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ANALYTICAL INVESTIGATION OF FACTORS AFFECTING THE
PERFORMANCE OF SINGLE-STAGE TURBINES HAVING
ROTOR-TIP DISCHARGE OF COOLING AIR

By Gordon T. Smith and Robert O. Hickel

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SUMMARY

An analysis of experimentally determined effects of rotor cooling air on single-stage turbine performance was made to determine some factors, which might account for the changes in turbine performance that had been observed when cooling air was discharged from the rotor blade tips.

The reaction of the turbine was assumed to be increased because discharge of the cooling air at the rotor blade tips increased the mass flow per unit flow area at the turbine exit. The cooling air was considered to displace the combustion gas, thus resulting in a reduction of the flow area available to the combustion gases. This displacement effect alone, however, did not account entirely for the changes in turbine performance caused by the presence of cooling air.

Therefore, the analysis also considered that the cooling air itself, after leaving the blade tips, may have contributed to the work output of the turbine. This was assumed to be accomplished by an acceleration of the cooling air in the tip region of the channels between adjacent blades. For the two turbines investigated, the additional work obtained from the cooling air could be expressed as a constant coefficient of the work required to pump the cooling air through the cooled turbine rotor. In this manner, the work output of the cooling air was related to a readily calculable parameter. The analytical procedure used satisfactorily described the effects of cooling air on turbine performance over a range of equivalent turbine speed, equivalent work, and cooling-air flow.

The analysis indicated that, near limiting loading, the effect of displacement alone can reduce the air-cooled-turbine efficiency 1 to 2 points relative to an uncooled turbine. The amount of work produced by the cooling air, when displacement effects were considered for the two turbines investigated, was 47 and 108 percent of the pumping work done on the cooling air within the turbine rotor, resulting in a net gain in turbine-blade efficiency.
INTRODUCTION

Experimental studies of the performance of two single-stage air-cooled turbines (ref. 1) indicate that the total work produced by the turbine blades increases, as a result of cooling air being discharged at the rotor blade tips. This investigation is an effort to systematically relate this additional turbine-blade work to easily calculable parameters and simultaneously acquire an understanding of the physical mechanisms involved.

Performance predictions for turbojet and turboprop engines utilizing air-cooled turbine rotors have been limited in the past by lack of specific knowledge regarding the effect of cooling-air flow from the rotor blade tips on the performance of the turbine. The experimental results of reference 1 indicate that the performance characteristics of air-cooled turbines may be more favorable than generally assumed in many previous analyses (e.g., refs. 2 to 4).

The purpose of this report is to further analyze the turbine performance results of reference 1 with the objectives of isolating the contribution of the cooling air to the turbine performance and establishing some systematic relation between these cooling-air effects and the turbine operational parameters. In order to accomplish this, the uncooled-turbine off-design performance was calculated by the methods of reference 5. These calculations were then modified by the assumption that the cooling air partly displaced the combustion-gas-flow area at the rotor exit. It was further assumed that the cooling air was accelerated in the rotor channel near the tip of the blade so that its final exit whirl velocity differed from the rotor tip speed. A comparison of the calculated performance trends with the experimental trends of reference 1 is used to indicate the extent to which the assumed cooling-air effects would account for the experimentally determined behavior of the two air-cooled turbines.

The two single-stage, air-cooled turbines of reference 1 are quite different geometrically. One turbine has nontwisted rotor blades and the other turbine has rotor blades which are twisted from root to tip. The nontwisted blades have impulse-type channel area distributions whereas the twisted blades have an impulse-type distribution at the hub varying to a reaction-type area distribution at the blade tips. In this investigation, the performance of the turbine with twisted blades was obtained analytically over an equivalent turbine-speed range from 80 to 110 percent of design value. The performance of the turbine with nontwisted blades was analytically investigated through an equivalent speed range from 50 to 100 percent of design speed. These ranges of equivalent speeds span the speed ranges that were investigated experimentally (ref. 1).
SYMBOLS

The following symbols are used herein:

\( c_p \) specific heat at constant pressure, Btu/(lb)(°R)

\( g \) acceleration due to gravity, 32.17 ft/sec\(^2\)

\( h' \) total enthalpy, Btu/lb

\( J \) mechanical equivalent of heat, 778 ft-lb/Btu

\( K \) viscous loss coefficient

\( K_p \) performance coefficient based on \( U_t \)

\( K_r \) performance coefficient based on \( W_{g,u,6,m} \)

\( N \) rotational speed, rpm

\( p' \) total pressure, lb/sq ft

\( R \) gas constant, ft-lb/(lb)(°R)

\( T' \) total temperature, °R

\( U \) blade tangential speed, ft/sec

\( V \) absolute velocity, ft/sec

\( W \) velocity relative to turbine rotor blade, ft/sec

\( w \) gas weight flow, lb/sec

\( \gamma \) ratio of specific heats

\( \eta \) turbine efficiency

\( \theta_{cr} \) equivalent turbine-inlet temperature ratio, \( \frac{\gamma}{\gamma + 1} \frac{gRT'}{(gRT')_{sl}} \)

