	8g	
R.M.S. LATERAL ACCN.	2g	
DESIGN ALTITUDE (FEET)	70,000	
RANGE (N. MILES)	87	

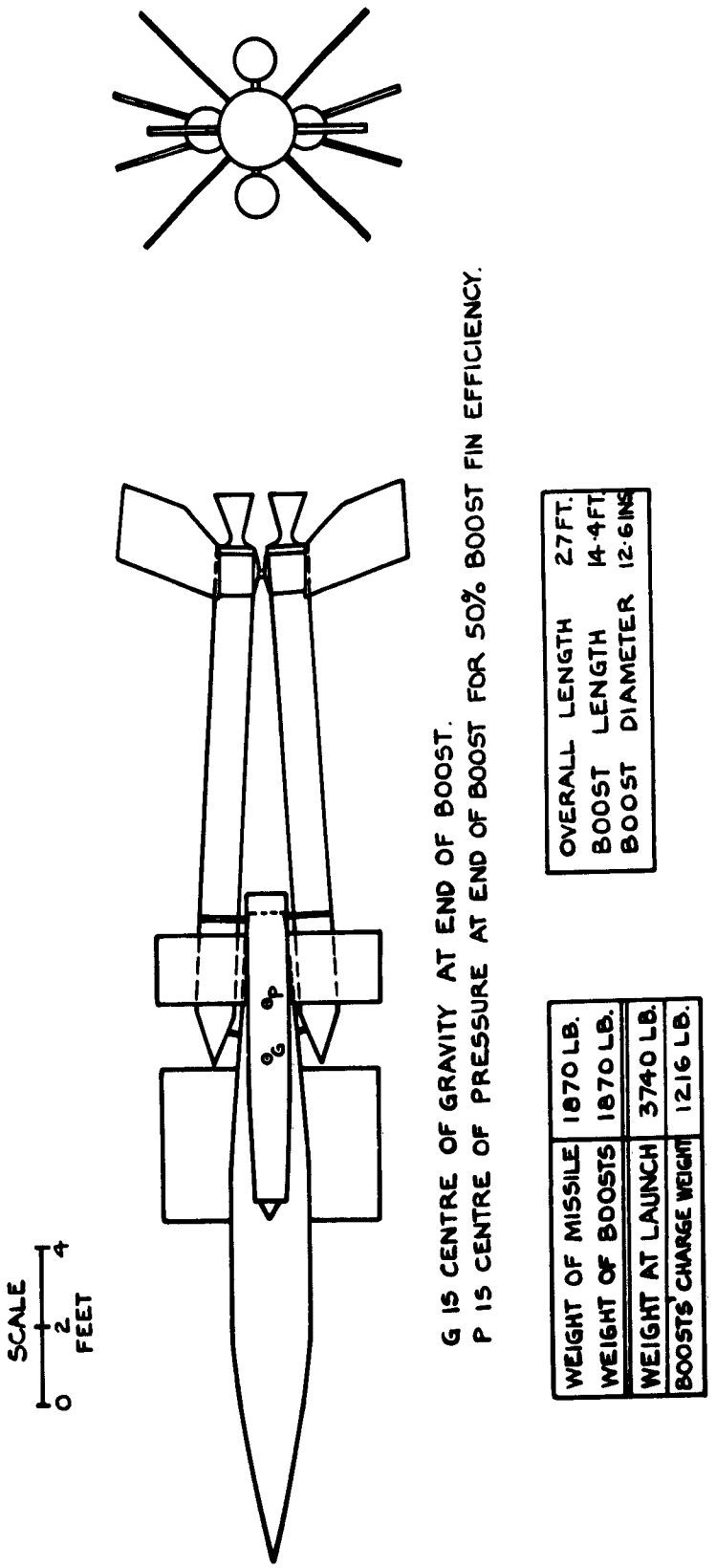
DRAG AT M=3 AT 70,000 FT. ALTITUDE

DRAG COMPONENT	C_D		%	
	FIXED WING	MOVING WING	FIXED WING	MOVING WING
NOSE WAVE	0·070	0·070	27	21
AFTERBODY WAVE	0·026	0·026	10	8
BASE WAVE	0·025	0·025	10	7
BODY SKIN FRICTION	0·036	0·040	14	12
STUB WINGS	0·006	0·006	2	2
WING WAVE	0·005	0·039	2	12
WING SKIN FRICTION	0·017	0·069	6	21
CONTROL OR FIN WAVE	0·017	0·005	6	2
CONTROL OR FIN SKIN FRICTION	0·038	0·019	14	6
ADDITION	0·025	0·031	9	9
TOTAL	0·265	0·330	100	100
	FIXED WING		MOVING WING	
ZERO-LIFT DRAG	485 LB.		603 LB.	
INDUCED DRAG	343 LB.		442 LB.	
TOTAL DRAG	828 LB.		1045 LB.	

