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Engineering Report
Feasibility Study
of the
GYROSCOPIC AIMING DEVICE

Contract DA-36-038-ORD-17921

CONTROL ENGINEERING CORPORATION

Date: June 4, 1954
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I. INTRODUCTION

A. Scope

This report is submitted in compliance with Army Ordnance Contract No. DA-36-038-ORD-17921, initiated March 29, 1954 for the purpose of investigating the feasibility of a gyroscopic aiming device for field artillery weapons.

The report presents the method of investigation, description and analysis of the system and its components, and conclusions as to the feasible accuracies of the gyroscopic aiming device.

Specifications for the gyroscopic aiming device are outlined in Fire Control Procurement Description No. 157 dated 1 December 1953, and feasibility conclusions are presented on the basis of these specifications.

B. Personnel

The following personnel, listed in the proper categories, contributed to the feasibility investigation and report.

System Analysis
F. R. Zatlin — Chief Electrical Engineer, 180 hours

Gyro Investigation
E. L. Swainson — Chief Components Engineer, 30 hours
R. S. Henderson — Senior Engineer, Gyro Design, 170 hours
R. G. Haagen — Engineer, Gyro Design, 40 hours

Gimbailing Configuration & Geometrical Analysis
J. Huff — Senior Mechanical Engineer, Stress Analysis, 60 hours

In addition to these engineers which were assigned to the project, company production end methods engineers contributed to the feasibility study.

C. Method of Investigation

Because of the limited scope of the program for the gyroscopic aiming device feasibility study, it was not possible to construct actual equipment to verify the results of this study. The procedures followed, upon which the conclusions of this report are based, are outlined below:

1. Gyro Investigation

Control Engineering Corporation has amassed considerable experience in the design and manufacture of gyro and gyro components. A portion of these, applicable to this study, are listed below:

a. Rate-integrating gyro (GI-K2) — angular momentum of $10^5$ dyne-cm-sec — now in production
b. Rate gyro — angular momentum of $10^4$ dyne-cm-sec — now in development stage.
c. Gyro wheel — used in two-degree of freedom gyro — angular momentum $7 \times 10^6$ dyne-cm-sec, now in production.
d. Signal generators, torque generators, and gyro wheels for the HIG U 10^5 gyro, now in production.
e. Various electromagnetic transducers.

The experience gained in the design, development, and production of the above units has been used as a basis for extrapolation of expected results of a production run of gyros having parameters compatible with the gyroscopic aiming device.

When the gyro wheel size was determined (on the basis of necessary threshold sensitivity), the gyro designers investigated the feasibility of manufacturing a gyro dictated by these specifications. In addition to conferring with the production staff of the Company, other manufacturers of gyros in this range of angular momentum were contacted. Since no manufacturer was engaged in large scale production of gyros of this type, laboratories engaged in prototype construction were also contacted.

Since the scope of the program did not allow for development of new techniques for gyro construction, the estimated production results presented in this report are necessarily an extrapolation of present prototype gyros based on experience gained by the evolution of smaller gyros from the design stage to the production stage.

(2) System Analysis

Much information is available for this report on the expected tolerances of gear trains, servo-mechanisms, amplifier characteristics, and the components which are a part of the gyroscopic aiming device reference system. This information is available on the basis of past experience of the personnel contributing to this report; experience gained in both design and production of this class of equipment.
Figure 1. Effect of Bearing on Component of Earth's angular velocity sensed by Gyro.
II. GENERAL SYSTEM DESCRIPTION

Since it is both desirable and advantageous to lay a field artillery weapon without requiring an unobstructed line of sight, there is a need for a device that will provide a self-contained azimuth reference reliably and with an accuracy compatible with present day artillery fire problems.

A. East Sensing Element

A single degree of freedom gyro will sense angular velocities about its input axis and furnish a precession torque about its output axis (and thereby a signal voltage) as a measure of the vector quantity of the angular velocity. By orientating the gyro so that its output axis is aligned with the earth's vertical, (thereby placing the input axis in the horizontal plane) the gyro will sense a component angular velocity of the earth and furnish a precession torque about its output axis proportional to that portion of the earth's angular velocity vector parallel to the input axis, i.e. torque about the output axis will be a maximum when the gyro input axis is in a plane which passes through the earth's rotational axis (North-South) and minimum when the input axis is perpendicular to this plane (East-West).

