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TECHNICAL REPORT 2044

THE BASIC METHOD OF
CALCULATING THE LETHAL AREA
OF A WARHEAD

BY
WILLARD R. BENSON

OCTOBER 1954

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SAMUEL FELTMAN AMMUNITION LABORATORIES
PICATINNY ARSENAL
DOVER, N. J.

ORDNANCE PROJECT TBI-0004
DEPT. OF THE ARMY PROJECT 572-01-004

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OCT 25 1954
THE BASIC METHOD OF CALCULATING THE LETHAL AREA OF A WARHEAD

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Technical Report 2044
Ordinance Project T81-0044
Dept of the Army Project 572-01-004

Confidential
This report presents the basic formulas of lethal-area calculation. The writer has primarily referred to various reports published by the Ballistic Research Laboratories of Aberdeen Proving Ground and the reader should refer to the references listed for further information on the various subjects discussed.

NOTICE: THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 and 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW.
Table of Symbols

\( E_h \) = Expected number of hits on the target.

\( \rho_n \) = Density of fragments normal to fragment path.

\( A_n \) = Presented area of target normal to the fragment path.

\( \phi_n \) = Fragments per unit area of the missile surface.

\( h \) = Height of burst.

\( \theta \) = Angle of fragment path from the vertical axis from the ground.

\( a \) = Radius of a spherical missile.

\( \rho_{mn} \) = Fragment distribution per unit area on a target normal to the fragment path \( R \) units from the center of burst.

\( r \) = Distance the fragment must travel from the center of burst to the intended target.

\( R \) = Distance the fragment travels from the center of burst to the experimental target.

\( E \) = Contribution to the kinetic energy of an exploding body by a unit of explosive mass.

\( \rho, \theta, \phi \) = Spherical coordinates.

\( m_q \) = Mass per unit volume of explosive.

\( V_0 \) = Initial fragment velocity.

\( V_q \) = Velocity of the gases.

\( C \) = Total weight of explosive.

\( M \) = Total weight of fragments

\( \beta E \) = Gurney constant.

\( F_d \) = Drag force.

\( m \) = Mass of a fragment.
\( V = \) Velocity.
\( A = \) Presented area of a fragment.
\( \rho = \) Density of air.
\( C_D = \) Drag coefficient.
\( P_{\text{hit}} = \) Conditional probability that if a single fragment hits the target it will disable the target.
\( E_{\text{hit}} = \) Expected number of disabling hits on the target.
\( P = \) The probability that the target will be disabled.
\( N = \) The number of targets disabled.
\( \sigma' = \) Number of targets per unit area.
\( A_L = \) Lethal area.
\( (\rho, \beta) = \) Polar coordinates.
INTRODUCTION:

1. The major function of any warhead is the elimination of its intended target. The probability of accomplishing this mission depends on the characteristics of the weapon system, the lethal potential of the warhead, and the resistance of the target. A projectile which has pinpoint accuracy is obviously useless if the lethality of the projectile is not enough to damage the target. Of course the converse is also true: that the lethal potential of a projectile is wasted if it does not reach the target. It is therefore concluded that a balance must be drawn to produce the optimum design of a weapon system.

2. One vital phase of weapon design which must be considered by the Ordnance designer is the determination of the lethality of the warhead and its utilization with other factors in the selection of optimum design parameters. The object of this report is to present the basic formulas necessary for the calculation of a warhead's lethality.

3. A convenient factor which is representative of a warhead's lethal potential is lethal area. Although lethal area has the units of area, it does not have the physical significance of area. It perhaps could have been more fittingly called "casualty index", since lethal area multiplied by the number of targets per unit area results in the number of targets disabled. It is expressed mathematically as \[ \int P_d dA \] where the limits of integration are determined by terrain limitations.

4. The principal use of lethal area is in comparing one weapon system with another. This comparison may be made in terms of lethal area per round, lethal area per pound, the lethal area a system can produce in a given amount of time, etc. The comparison used will be determined by the tactical situation most commonly encountered with the weapon being considered.

5. To calculate lethal area, certain information must be known. The fragment distribution and the area of the target must be known so that the expected number of hits on the target can be determined. In addition, the impact velocity of the fragment must be known. This requires a knowledge of the initial fragment velocity and the velocity decay over the distance traveled by the fragment.
Finally, a damage criterion must be established. The damage criterion for personnel targets is expressed as the conditional probability that a single fragment will strike the target and is a function of the mass, shape, and velocity of the fragment.