Subscripts:

- \( a \) cooling air
- \( c \) compressor

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d refers to turbine calculations based on displacement of combustion gases

g combustion gas entering turbine stator

id ideal

m mean section

s1 NACA standard sea-level air conditions

T turbine

t turbine rotor blade tip

u tangential

x axial

0 stator inlet station

1 stator throat station

2 stator exit station (immediately inside trailing edge)

3 rotor inlet station

4 rotor throat station

5 rotor exit station (immediately inside trailing edge)

6 rotor exit station (downstream of trailing edge)

ANALYSIS.

The power developed by the gas forces acting on the outside surface of turbine rotor blades is the total power output of a turbine. This total power output supplies the power necessary to drive the engine compressor and propeller shaft (if one is present) and, in addition, supplies the power necessary for the following engine functions:

- Air through the rotor blades

- Power to drive the accessories

- Friction and bearing losses (bearing, gear, etc.)

Negligible and is disregarded in
the experimental results of reference 1. The principal data presentation of reference 1 expresses the turbine performance parameters of equivalent work and turbine efficiency in terms of the total blade power. The calculation of the turbine power by the methods of reference 5 gives results in terms of this total power generated by the turbine through the action of the external gas pressure forces on the turbine blades. All turbine performance parameters, both calculated and experimental, subsequently discussed herein will be based on the total blade power rather than on the net power supplied to the turbine shaft. The experimental equivalent turbine work and turbine efficiency of reference 1 are obtained from the following equations:

\[
\frac{\Delta h'_{c}}{\theta_{cr,T}} = \frac{w_{c} \Delta h'_{c} + w_{a} (U_{c}^{2}/gJ)}{w_{c} \theta_{cr}} \quad (1)
\]

\[
\eta = \frac{\frac{(\Delta h'/\theta_{cr})_T}{(\Delta h'/\theta_{cr})_{T,id}}}{(2a)}
\]

where

\[
\frac{(\Delta h'_{c})_{T,id}}{\theta_{cr}} = c_{p,g,0} T_{0} \left[ 1 - \left( \frac{p'_{0}^{r-1}}{p_{0}^{r}} \right) \right] \quad (2b)
\]

The ideal equivalent work is based solely on the combustion-gas mass flow entering the turbine.

This report endeavors to account for the increase in the turbine efficiency (defined by eqs. (2)) that was noted in reference 1 when the cooling-air flow was increased. If the uncooled-turbine performance of the two turbines of reference 1 can be sufficiently well described by the theory of reference 5, it may be possible to describe the effect of cooling air on the performance of these two turbines by consideration of certain effects, which would be imposed by the presence of cooling air emerging from the tips of the rotor blades.

The first effect, which the presence of the cooling air might impose on the turbine, can be referred to as a displacement effect. Cooling air, emerging from the tips of the rotor blades, will increase the total mass flow of fluid leaving the rotor exit and, consequently, displace the combustion gas from a portion of the region that it would otherwise occupy. This effect would operate to increase the reaction of the turbine rotor, resulting in higher rotor-exit Mach numbers and lower rotor-inlet Mach numbers for particular equivalent speed and equivalent work (map point). Also, the rotor incidence angles would be decreased at a given map point as a consequence of the reduced inlet velocities. The combination of the Mach number changes and incidence angle changes might account for part, or all, of the experimentally determined effect of the cooling air on the two turbines of reference 1.
A second effect, which might be associated with the effect of cooling air in the tip region of the rotor, would be an acceleration of the cooling air in the channels between the blade tips. The cooling air will leave the tips of the rotor blades with a total pressure greater than the pressure immediately downstream of the rotor (station 6, fig. 1). Since this condition exists, the cooling air can experience an acceleration through the channels in the tip region of the rotor because of its intrinsic energy and acquire a whirl velocity (different from the rotor tip velocity) before passing downstream of the rotor. When this condition exists, the work output of the turbine will be increased because of the forces the cooling air exerts on the blades as the air is accelerated. If the cooling air produces work in this manner, it may be considered as a work-absorbing medium while within the passages of the rotor and blades and as a work-producing medium after leaving the blade tips.

The presence of cooling air may also cause a significant change in the viscous losses of the turbine rotor. Reference 6 presents some detailed flow-visualization studies of some viscous effects that are present in turbine rotors. Large flow disturbances in the tip region of the rotor blades, associated with the scraping action of the blade tips on the shroud boundary layer, are illustrated. Unpublished data indicate that the discharge of cooling air from the tips of the rotor blades has the effect of reducing or removing these tip disturbances; similar effects may be of importance in causing an increase in the turbine blade work, but these phenomena are not sufficiently well understood at present to describe turbine performance analytically. The cooling-air effects, which are subsequently utilized herein to describe the performance of the two air-cooled turbines, were restricted to the displacement and acceleration effects of the cooling air. No changes in the viscous loss characteristics of the rotor due to cooling air were considered.