Figure 1 shows the relationship of the gyro input axis to the earth's rotational axis. It can be seen that the component of the earth's angular velocity vector parallel to the gyro input axis will vary as the cosine of the angle between the earth's rotational axis and the gyro input axis which is the bearing angle.

From Figure 2 it can be seen that the component of the earth's angular velocity vector parallel to the input axis will also vary as the cosine of latitude.

Therefore, to indicate a North reference (gyro input axis east) to an accuracy of 0.03 degrees at 40° Latitude, it will be necessary to have a gyroscopic element capable of sensing angular velocities compatible with equation 1.

\[
\text{Threshold Sensitivity} = W_e \cos \theta \cos \text{Lat.} \tag{1}
\]

where \( W_e = \) earth's rate, 15°/hr

\( \theta = \) angle between true North and the gyro input axis.

Threshold Sens. = 15°/hr x cos 89.97° x cos 40.0°

B. Servo Control System

To provide an azimuth reference, using the single degree of freedom gyro as the sensing element, it will be necessary to control the gyro gimbal so that the input axis of the gyro is continuously maintained perpendicular to a plane passing through the earth's rotational axis. This is accomplished by driving the gyro about its output axis, until the control system is at null. This will necessitate a servo control system consisting of an amplifier, servo motor, support gimbal and inner gimbal which will contain the gyroscopic element. The angle between the inner gimbal and the support gimbal will then indicate true bearing. A two speed synchro system (1 and 36 speed) will be used to transmit bearing angle to the gun operator.
NORTH EARTH'S ANGULAR VELOCITY VECTOR

NOTE ZERO ANGULAR VELOCITY AT 90° LATITUDE

PLANE OF INPUT AXIS

VERTICAL REFERENCE

EARTH'S ROTATIONAL AXIS

EQUATOR

LATITUDE ANGLE

COMPONENT OF EARTH'S ANGULAR VELOCITY AFFECTING PLANE OF INPUT AXIS. (COSINE OF LATITUDE TIMES EARTH'S ANGULAR VELOCITY)

NOTE FULL ANGULAR VELOCITY AT 0° LATITUDE

Figure 2. Effect of Latitude on Component of Earth's angular velocity sensed by Gyro.
C. Geometrical Considerations

In the general case, the gun trunnions (elevation axis) will not be level and the gun barrel will elevate in a plane normal to the deck plane. The gun mount, being canted, will traverse in the deck plane and not in the horizontal plane. It is, therefore, necessary to provide a means to convert traverse angle to azimuth angle so that the aiming device will lie in a horizontal plane and read Bearing.

The present M18A1 Compensating mount is a device which, by means of a Hooke's joint mechanical linkage will convert traverse angle to bearing, and elevation angle to cross-level.

Since it is herein proposed that the present M18A1 Compensating mount, with minor modifications, will be used to mount the gyroscopic aiming device, further discussion of the geometrical problem will be limited to the mechanical modifications necessary to allow mounting of the gimbaling system on the present compensating mount.
Figure 3. Mechanical Schematic – Rate integrating – Gyro.
III. DETAILED SYSTEM DESCRIPTION AND ANALYSIS

A. Gyro

1. Gyro Characteristics

The basic gyro used as the earth's rate sensing element for the gyroscopic aiming device will be a viscously floated single degree of freedom rate-integrating gyro. Figure 3 is a schematic showing the various components of the rate-integrating gyro.

The single degree of freedom rate-integrating gyro functions to sense angular velocity about its input axis and produce a precession torque to rotate the gyro wheel gimbal about its output axis. This torque, opposed by the viscous shear force of the damping fluid, results in the gyro gimbal rotating about its output axis at an angular velocity proportional to the sensed velocity about its input axis. In like manner, when the gyro is rotated through a given angle, the output axis will rotate through a proportional angle. The output angular position of the gyro gimbal is, therefore, the time integral of the velocity about the input axis; hence the name - rate-integrating gyro.