**EXPECTED NUMBER OF HITS ON A TARGET**

6. The expected number of hits on a target is the product of the number of fragments per unit area at the target and the projected area of the target normal to the fragment array. This can be expressed as

\[ E_h = \rho_N \cdot A_n \]

where

- \( \rho_N \) = Fragment density normal to the fragment path.
- \( A_n \) = Presented area of target normal to the fragment path.
- \( E_h \) = Expected number of hits on the target.

This equation for \( E_h \) is good, providing \( \rho_N \) is known. Before fabricating any models, one must resort to a mathematical expression for \( \rho_N \). \( \rho_N \) is primarily a function of the number of fragments per unit area of missile surface and the shape of the missile surface.

*The mathematical calculation of fragment density is given extensive treatment in a report, now in rough draft form and not yet ready for release, entitled The Influence of the Surface Contour of an Exploding Body on Fragment Distribution by Millard R. Benson, Picatinny Arsenal, Dover, N. J. See Figures 1 and 2 for a summary of that report.*
7. For an air-burst sphere, \( \rho \) can be expressed as

\[
\rho_n = R_0 \left( \frac{\pi}{h} \cos \theta \right)^2
\]

(2)

where

(See Fig 3)

- \( \rho_s \) = Fragments per unit area of missile surface
- \( a \) = Radius of the sphere
- \( h \) = Height of burst
- \( \theta \) = Angle of fragment path from the vertical axis

8. Obviously \( \rho \) can also be experimentally determined once a model is fabricated, and it is considered advisable to do so to check the theoretical predicted fragment distribution. It is more convenient in this case to express \( E_h \) as

\[
E_h = \frac{(\rho_a R^2)}{r_s} A_n
\]

(3)

where

- \( \rho_a \) = Fragment distribution per unit area on a target, normal to the fragment path, \( R \) units from the center of burst
- \( R \) = Distance the fragment travels from the center of burst to the experimental target
- \( r_s \) = Distance the fragment must travel from the center of burst to the intended target

9. The average presented area \( A_n \) of a human target unprotected by material obstruction can be approximated as

\[
A_n = 3.1 \times 1.10 \cos \theta \text{ sq ft}
\]

(4)

For a ground burst, then, \( A_n \) = 3.1 sq ft.
10. In the presentation of the above information two assumptions were made that are not always valid:

(a) The velocity of the warhead is small in comparison with the fragment velocity.

(b) The target is completely exposed to the fragment spray.

11. The assumptions were purposely made so that the general approach of the report would not become mathematically over-complicated. It is possible, however, to include the above influence in a neat and orderly fashion (Ref 1).

**FRAGMENT VELOCITY**

12. One of the most important parameters affecting lethal area is the striking velocity of the fragments upon contacting the target. To determine the striking velocity, it is necessary to know the initial fragment velocity and the velocity decay along its trajectory.

13. The initial fragment velocity for a conventional shell is closely predicted by the Gurney formula (Ref 2). In his thesis Gurney assumed that the contribution to the kinetic energy of the explosive gases and metal fragments made by the detonation of each unit mass of explosive is the same in all types of projectiles.

14. Consider the application of this statement to a spherical shell where

Diagram 1

\[ E = \text{Contribution to the kinetic energy by a unit of explosive mass.} \]

\[ \rho, \theta, \phi = \text{Spherical coordinates shown in Diagram 1.} \]

\[ M = \text{Mass per unit volume of explosive.} \]

\[ V_0 = \text{Initial fragment velocity.} \]

\[ V_g = \text{Velocity of the gases.} \]
\[ C = \text{Total weight of explosive.} \]

\[ a = \text{Inner radius of the sphere.} \]

\[ M = \text{Total weight of fragments.} \]

There results

\[ CE = \frac{1}{2} MV_o^2 + \frac{1}{2} \int M_9 V_9^2 dV \quad (5) \]

It is tacitly assumed that \( V_9 \) is zero at the center of the missile, equal to the fragment velocity \( V_o \) at the surface of the missile, and varies linearly between these two extreme points. Mathematically, this statement can be expressed as

\[ V_9 = \frac{a}{a} V_o \quad (6) \]

where \( 0 \leq \rho \leq a \)

\( dV \) in spherical coordinates is

\[ dV = \rho^2 \sin \theta d\rho d\phi d\theta \quad (7) \]