PROCEDURE

The calculation procedures used to determine the performance of the two turbines of reference 1 are identical and the subsequent discussion applies to both. The procedures used to establish analytically the uncooled-turbine performance, to determine the cooling-air displacement effect, and to establish the work produced as a result of acceleration of the cooling air after leaving the blade tips are discussed herein.

Calculation of Uncooled-Turbine Performance

The uncooled-turbine performance was calculated by the methods of reference 5, utilizing a single experimental datum point to establish the required viscous loss coefficient. This method involves the following principal assumptions:
(1) The sum of the losses due to friction, secondary flow, and tip clearance can be combined into a single over-all viscous loss parameter K.

(2) Any component of rotor inlet velocity not parallel to the direction of the mean camber line at the rotor blade inlet represents a total pressure loss.

(3) The tangential velocity of the gas stream remains constant as it passes from a point immediately inside a blade trailing edge to a point downstream of the blade trailing edge.

(4) At supercritical pressure ratios across either the nozzle or the rotor blade row, continuity within the blade row is used to obtain the supersonic expansion.

(5) A normal shock loss occurs at the exit of a blade row when the exit velocity is supersonic.

These assumptions are found to give reasonable predictions of the performance of small-scale cold-air turbines (e.g., ref. 5).

The calculations are begun by assuming a series of values of the viscous loss coefficient K and then determining, for each value of K, the performance at an equivalent speed corresponding to an experimental operating point. After the experimental equivalent turbine work has been spanned for each of the assumed values of K, a cross plot can be constructed, which will indicate the value of K necessary to give a very close approximation of the desired value of turbine efficiency. After obtaining a value for K, a series of working curves are constructed to aid the calculations. Typical examples of such curves and a detailed sample calculation are given in reference 5.

The proper value of K for each of the turbines was obtained as previously indicated for one uncooled-turbine experimental-operating point. For the turbine with twisted blades, hereinafter referred to as turbine A, the map and the data were matched at 99 percent of design equivalent turbine speed and an equivalent work of 20.94 Btu per pound. For the turbine with nontwisted blades, hereinafter referred to as turbine B, the match point was at 87 percent of design equivalent turbine speed and an equivalent turbine work of 20.30 Btu per pound. The 99- and 87-percent equivalent speed points were selected for turbines A and B because they most closely approximate the design speeds for which experimental data are available. Turbine B failed to reach the design equivalent turbine speed because of inlet temperature limitations. After the uncooled-turbine maps were calculated, the performance predicted by the turbine maps was checked against other uncooled-turbine experimental-operating point data to determine the validity of the procedure and assumptions when applied to the hot, full-scale turbines of reference 1.
Calculation of Cooled-Turbine Performance

The uncooled-turbine calculations were then modified by first accounting for the displacement effect of the cooling air. The mass flow leaving the turbine rotor was increased over the mass flow entering the rotor by 4 and 8 percent to determine the displacement maps for coolant-to-combustion gas flow ratios of 0.04 and 0.08, respectively. The viscous loss coefficient $K$ that was determined for the uncooled-turbine performance was used. The resultant maps were then checked against the corresponding experimental data by superposition of the experimental operating lines (determined by equivalent speed and equivalent work) for coolant-flow ratios of 0.08 and 0.04. The resulting comparison between the map efficiency and the experimental efficiency was taken as an indication of the degree to which the displacement effect accounted for the experimental trends of turbine performance. Although the turbine efficiency was used for the comparison, a discrepancy in pressure ratio (see eq. (2b)) would also be present. Since the assumed displacement effect was certainly present, at least in part, during the experimental operation of the turbines, any discrepancies remaining between the calculated performance and experimental performance were assumed to be accounted for by some additional mechanism.

The additional mechanism involved was assumed to be acceleration of the cooling air in the channel between the turbine blades in the tip region of the rotor. Since the total pressure of this cooling air is higher than the ambient combustion gas pressure (the cooling air has forced itself into this region), the cooling air is capable of performing work as it expands through the blade channel to the downstream pressure. The assumption was made that the entire amount of the differences in the performance between the displacement maps and the experimental results were due to such cooling-air work. As previously mentioned, the assumption that the entire amount of the extra blade work is due to cooling-air acceleration may mask actual improvement in aerodynamic efficiency of the combustion gases because of reduction of the usual flow disturbance in the blade tip region.