ZERO-LIFT DRAG COEFFICIENT, C_D, BASED ON BODY FRONTAL AREA.

FIG. 41. COMPARISON OF FIXED-WING AND MOVING-WING MISSILES.

FIG. 42.



G IS CENTRE OF GRAVITY AT END OF BOOST.
P IS CENTRE OF PRESSURE AT END OF BOOST FOR 50% BOOST FIN EFFICIENCY.

WEIGHT OF MISSILE	1870 LB.
WEIGHT OF BOOSTS	1870 LB.
WEIGHT AT LAUNCH	3740 LB.
BOOSTS' CHARGE WEIGHT	1216 LB.

OVERALL LENGTH	27 FT.
BOOST LENGTH	14·4 FT.
BOOST DIAMETER	12·6 INS.

FIG. 42. POSSIBLE BOOST LAYOUT FOR STANDARD MISSILE.

DETACHABLE ABSTRACT CARDS

These abstract cards are inserted in RAE Reports and Technical Notes for the convenience of Librarians and others who need to maintain an Information Index.

Detached cards are subject to the same Security Regulations as the parent document, and a record of their location should be made on the inside of the back cover of the parent document.

SECRET

Royal Aircraft Estab. Technical Note No. GW 389 — 623.451-519:
1955.11 — 621.454.018:
King-Hele, D. G. and Hiller, H. — 531.565

WEIGHT ESTIMATES FOR LONG-RANGE SURFACE-TO-AIR GUIDED MISSILES

Weight estimates are made for high-altitude surface-to-air missiles having ranges between 30 and 200 n.miles, and the effects of eleven design parameters are investigated. Propulsion is by ramjet, and guidance is tacitly assumed to include a mid-course phase, followed by radar homing in the terminal phase. The 'standard' missile, after rocket boost to M = 2 at sea level, climbs under ramjet power, steeply at first and then more gently in the stratosphere, so that it reaches its design altitude of 70,000 ft at a Mach number M of 3 and a ground range of about 25 n.miles. This standard missile carries a payload (warhead + guidance) of 700 lb, develops a maximum lateral acceleration of 8g at design altitude, and is assumed to suffer an r.m.s. lateral acceleration of 2g in its mid-course

SECRET — P.T.O.

SECRET

Royal Aircraft Estab. Technical Note No. GW 389 — 623.451-519:
1955.11 — 621.454.018:
King-Hele, D. G. and Hiller, H. — 531.565

WEIGHT ESTIMATES FOR LONG-RANGE SURFACE-TO-AIR GUIDED MISSILES

Weight estimates are made for high-altitude surface-to-air missiles having ranges between 30 and 200 n.miles, and the effects of eleven design parameters are investigated. Propulsion is by ramjet, and guidance is tacitly assumed to include a mid-course phase, followed by radar homing in the terminal phase. The 'standard' missile, after rocket boost to M = 2 at sea level, climbs under ramjet power, steeply at first and then more gently in the stratosphere, so that it reaches its design altitude of 70,000 ft at a Mach number M of 3 and a ground range of about 25 n.miles. This standard missile carries a payload (warhead + guidance) of 700 lb, develops a maximum lateral acceleration of 8g at design altitude, and is assumed to suffer an r.m.s. lateral acceleration of 2g in its mid-course

SECRET — P.T.O.

SECRET

Royal Aircraft Estab. Technical Note No. GW 389 — 623.451-519:
1955.11 — 621.454.018:
King-Hele, D. G. and Hiller, H. — 531.565

WEIGHT ESTIMATES FOR LONG-RANGE SURFACE-TO-AIR GUIDED MISSILES

Weight estimates are made for high-altitude surface-to-air missiles having ranges between 30 and 200 n.miles, and the effects of eleven design parameters are investigated. Propulsion is by ramjet, and guidance is tacitly assumed to include a mid-course phase, followed by radar homing in the terminal phase. The 'standard' missile, after rocket boost to M = 2 at sea level, climbs under ramjet power, steeply at first and then more gently in the stratosphere, so that it reaches its design altitude of 70,000 ft at a Mach number M of 3 and a ground range of about 25 n.miles. This standard missile carries a payload (warhead + guidance) of 700 lb, develops a maximum lateral acceleration of 8g at design altitude, and is assumed to suffer an r.m.s. lateral acceleration of 2g in its mid-course

SECRET — P.T.O.