Substituting equations (6) and (7) in equation (5) there results

\[ CE = \frac{1}{2} MV_o^2 + \frac{1}{2} \int_0^a \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{M_9 V_o^2}{\rho^2} \sin \theta d\rho d\phi d\theta \quad (8) \]

Integrating equation (6) there results

\[ CE = \frac{1}{2} MV_o^2 + \frac{4}{3} \pi a^3 M_9 \cdot \frac{1}{2} \cdot \frac{3V_o^2}{2} \quad (9) \]

since \( C = \frac{4}{3} \pi a^3 M_9 \) the above equation can be expressed as

\[ CE = \frac{1}{2} \left( MV_o^2 + \frac{2}{3} CV_o^2 \right) \quad (10) \]
Solving for $V_0$,

$$V_0 = \sqrt{2E} \sqrt{\frac{\rho}{1 + 0.9 \frac{u}{E}}} \quad (11)$$

Equation (11) gives the initial fragment velocity for a spherical shell. If the same procedure is followed in considering a cylindrical shell there results

$$V_0 = \sqrt{2E} \sqrt{\frac{\rho}{1 + 0.9 \frac{u}{E}}} \quad (12)$$

Figure 4 is a graph of these functions.

15. The value $\sqrt{2E}$ is determined experimentally and varies with the explosive. Values commonly used are:

<table>
<thead>
<tr>
<th>Explosive</th>
<th>$\sqrt{2E}$</th>
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</thead>
<tbody>
<tr>
<td>TNT</td>
<td>8000 f/s</td>
</tr>
<tr>
<td>Comp C3</td>
<td>8800 f/s</td>
</tr>
<tr>
<td>Comp B</td>
<td>9000 f/s</td>
</tr>
</tbody>
</table>

16. As stated previously the Gurney formula is applicable in the velocity analysis of a homogeneous shell. Recent experiments conducted to determine fragment velocity of shell with controlled fragment masses have resulted in velocities as low as 70% of the value predicted by the Gurney formula. It is currently believed that the reduced velocity is the result of gas blow-by between the fragments. This gas blow-by occurs sooner with a controlled fragment shell than with a conventional shell, since the conventional shell will stretch, due to its ductility, and obstruct the gases longer than the controlled fragment shell which possesses little, if any, shell strength. In utilizing the Gurney formula for shell with controlled fragments, the designer should not be too optimistic in expecting theoretical calculated velocity.

* Recently published values which differ from the above listed values are presented in NAVORD Report 2935 titled Explosive Comparison for Fragmentation Effectiveness 11 August 1958. This report also gives values for the Gurney constant for many other types of explosives.
17. In addition to the initial fragment velocity, it is necessary to know the fragment velocity decay as a function of the mass, shape, and path of the fragment.

18. A mathematical expression for velocity decay is quite easily obtained by considering Newton's Law of Action

\[ m \frac{dv}{dt} = -F_d \]  \hspace{1cm} (13)

where

\[ F_d = \text{Drag Force} \]
\[ m = \text{Mass of the fragment} \]
\[ v = \text{Velocity} \]

\( F_d \) is commonly expressed as

\[ F_d = C_D A \sigma v^2 \]  \hspace{1cm} (14)

where

\[ A = \text{Presented area of the fragment.} \]
\[ \sigma = \text{Density of air.} \]
\[ C_D = \text{Drag coefficient.} \]

Substituting eq (14) in eq (13) there results

\[ \frac{dv}{dt} = -\frac{F_d v^2}{m} \]  \hspace{1cm} (15)

where

\[ \beta = C_D \sigma \]

consider further that

\[ \frac{dv}{dt} = \frac{dv}{dr} \frac{dr}{dr} = v \frac{dv}{dr} \]  \hspace{1cm} (16)
Equation (15) becomes

\[ \frac{dV}{V} = -\frac{BAr}{m} \]  

(17)

Integrating

\[ \ln V = -\frac{BAr}{m} \ln E + C' \]  

(18)

Where \( C' \) is the constant of integration

or

\[ V = C \frac{BAr}{m} \]  

(19)

From the initial condition \( t = 0, V = V_0 \)

(20)

Hence

\[ V = V_0 C \frac{BAr}{m} \]  

(21)

Equation 21 can also be expressed with a base 10 as

\[ V = V_0 10^{\frac{BAr}{m}} \]  

(22)

It has been experimentally found that a good representative value of \( B^I \) for supersonic velocities around Mach 5 is

\[ 0.0091 \] (Ref 3).