As an azimuth reference the rate-integrating gyro alone is not suitable. If the wheel is not in the null or zero position when power is applied, the azimuth reference will change from its original position. Therefore, it is necessary to include provisions for restraining the gyro in its zero position; i.e. simulating the characteristics of the rate gyro. Figure 4 is a schematic diagram of the method used to, in effect, produce a rate gyro while still retaining the desirable characteristics of the rate-integrating gyro.

![Block Diagram Rate Gyro Loop Configuration](image)

**Figure 4. Block Diagram Rate Gyro Loop Configuration.**

Where:

- $W_o$ — earth's angular velocity input
- $K_1$ — cos Latitude $\times$ cos Bearing Angle
- $T_0$ — precession torque applied to rotate the output axis
- $T_1$ — torque supplied by Torque Generator to rotate the output axis
- $C$ — damping coefficient
- $I$ — inertia of the gyro gimbal (about output axis)
(2) Derivation of Gyro Parameter Equations

The applied torque will be a cross product of the angular momentum of the gyro wheel and the sensed velocity about the gyro input axis.

\[ T_a = H K_1 \omega \]

The opposing or resisting torque will then be the summation of the effect of output gimbal rotation on the inertia and damping plus the feedback torque from the gyro torque generator.

\[ T_R = \frac{I d^2 \theta_o}{dt^2} + \frac{C d \theta_o}{dt} + K_1 K_2 K_3 \theta_o \]

or, in the transformed equation: (p is used to denote the function d/dt)

\[ T_R = I p^2 \theta_o + C p \theta_o + K_1 K_2 K_3 \theta_o \]

From Newton’s second law applied to torque equilibrium; the summation of forces acting on a body will be zero, the equation may be written as follows:

\[ HK_1 \omega = I p^2 \theta_o + C p \theta_o + K_1 K_2 K_3 \theta_o \]

Transposing:

\[ \frac{\theta_o}{K_1 \omega} = \frac{H}{I p^2 + C p + K_1 K_2 K_3} = \frac{H K_1 K_2 K_3}{1 + p^2 + \frac{C}{K_1 K_2 K_3}} \]

(2)

From this equation it can be seen that the gyro now possesses the characteristics of a rate gyro; namely, in the steady state, a constant velocity sensed about the input axis will result in an angular position about the output axis (and, therefore, a voltage proportional to sensed rate).

The gyro sensitivity will be \( H/K_1 K_2 K_3 \)

The dimensions will be

\[ \frac{H \text{ dyne-cm-sec.}}{K_1 \text{ Volts} \times K_2 \text{ mamp} \times K_3 \text{ dyn cm}} = \frac{\text{Angle}}{\text{Rad./sec}} = \frac{\text{Position}}{\text{Velocity}} \]
The characteristic equation of the gyro and its feedback loop as shown in equation (2) is

\[
\theta_o = \frac{1}{w_{input}} = \frac{1}{K_1 K_2 K_3 p^2 + C K_1 K_2 K_3 p + 1}
\]

therefore, the undamped natural frequency \( w_n \) can be determined:

\[
w_n = \sqrt{\frac{K_1 K_2 K_3}{1}}
\]

and the damping ratio \( \zeta \) will be:

\[
\zeta = \frac{C w_n}{2 (K_1 K_2 K_3)}
\]

(3) **Determination of Gyro Parameters**

As previously stated the threshold sensitivity of the gyro element is determined by the equation:

\[
\text{Threshold Sensitivity} = 15^\circ/hr \times \cos 89.97^\circ \times \cos 40.0^\circ
\]

The gyro must be able to sense angular velocities of

\[
7.29 \times 10^5 \text{ rad. sec.} \times .00052 \times .766
\]

\[
(\omega_s) (\cos 89.97^\circ) (\cos 40.0^\circ)
\]

\[
= 29.04 \times 10^{-9} \text{ rad/sec.}
\]

The uncertainties of drift rates for a gyro are determined by friction level, uncertainty in flex lead torque, rectification of pivot vibration, and uncertainty in signal generator measurements. Under laboratory conditions it is possible to keep the sum of these uncertainties to a level of 1.3 dyne-cm. This has been verified by laboratory tests of gyros being produced in prototype quantities.