Considering the direction of the wheel rotation to be positive, the equivalent work done by the cooling air can be expressed as

$$\left(\frac{\Delta h'}{\beta_{cr_a}}\right) = W_a \left(\frac{-W_{a,u,6} U_t}{g \beta_{cr}}\right)$$

Since the cooling air enters the channel with a whirl velocity equal to the blade tip velocity $U_t$, $W_{a,u,6}$ is the change in the whirl velocity of the cooling air relative to the rotor tip velocity. If the expansion of this air is to produce useful work, it must be accelerated in a direction opposite the direction of wheel rotation; hence, the minus sign.
Figure 1 illustrates typical assumed velocity diagrams for the combustion gas and the cooling air at the blade tip and indicates the station notation used in the analysis.

This cooling-air work can be expressed in terms of the equivalent pumping work done by the turbine rotor on the cooling air, as follows:

\[
\frac{\left(\Delta h^{'}/\theta_{cr}\right)_{a}}{w_{a}} = \left(\frac{U_{t}^{2}}{g\theta_{cr}}\right)\left(-\frac{V_{a},u_{6}}{U_{t}}\right) = \left(\frac{w_{a}}{w_{g}}\right)\left(\frac{U_{t}^{2}}{g\theta_{cr}}\right)\left(1 - \frac{V_{a},u_{6}}{U_{t}}\right)
\]

\[
= \left(\frac{w_{a}}{w_{g}}\right)\left(\frac{U_{t}^{2}}{g\theta_{cr}}\right)K_{p}
\]

where the pumping coefficient \(K_{p} = \frac{-V_{a},u_{6}}{U_{t}} = 1 - \frac{V_{a},u_{6}}{U_{t}}\).

After the assumed cooling-air displacement effects have been accounted for, the cooled-turbine efficiency can be expressed by the following equation:

\[
\eta = \frac{(\Delta h^{'}/\theta_{cr})_{d}}{(\Delta h^{'}/\theta_{cr})_{d, id}} + \frac{K_{p}\left(\frac{w_{a}}{w_{g}}\right)\left(U_{t}^{2}/g\theta_{cr}\right)}{(\Delta h^{'}/\theta_{cr})_{d, id}}
\]

At any fixed value of equivalent work and equivalent speed, \((\Delta h^{'}/\theta_{cr})_{d}\), \((\Delta h^{'}/\theta_{cr})_{d, id}\), and \((U_{t}^{2}/g\theta_{cr})\) are set by the displacement calculations and, for any coolant-flow ratio \(w_{a}/w_{g}\), the \(\eta\) will vary in a linear manner with \(K_{p}\). For the displacement performance calculations, \(K_{p}\) is zero. The value of \(K_{p}\) necessary to obtain any required value of cooled-turbine efficiency \(\eta\) can be obtained by rewriting equation (5) as follows:

\[
K_{p} = \frac{(\Delta h^{'}/\theta_{cr})_{d, id}}{\left(\frac{w_{a}}{w_{g}}\right)\left(\frac{U_{t}^{2}}{g\theta_{cr}}\right)}
\]

The cooled-turbine performance was calculated by determining a value of \(K_{p}\) from equation (6) using an experimental value of \(\eta\) obtained at a coolant-flow ratio of 0.08.

The performance was determined for the coolant-flow ratio of 0.08 by holding \(K_{p}\) constant over the entire region of the map. The same value of \(K_{p}\) used at the coolant-flow ratio of 0.08 was then applied for the calculation of the cooled-turbine performance at a coolant-flow ratio of 0.04. The validity of assuming \(K_{p}\) constant will be determined by comparison between the calculated and experimental turbine efficiency variations.
RESULTS AND DISCUSSION

The calculated turbine performance maps, with the experimental operating lines superimposed, were used to obtain the variation of the calculated turbine efficiencies along the operating line of the experimental data. The experimental operating lines were located by the experimental values of equivalent speed and equivalent blade work. These calculated variations were then compared with the experimental turbine efficiency variations along the same operating line.

Uncooled-Turbine Performance

Calculated efficiency. - The calculated uncooled performance of the two turbines of reference 1 is shown in figure 2. The short horizontal lines extending off the primary operating line to the left were obtained by bleeding the compressor discharge air while the engine speed remained constant (ref. 1). For turbine A (fig. 2(a)), the match point for experimental and calculated turbine performance was at 99 percent of design equivalent turbine speed and an equivalent turbine work of 20.94 Btu per pound. For turbine B (fig. 2(b)), the match point was at 87 percent of design equivalent turbine speed and an equivalent turbine work of 20.30 Btu per pound. The different geometric characteristics of the two turbines result in different calculated performance characteristics. The greatest difference in the general trends of the calculated performance maps for the two turbines is that the efficiency contours of turbine A have considerable curvature, whereas turbine B (fig. 2(b)) exhibits efficiency contours that have little curvature throughout most of the turbine operating range covered by the calculations. The change in pressure ratio over a given range of equivalent turbine speed for a constant amount of equivalent blade work is generally slightly greater for turbine B than for turbine A.