Therefore

\[ V = V_0 10^{-0.0091} \frac{Ar}{m} \]  

(22)

where

\[ \frac{Ar}{m} \] is in cgs units

and

\[ V_0 \] is in f/s and \( r \) is in ft

Table 1 gives values of \( Ar/m \) for various shaped fragments.

Figure 5 is a plot of the velocity decay \( (V/V_0) \) for various sized spheres.

11
19. The user of Figure 5 should keep in mind that a constant drag coefficient was assumed in the plotting of the curves. Provided fragment velocity remains above Mach 1.6, the assumption of a constant drag coefficient will probably satisfy most engineering applications.

TARGET DAMAGE CRITERION

20. The resulting equations of the subjects discussed to this point are perfectly general and applicable to any type of target. The type of target is introduced when the vulnerability of the target to impacting fragments is considered. As an example of the mathematical treatment that is applied to target vulnerability, the case of personnel targets will be considered.

21. Before recent experimentation, a human target was considered incapacitated if it was struck with at least one fragment having a kinetic energy of 58 foot-pounds or greater. This has been superseded by a more refined and logical theory (Ref 4 and 5). The present damage criterion is expressed as the conditional probability ($P_{hk}$) that a single fragment will disable the target and it is a function of $\frac{mV}{A}$, where $m$ is the mass of the fragment, $V$ is the striking velocity of the fragment, and $A$ is the cross-sectional area of the fragment. Furthermore, time is introduced into the criterion by plotting the probability of disabling the target within 5 seconds, within 30 seconds, within 5 minutes, and within an unlimited amount of time (Type B disablement).

22. The current antipersonnel disablement curves are shown in Figure 6. It can be readily observed that nothing in terms of lethal potential is gained by allowing $\left( \frac{mV}{A} \right)$ to become greater than 2.5. Furthermore, no damage will result if $\left( \frac{mV}{A} \right)$ is less than 1.0. *

23. Figure 7 graphically shows the difference between the $mV/A$ criterion and the kinetic-energy criterion for disabling. For instance a 1/16-diameter fragment is on the threshold of disabling if it has a fragment velocity of 4000 f/s or a kinetic energy of 9 ft-lbs. The same fragment has nearly reached its lethality saturation value if it has a velocity of 8000 f/s which corresponds to a kinetic energy of 35 ft-lbs.

* The $P_{hk}$ curves presented in this report are by no means fixed and are subject to change. One should therefore determine, prior to an analysis, the status of the disablement criterion he intends to use.
LETHAL AREA

24. $P_K$ is, as stated, the single-hit probability of disabling the target. What is needed, however, is the overall probability (denoted as $P_K$) of disabling the target. This will, in effect, consider the fact that the target will receive a random number of hits. If then $P_K$ is the probability that a single hit will disable the target and $E_h$ is the expected number of disabling hits, $E_K$, is simple

$$E_K = E_h P_K$$  \hspace{1cm} (23)

The probability, $P_K$, that the target will be disabled is closely approximated by the Poisson probability function as

$$P_K = e^{-E_K}$$  \hspace{1cm} (24)

Figure 8 is a plot of this function.

25. The number of targets disabled, $N_K$, in detonating a single missile is then:

$$N_K = \int_0^\infty \int_0^\infty \sigma' P_K \, dx \, dy$$  \hspace{1cm} (25)

where $\sigma'$ is the number of targets per unit area.

If $\sigma'$ is considered constant, equation (25) can be written

$$N_K = \sigma' \int_0^\infty \int_0^\infty P_K \, dx \, dy$$  \hspace{1cm} (26)

By definition, the integral term is the lethal area, $A_L$. Hence

$$A_L = \int_0^\infty \int_0^\infty P_K \, dx \, dy$$  \hspace{1cm} (27)
If the fragment distribution has circular symmetry, the lethal area can be expressed in polar coordinates \((\eta, \beta)\) as

\[
A_L = 2\pi \int_0^\infty P_K \eta d\eta
\]

(28)

26. In a practical problem, the limits of integration will not extend to infinity but will be governed by the imposed terrain limitation. Furthermore, \(P_K\) falls off to zero at a finite value of \(\eta\).

27. It should be pointed out that the lethal area divided by the physical area covered by fragments results in the average probability of kill over the area covered by fragments.