From the equation, Precession Torque \( (T_g) = H \omega \), it is then possible to determine the necessary angular momentum of the gyro wheel for the case where torques of 1.3 dyne-cm must be produced for the required rate of 29.04 \( \times 10^{-9} \) rad/sec.

\[
T_g = H \omega
\]

\[
H = \frac{T_g}{\omega} = 1.3 \frac{\text{dyne-cm}}{29.04 \times 10^{-9} \text{ rad/sec.}} = 44.8 \times 10^6 \text{ dyne-cm-sec/rad.}
\]

Let \( H = 45.0 \times 10^6 \text{ dyne-cm-sec/rad.} \)
Assuming a torque generator sensitivity \( (K_2) \) of \( \frac{100 \text{ dyne-cm}}{\text{m-str}} \) and a signal generator sensitivity \( (K_1) \) of \( \frac{100 \text{ Mvolts}}{\text{M-radian}} \) (these are nominal values which are well within the range of practicality in production equipment) it will be possible to determine the remaining parameters of the gyro.

The feedback amplifier sensitivity \( (K_2) \) can be determined in the following manner:

From equation (2) –

In the steady state, \( (\text{time} = \infty) \)

\[
\frac{\theta_o}{K_1W_s} = \frac{H}{K_1 K_2 K_3}
\]

or:

\[
\theta_o = \frac{HK_1W_s}{K_1 K_2 K_3}
\]  (7)

at 89.97° angle and 40° latitude the angle through which the output gimbal will rotate is:

\[
\theta_o = \frac{HW_s \cos 89.97° \times \cos 40°}{K_1 K_2 K_3}
\]

the equation may be written in terms of the signal generator voltage \( (E_o) \)

\[
E_o = \frac{HW_s \cos 89.97° \times \cos 40°}{K_2 K_3}
\]

where \( E_o = K_1 \theta_o \)

A signal generator of this type (sensitivity 100 millivolts/milliradian of output angle) provides angular resolution corresponding to .001 milliradian or 0.1 millivolt of output.

Therefore, \( E \) must be not greater than 0.1 millivolt.

Solving for \( K_2 \):

\[
K_2 = \frac{HW_s \cos 89.97° \times \cos 40°}{K_3 \times E}
\]
The gyro gimbal having a wheel of $45 \times 10^6$ dyne-cm-sec and rotating at 12,000 rpm will have an inertia $I$

$$I = 55,000 \frac{\text{dyne-cm-sec}^2}{\text{rad.}^2}$$

Then $\omega_n = \sqrt{\frac{K_1 K_2 K_3}{I}} = \sqrt{\frac{100 \text{ volts}}{\text{rad.}}} \cdot \frac{130.66 \text{ mamp}}{100 \text{ dyne-cm}} \cdot \frac{100 \text{ dyne-cm}}{\text{volt}}$

$$= \sqrt{23.76 \text{ rad}^2/\text{sec}^2} = 4.88 \text{ rad/sec}$$

Assuming damping ratio ($\zeta$) = 1.00, the gyro damping coefficient ($C$) can then be computed from the equation:

$$\zeta = \frac{C \omega_n}{2(K_1K_2K_3)}$$

or:

$$C = \frac{2.0 (K_1K_2K_3)}{\omega_n} = \frac{2.0 \frac{100 \text{ v}}{\text{rad}} \times \frac{130.6 \text{ ma/\text{v}}}{\text{mamp}} \times \frac{100 \text{ dyne-cm}}{\text{mamp}}}{4.88 \text{ rad/sec}}$$

$$= 5.34 \times 10^5 \text{ dyne-cm/rad/sec}$$
Gyro Sensitivity

From Equation 1 the sensitivity of the gyro is derived as

\[
\frac{\theta}{W_{\text{input}}} = \frac{H}{K_1 K_2 K_3}
\]

Since \( E_o = \frac{\theta}{K_1} \)

\[
\frac{E_o}{W_{\text{input}}} = \frac{H}{K_2 K_3} = \frac{45 \times 10^6 \text{ dyne-cm-sec/rad}}{130.66 \frac{\text{ mamp}}{\text{volt}} \times 100 \frac{\text{ dyne-cm}}{\text{mamp}}}
\]

\[= 3.45 \text{ volts/milliradian/sec} \]

Listed below is a compilation of the gyro parameters based on the preceding computations.