Comparison of experimental and calculated efficiencies. - Figure 3 presents the comparisons between the experimental uncooled-turbine efficiencies and the turbine efficiencies predicted by each of the two calculated uncooled-turbine maps. The comparison of efficiencies covers a range of operating conditions varying from about 90 to 103 percent of rated equivalent turbine speed for turbine A (fig. 3(a)) and varying from about 70 to 87 percent of rated equivalent turbine speed for turbine B (fig. 3(b)). A tolerance of ±1/2 percentage point in efficiency was used at the match points.

For turbine A (fig. 3(a)), the calculated efficiency values average about 0.8 percentage point below the experimental values. Since most of the experimental data was within 1 percentage point of the predicted value, no revision of the viscous loss coefficient was made. The value of the viscous loss coefficient \( K \) was 0.8070 for turbine A.

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For turbine B (fig. 3(b)), the agreement between experimental and calculated efficiency values was generally within ±0.5 percentage point. The best agreement was obtained at the highest efficiencies which occur at the highest equivalent turbine speeds (see fig. 2(b)). The value of viscous loss coefficient $K$ used for turbine B was 1.125.

From the results shown in figure 3, it was concluded that the methods of reference 5 can give satisfactory results when applied to full-scale uncooled turbines operating at current turbine-inlet temperature levels, provided a satisfactory value of the viscous loss coefficient is obtained.

**Cooled-Turbine Performance**

**Effect of combustion gas displacement.** - If the assumption is made that the fluid leaving the turbine rotor exit (stations 4 and 5, fig. 1) is increased by the amount of the cooling-air flow, the displacement effect of the cooling air can be calculated. The performance of both turbines was calculated with the foregoing assumption for coolant-flow ratios of 0.04 and 0.08.

The effects of the cooling-air displacement on the calculated turbine efficiency at 90 and 100 percent of design equivalent turbine speeds for turbines A and B are presented in figures 4(a) and (b), respectively. The difference between the calculated, uncooled-turbine efficiency and that which is obtained when displacement effects are considered for a coolant-flow ratio of 0.08 is plotted against equivalent work for two constant values of equivalent speed. For values of equivalent work near limiting loading, the loss in efficiency of turbine A (fig. 4(a)) is about 1 and 2 points for design equivalent speeds of 100 and 90 percent, respectively. When the equivalent work is 2 to 3 Btu per pound lower than the limiting loading value, the decrease in efficiency is negligible and, when the equivalent work is about 4 to 5 Btu per pound below limiting loading, slight increases in turbine efficiency are apparent. Similar trends are indicated on figure 4(b) for turbine B. A cross-over of the 90- and 100-percent-equivalent-speed lines occurred for turbine A as limiting loading was approached. However, the 100-percent-equivalent-speed line of turbine B was more favorably affected by displacement than the 90-percent-equivalent-speed line throughout the entire range of the comparison. Since high-output turbines for aircraft engines are often designed to operate near limiting loading, the effects of displacement may be of importance in air-cooled-turbine performance and should be considered in the initial turbine design.

The experimental cooled-turbine efficiencies are compared in figure 5 with those predicted from the displacement calculations for a coolant-flow ratio of 0.08. Figure 5 clearly indicates that the experimental trends of reference 1 are not accounted for by the displacement effects alone. For
turbine A (fig. 5(a)), both the predicted and experimental turbine efficiencies were essentially independent of equivalent turbine speed, but the experimental values were about 7 percentage points greater than those predicted by the displacement calculations. Furthermore, turbine A of reference 1 actually operated in a region beyond the predicted limiting loading point (fig. 6). The operating line of turbine A extends through the limiting loading line and a considerable portion of the actual operation was outside the operating region predicted by the displacement calculations. No similar obvious misplacement of the operating line was evident on the map of turbine B. Turbine B (fig. 5(b)) also shows a disagreement between the calculated and experimental turbine efficiencies. The general trend of increasing efficiency with increasing equivalent turbine speed was common to both the calculated and experimental values of efficiency, but the experimental values are from about 1.5 to 3 percentage points greater than the predicted efficiency values. The calculated values of turbine efficiency at a coolant-flow ratio of 0.08 for turbine B compared more favorably with the experimental values than did those for turbine A. Since the experimental turbine efficiencies of both turbines were consistently higher than those predicted by the displacement calculations, some additional work-producing effect must have been present during the cooled operation.

**Effect of pumping coefficient.** - When the assumption is made that the apparent increase in turbine blade work beyond that predicted by the displacement calculations is entirely a result of an acceleration of the cooling air subsequent to leaving the blade tips and prior to passing axially downstream of the rotor-blade exit station, a coefficient $K_p$ that describes the amount of tangential acceleration of the cooling air because of its own energy level relative to conditions at station 6 can be established. The magnitude of $K_p$ can be established so that the calculated and experimental turbine performance are coincident at any single value of equivalent turbine speed, equivalent work, and coolant-flow ratio. If this coefficient were essentially independent of equivalent turbine speed, work, and coolant-flow ratio, it would be a useful parameter for describing the performance characteristics of a cooled turbine. Figure 7 presents the calculated, cooled-turbine performance maps obtained for turbines A and B at a coolant-flow ratio of 0.08. For turbine A, the pumping coefficient was 1.08 and, for turbine B, the pumping coefficient of 0.465 was used.