28. The basic information for the calculation of lethal area has been presented in the foregoing sections.

29. One can utilize the outlined approach to determine such optimum design parameters as fragment size, bomb size, height of burst, etc. When all except one parameters are held constant, the variation in lethal area can be determined as a function of the varied parameter and an optimum value for this parameter can be established.
REFERENCES:


INCLUSIONS:

1. Appendix.

2. Tables 1-2.

Appendix

Example

The following problem is offered to illustrate the use of the computing form (Table 2).

Given:

A three-inch-diameter missile with 1/4-inch-diameter steel fragments.

\[ C = 220.0 \text{ grams} \]
\[ n = 455.0 \text{ grams} \]
\[ A/m = 300 \text{ (age units)} \]

Explosive = TNT

To Find:

The lethal area of the missile considering a 15-ft terrain limitation, Type B disablement, and ground-burst functioning.

The number of fragments per square inch of missile surface can be approximated by the formula \( \frac{C}{d} \) where \( d \) is the diameter of the fragment in inches. \( C \) can then be calculated from equation (3) by letting \( R = \frac{C}{12} \left( \frac{37}{4} - \frac{d}{2} \right) \times \frac{L}{2} \text{ Fr} \).

The rest of the form is filled out as indicated by the column heads.

The symbol \( \int \) is used by the author as a notation indicating integration by the trapezoidal rule.

Diagrammatically:

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<th>Column (19)</th>
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<tr>
<td>( a_2 )</td>
<td>+ ( a_1 + a_2 )</td>
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<tr>
<td>( a_3 )</td>
<td>+ ( a_1 + 2a_2 + a_3 )</td>
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<td>( a_4 )</td>
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<td>( a_5 )</td>
<td>+ ( a_1 + 2a_2 + 2a_3 + 2a_4 + a_5 )</td>
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<table>
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<tr>
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<td>$\frac{A}{m}$ FOR VARIOUS SHAPES OF FRAGMENTS</td>
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<td>(cgs system of units)</td>
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<td>Steel Cubes</td>
<td>$\frac{A}{m} = 0.38 \ m^{-\frac{1}{3}}$</td>
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### TABLE 2

**LETHAL AREA CALCULATION TABLE**

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13

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TABLE 2
LETHAL AREA CALCULATION TABLE

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<td>( V_T/V_o )</td>
<td>( V_T )</td>
<td>( mV_T )</td>
<td>( P_{HK} )</td>
<td>( E_{HK} )</td>
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\[ A = 389.25 \text{ ft}^2 \]
\[ P_n = \frac{\rho_0 x [1 + y^2]}{[x + y^2(y-h)][1 + (y)^2 + y^2(y-h)]} \]

and

\[ \eta = x + y(y-h) \]

where

\[ y' = \frac{dy}{dx} \]
\[ y'' = \frac{d^2y}{dx^2} \]

and

\[ h \] is burst height

FIG. 1
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NOTE:

\( P_{n'} \) - fragment distribution on \( x'o'y' \) plane
\( P_n \) - fragment distribution on \( xoy \) plane
(The equation for \( P_n \) for any given surface is listed on Figure 1.)

\[
P_n = \frac{P_n \cos^2(\theta + \varphi)}{\cos^2 \varphi \cos \psi}
\]
\[
\eta = \frac{\rho \cos^2 \theta}{\cos(\theta + \varphi)}
\]
\[
\xi = \frac{\cos \theta \cos \varphi}{\cos(\theta + \varphi)}
\]

FIG. 2
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r = radius of sphere
h = height of burst
P*/A = frag/unit area of missile surface
\( \theta \) = angle of fragment path from the vertical
P*/A = frag/unit area of target surface normal to the path (n)
P*/A = frag/unit area on the target surface

\[
P_{n} = \frac{P_{o}}{h_{n}} = \cos^{2} \theta = \frac{h_{n}}{h}
\]

\[
P_{n}/A = \cos^{2} \theta
\]

\[
W_{o}/A = \cos^{2} \theta
\]

FRAGMENT DISTRIBUTION FOR A SPHERICAL SURFACE

FIG. 3
### Fragment Velocity ($V_f$) = Initial Fragment Velocity ($V_i$) vs Distance Traveled ($V$)

**Figure 3**

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KINETIC ENERGY VS MV/A FOR SPHERICAL FRAGMENTS

Fig. 7
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