SECRET

Royal Aircraft Estab. Technical Note No. GW 389 — 623.451-519:
1955.11 — 621.454.018:
King-Hele, D. G. and Hiller, H. — 531.565

WEIGHT ESTIMATES FOR LONG-RANGE SURFACE-TO-AIR GUIDED MISSILES

Weight estimates are made for high-altitude surface-to-air missiles having ranges between 30 and 200 n.miles, and the effects of eleven design parameters are investigated. Propulsion is by ramjet, and guidance is tacitly assumed to include a mid-course phase, followed by radar homing in the terminal phase. The 'standard' missile, after rocket boost to M = 2 at sea level, climbs under ramjet power, steeply at first and then more gently in the stratosphere, so that it reaches its design altitude of 70,000 ft at a Mach number M of 3 and a ground range of about 25 n.miles. This standard missile carries a payload (warhead + guidance) of 700 lb, develops a maximum lateral acceleration of 8g at design altitude, and is assumed to suffer an r.m.s. lateral acceleration of 2g in its mid-course

SECRET — P.T.O.

SECRET

flight. The estimated weight of the missile for 100 n.miles range at 70,000 ft altitude is about 1900 lb without boosts. Estimates are made of the changes in weight resulting from changes in design altitude, range, missile diameter, payload weight, payload density, maximum lateral acceleration, r.m.s. lateral acceleration, boost Mach number, propulsion and layout. The propulsion range of the standard missile (designed for 70,000 ft altitude) on various trajectories - up-and-along, up-along-down-along, and beam-riding, to target altitudes between 36,000 and 70,000 ft - is also given.

SECRET

flight. The estimated weight of the missile for 100 n.miles range at 70,000 ft altitude is about 1900 lb without boosts. Estimates are made of the changes in weight resulting from changes in design altitude, range, missile diameter, payload weight, payload density, maximum lateral acceleration, r.m.s. lateral acceleration, boost Mach number, propulsion and layout. The propulsion range of the standard missile (designed for 70,000 ft altitude) on various trajectories - up-and-along, up-along-down-along, and beam-riding, to target altitudes between 36,000 and 70,000 ft - is also given.

SECRET

SECRET

flight. The estimated weight of the missile for 100 n.miles range at 70,000 ft altitude is about 1900 lb without boosts. Estimates are made of the changes in weight resulting from changes in design altitude, range, missile diameter, payload weight, payload density, maximum lateral acceleration, r.m.s. lateral acceleration, boost Mach number, propulsion and layout. The propulsion range of the standard missile (designed for 70,000 ft altitude) on various trajectories - up-and-along, up-along-down-along, and beam-riding, to target altitudes between 36,000 and 70,000 ft - is also given.

SECRET

SECRET

flight. The estimated weight of the missile for 100 n.miles range at 70,000 ft altitude is about 1900 lb without boosts. Estimates are made of the changes in weight resulting from changes in design altitude, range, missile diameter, payload weight, payload density, maximum lateral acceleration, r.m.s. lateral acceleration, boost Mach number, propulsion and layout. The propulsion range of the standard missile (designed for 70,000 ft altitude) on various trajectories - up-and-along, up-along-down-along, and beam-riding, to target altitudes between 36,000 and 70,000 ft - is also given.

SECRET

Information Centre
Knowledge Services
[dstl] Porton Down
Salisbury
Wilts
SP4 0JQ
22060-6218
Tel: 01980-613753
Fax 01980-613970

Defense Technical Information Center (DTIC)
8725 John J. Kingman Road, Suit 0944
Fort Belvoir, VA 22060-6218
U.S.A.

AD#: AD093330

Date of Search: 5 August 2008

Record Summary: AVIA 6/17312
Title: Weight estimates for long range surface-to-air guided missiles
Availability Open Document, Open Description, Normal Closure before FOI Act: 30 years
Former reference (Department) TECH NOTE'S GW 389
Held by The National Archives, Kew

This document is now available at the National Archives, Kew, Surrey, United Kingdom.

DTIC has checked the National Archives Catalogue website (http://www.nationalarchives.gov.uk) and found the document is available and releasable to the public.

Access to UK public records is governed by statute, namely the Public Records Act, 1958, and the Public Records Act, 1967.
The document has been released under the 30 year rule.
(The vast majority of records selected for permanent preservation are made available to the public when they are 30 years old. This is commonly referred to as the 30 year rule and was established by the Public Records Act of 1967).

This document may be treated as UNLIMITED.