The limiting loading lines of these maps occur at the same pressure ratios as the limiting loading lines of the cooling-air displacement calculations. The equivalent blade work has been increased over that given by the displacement calculation at each equivalent speed by an amount indicated by equations (4). The efficiency level at a particular pressure ratio has increased a corresponding amount. All of the experimentally determined turbine operating lines now lie within the operating region
predicted by the calculations. Comparisons between calculated and experimental turbine efficiency values of turbine A can now be made along the entire range of the experimental operating line rather than along the restricted range possible from figure 5(a).

The behavior of the cooling air after leaving the blade tips is not known. The whirl velocity of the cooling air leaving the blade row might be expected to be a function of equivalent turbine speed and work. Both of these parameters would tend to affect the pressure ratio through which the cooling air could expand in moving from the blade tips to station 6, downstream of the turbine. The coolant-flow ratio would also be expected to affect $K_p$ since it would affect the pressure of the cooling air emerging from the blade tips. All of these effects were present in the data of reference 1. Comparisons between calculated efficiency variations (which assume a constant $K_p$) and the experimental efficiency trends should indicate the degree to which the net effect (if any) of these variations influence $K_p$. These comparisons will first be made over a range of equivalent speed and work (along the operating line of the data) and then at two values of the coolant-flow ratio. The first comparisons will indicate the net effect of speed and work; and the second comparisons will indicate the effect of coolant-flow ratio.

Figure 8 presents a comparison of the efficiency values obtained from the calculated maps with the experimentally established turbine efficiency values. For turbine A (fig. 8(a)), at a coolant-flow ratio of 0.08, the maximum deviation between the calculated efficiency and the experimentally determined efficiency, at the same equivalent speed and equivalent work, is about 0.2 percentage point. For a coolant-flow ratio of 0.04 for turbine A (fig. 8(b)), the discrepancy between the calculated and measured turbine efficiency values varies from about 0.1 to 0.5 percentage point over the speed range covered by the experimental data.

For turbine B (figs. 8(c) and (d)), there was exact agreement between the calculated and experimental values of efficiency at only one value of equivalent speed for both coolant-flow ratios considered. The maximum deviation between the calculated and experimental efficiency values was about 1.5 percentage points at a coolant-flow ratio of 0.08 and an equivalent speed of 2820 rpm (fig. 8(c)), and about 1.6 percentage points at a coolant-flow ratio of 0.04 and an equivalent speed of 2850 rpm (fig. 8(d)). For both turbines A and B, the use of the pumping coefficient $K_p$ established at a coolant-flow ratio of 0.08, for the calculations of efficiency trends for the coolant-flow ratio of 0.04, resulted in an upward displacement of the calculated turbine-efficiency line with respect to the experimentally determined efficiencies. This effect may appear to indicate that $K_p$ increases slightly with increasing coolant-flow ratio; however, the accuracy of the experimental data available for this investigation is not sufficient to justify such a conclusion with respect to the
small change between the calculated and experimental data when moving from a coolant-flow ratio of 0.08 to 0.04.

A summary of the effects of the various assumptions used in calculating the performance of the cooled turbines at a coolant-flow ratio of 0.08 are presented in figure 9. Turbines A and B are illustrated in figures 9(a) and (b), respectively. Also shown in figure 9 is the comparison between the experimental cooled- and uncooled-turbine efficiency data of the two turbines. These figures were constructed so that efficiency comparisons, made at a particular value of equivalent turbine speed, are also made at a common value of equivalent blade work. The experimental operating line for a coolant-flow ratio of 0.08 was used to establish these comparisons.

A comparison between the experimental efficiencies obtained for the uncooled turbine and the efficiencies obtained from calculations by the methods of reference 5 indicates that the efficiency trends for uncooled turbines were better calculated for turbine A than for turbine B. When the turbines are assumed cooled, using a coolant-flow ratio of 0.08, the deviation of the calculated and measured uncooled-turbine efficiencies for turbine A is a maximum of about 0.3 percentage point. For turbine B the discrepancy is a maximum of about 1.5 percentage points at an equivalent speed of 2800 rpm. Since the uncooled-turbine calculations (by methods of ref. 5) constitute the foundation upon which the cooling-air displacement and pumping coefficient modifications are imposed, the cooled-turbine map calculation would be expected to reproduce the cooled-turbine efficiency data trends much better for turbine A than for turbine B. This is clearly the case, as can be seen from both figures 8 and 9.

The effect of cooling-air displacement was considerably greater for turbine A than for turbine B, probably because turbine A operated in a region near the limiting loading line where the effects of the cooling-air displacement were greatest. The maximum effect of the cooling-air displacement was to reduce the calculated efficiency of turbine A about 1.3 percentage points at an equivalent speed of 5640 rpm. The effect of the cooling-air displacement on the calculated efficiencies was negligible for turbine B (maximum of 0.2 percentage point).

The amount of work required by the acceleration of the cooling air at the blade tips was the amount required to increase the efficiencies predicted for the displacement calculations to the efficiency level of the experimental data. As mentioned previously, in the discussion of figure 7 for turbine A, the addition of the cooling-air work to the displacement work extends the equivalent work and speed range of the cooled-turbine efficiency calculations relative to that of displacement calculations, so that comparison between calculated and experimental efficiencies can then be made over the entire speed range of the cooled-turbine data. For turbine A, the value of \( K_p \), used to obtain the calculated cooled-turbine efficiency variation, was 1.08. For turbine B, a value of \( K_p \)
of 0.465 was required. An investigation of the value of $K_p$ along the experimental operating line, which would result in exact agreement between the calculated and measured cooled-turbine efficiency values, was not made because of the large discrepancy that existed between the experimental uncooled-turbine data and the calculated uncooled-turbine map in the case of turbine B. This initial discrepancy would result in very large variations of $K_p$ with equivalent speed and equivalent work, since there is a large variation between the displacement map efficiency values and the uncooled-turbine data values. Because the separation between the cooled- and uncooled-turbine experimental data for turbine B was relatively constant throughout the speed range of the cooled-turbine data and the difference between the displacement efficiency values and the calculated cooled-turbine efficiency values was also relatively constant, the use of a constant value of $K_p$ apparently does account for the differences between the cooled- and uncooled-turbine efficiency trends. If better agreement between the uncooled-turbine data and the uncooled-turbine map had been obtained, results comparable to the results of turbine A apparently might have been achieved. In the case of turbine A, a constant value of $K_p$ certainly predicts the measured turbine efficiency trend within the accuracy of the data available.

The reason for the disagreement between the uncooled-turbine map calculation and experimental data of turbine B is not completely understood. This turbine was of an unorthodox design and operated at a very low efficiency level. Turbine A was a more conventionally designed turbine and the methods of reference 5 gave more reasonable results. Possibly, the assumption of a constant viscous loss coefficient becomes less accurate when the magnitude of the turbine losses becomes large, as was the case for turbine B ($K, 1.125$).

**General Discussion**

The magnitude of the pumping coefficient for turbine B was 0.465 as compared with the value of 1.08 obtained for turbine A. The reasons for this considerable difference in the recovery of cooling-air pumping work are not completely understood. Reasoning from the viewpoint that the majority of additional blade work is produced by the cooling air accelerating through the combustion gas channel between the turbine blade tips, it would seem likely that a turbine with a high degree of reaction at the blade tips would realize a greater recovery of work from the cooling air than a turbine that had essentially an impulse-type characteristic at the blade tips. In the high reaction-type turbine, the cooling air would have a large pressure gradient available to provide the acceleration necessary for acquiring a large exit whirl velocity in a direction opposite the wheel rotation.
With no cooling air present, turbine A had a considerable degree of tip reaction, whereas turbine B had essentially an impulse-type tip design. Undoubtedly, the reaction effect at the rotor tips was increased in both turbines as a result of the displacement effects of the cooling air, but the degree of reaction for turbine A certainly was always higher than for turbine B. The high recovery of work indicated by the pumping coefficient of turbine A relative to turbine B, therefore, appears reasonable. This reasoning suggests that a high degree of tip reaction may be desireable for air-cooled turbines which dispose of the cooling air by discharging it from the blade tips.

The pumping coefficient \( K_p \), which has been previously utilized herein to describe the performance of the two air-cooled turbines of reference 1, is certainly not the only such coefficient which could be devised. Reference 1 suggested that the whirl velocity of the cooling air might approach the whirl velocity of the combustion gas. Since the increased reaction of the turbine rotor associated with the addition of cooling air should cause an increase in the exit whirl velocity of both the combustion gas and the cooling air, a coefficient relating these whirl velocities might be expected to be a useful performance coefficient.

When the exit velocity of the cooling air at the blade tip is assumed to be proportional to the exit whirl velocity of the combustion gas at the mean span position, the work done by the cooling air can be expressed by

\[
\frac{\Delta h'}{\theta_{cr}} = \frac{w_a}{w_g} \frac{U_t}{\omega} (U - \bar{V}_g, \bar{u}, \bar{S})_m K_r = \frac{w_a}{w_g} \frac{U_t^2}{\omega} \frac{(-\bar{V}_g, \bar{u}, \bar{S})_m}{U_t} K_r
\]

(7)

A constant value of the performance coefficient \( K_r \) would then result in a variation of the coefficient \( K_p \) along a constant equivalent turbine speed line since

\[
K_p = K_r \left( \frac{w_g, \bar{u}, \bar{S}, \bar{m}}{U_t} \right)
\]

Calculations of this type were made for turbine A, using a constant value of 0.9 for \( K_r \). The predicted efficiency trends did not appear to follow the experimental trends as closely as the calculations in which a constant value of \( K_p \) was assumed. The accuracy of the available experimental data, however, was not considered sufficient to indicate a definite conclusion that \( K_p \) was superior to \( K_r \), even for this one turbine.

The degree to which the results of this investigation can be generalized is not known. More experimental data are needed on a variety of types of air-cooled turbines before the most useful type of performance coefficient can be established and correlated in order that an accurate
prediction of a cooled-turbine performance coefficient can be made while the turbine is still in the initial design stage.

Regardless of the type of performance coefficient that is eventually found to be most useful, the foregoing considerations appear to indicate some design features which may contribute to improvements in air-cooled-turbine performance. These are summarized as follows:

1. Allowance should be made for the displacement effect of the cooling air, particularly if the turbine is to be highly loaded.

2. A high degree of reaction at the blade tips may be conducive to a high recovery of work from the cooling air.

3. Modification of the blade tips, which would aid in redirecting the cooling-air velocity in a direction opposite the wheel rotation, may improve performance. German investigators (ref. 7) effected a recovery of approximately two-thirds of the rotor cooling-air pumping work by shortening the pressure surfaces of the turbine blades of a cooled-turbosupercharger unit.

**SUMMARY OF RESULTS**

The principal results of an analytical investigation of the performance of two air-cooled turbines can be summarized as follows:

1. The effect of cooling air on the performance of two air-cooled turbines was reasonably well calculated by assuming a displacement of combustion gas by cooling air at the rotor exit proportional to the coolant-flow ratio and a production of additional work from the cooling air in an amount proportional to the rotor tip speed.

2. The effect of combustion gas displacement alone was estimated to result in a maximum loss in turbine efficiency of from 1 to 2 points. This maximum loss occurred in the vicinity of turbine limiting loading. Since high-output turbines are often designed to operate near limiting loading, the effects of displacement should be considered in the initial turbine design.

3. The additional blade work obtained from the cooling air, necessary to account for the cooled-turbine performance, was estimated to be about 47 and 108 percent of the pumping work done by the turbine rotor on the cooling air for the two turbines investigated.

4. The additional blade work may depend, at least in part, on the degree of reaction of the turbine rotor tip section. The turbine with the highest tip reaction gave the greatest recovery of work from the cooling air.

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5. The additional blade work produced by the air-cooled turbines is believed to result primarily from acceleration of the cooling air subsequent to leaving the interior of the blade tips and prior to passing downstream of the rotor trailing edge, although reduction in the viscous losses within the rotor may make some contribution.

6. No effect of coolant-flow ratio on the magnitude of the pumping coefficient, which was used to express the additional work produced by the cooling air, could be established from the data available.

**REFERENCES**


Figure 1. - Station designations and velocity diagrams used in turbine analysis.
(a) Turbine A.

Figure 2. - Calculated performance map for uncooled turbine.
Figure 2. Concluded. Calculated performance map for uncooled turbine.
(a) Turbine A. Viscous loss coefficient $K$, 0.8070.

(b) Turbine B. Viscous loss coefficient $K$, 1.125.

Figure 3. - Comparison of calculated and experimental turbine efficiencies for uncooled turbines.
Figure 4. - Effect of cooling-air displacement on turbine efficiency. Coolant-flow ratio, 0.08.
Figure 5. - Comparison of calculated and experimental air-cooled turbine efficiency for a range of equivalent turbine speeds. Only displacement effects considered in calculated efficiency. Coolant-flow ratio, 0.08.
Figure 6. Calculated performance map for air-cooled turbine based on consideration of cooling-air displacement effects. Coolant-flow ratio, 0.08; turbine A.
Figure 7. - Calculated performance map for air-cooled turbine based on consideration of cooling-air displacement and pumping coefficient effects. Coolant-flow ratio, 0.08.
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Figure 7 (Continued) Calculated performance with standard-cooled blades based on consideration of cooling-air displacement and pumping coefficient effects. Cooling-fan ratio 0.08.

(b) Turbine B; pumping coefficient \( K_p = 0.465 \).
Figure 8 - Comparison of calculated and experimental turbine efficiency with cooling-air displacement and pumping coefficient $k_p$ considered.
Figure 9. - Summary of turbine-performance calculations for coolant-flow ratio of 0.08.
An analysis of experimentally determined effects of rotor cooling air on single-stage turbine performance was made. The analysis assumed that the reaction of the turbine was increased because discharge of the cooling air from the blade tips increased the mass flow per unit area at the turbine exit and that the cooling air, after being discharged from the blade tips, did work upon the turbine blades. The work done by the cooling air was related to an easily calculable parameter.
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