Angular Momentum (H) = 45 \times 10^6 \text{ dyne-cm-sec}

Damping Coefficient (C) = 5.34 \times 10^5 \text{ dyne-cm/rad/sec}

Gimbal Inertia (I) = 55,000 \text{ gr-cm}^2

Signal Generator Sensitivity (K_1) = \frac{100 \text{ millivolts}}{\text{milliradian}}

Torque Generator Sensitivity (K_3) = 100 \frac{\text{dyne-cm}}{\text{mamp}}

Wheel speed = 12,000 \text{ rpm}

Sensitivity = 3.45 \text{ volts/milliradian/sec.}

Size = 7 \text{ inches diameter, 12 inches length} (estimated)

Weight = 25 \text{ lbs.}

Note: These are the parameters for one basic gyro compatible with specifications for the gyro aiming device. The parameters may be varied, and remain compatible, by altering C, K_1, K_2, K_3 within the bounds of the defining equations.

Although these parameters are compatible with the specifications, the parameters listed are not necessarily compatible with the present state of the art of gyro construction.
B. Servo Control System

(1) Servo System Configuration

Figure 5 is a block diagram of the servo control system that functions to hold the gyro platform (inner gimbal) at null position; i.e., gyro input axis East.

From the diagram it can be seen that when the gyro senses Earth's angular velocity, the gyro gimbal will rotate to an angular position proportional to this velocity and the gyro signal generator will provide an error signal \( E \) corresponding to this position. The gyro error signal is then amplified (the preamplifier would actually be located at the gyro so that signal transmission could be accomplished at a reasonably high level) and detected with respect to its phase. Phase sensitive detection is necessary in order to avoid East West ambiguity.

The passive network is included for phase margin compensation to provide a stable servo system. Power amplification to energize the control phase of a two phase servo motor is furnished by the power amplifier stage of the amplifier. The motor, through its gearing will drive the inner gimbal, and hence the gyro, until the system is at null condition. Figure 6 shows the proposed method of mechanizing the gyro and gimbaling system.

(2) Servo System Parameters

The inner gimbal, will present a static friction level of approximately 15 in-oz. to the system (experience derived from production of similar gimbal configurations indicates that this is a pessimistic figure allowing a considerable factor of safety).

The motor will rotate at a maximum no-load speed of 5000 RPM and a maximum speed of 2800 RPM at a maximum power output. F.C.P.D. No. 157 specifies that the aiming device must reach null within five minutes after the system is energized. Therefore, the inner gimbal must rotate at a speed of \( \frac{180}{5} \) minutes. A gear ratio of 2000:1 will be well within the velocity specifications of the system.

Using a gear ratio of 2000:1, the static friction torque reflected to the motor will be

\[
\frac{15 \text{ in-oz.}}{2000} = 0.0075 \text{ in-oz.}
\]

(It is noted here that the system parameters are being computed on a static or position basis since the accuracy of the device when subjected to velocities or accelerations is of consideration only with regard to system stability and not system accuracy.)

It is, therefore, necessary to supply approximately 1.0 volt to the servo motor control phase (with 115 volts supplied to the reference phase) to break the static friction. Since the error voltage

---
from the gyro is 0.1 millivolts at the specified azimuth accuracy of 0.03° at 40° Latitude, the amplifier will require an overall gain of \( \frac{1.0}{0.001} \) volts/volt or 10,000 Volts/volt to provide the necessary system sensitivity about the gyro null position.

The system will saturate when the gyro output is 11.5 millivolts (115 volts on the servo motor control phase) and will drive at maximum speed. Therefore, since the gyro sensitivity is 3.45 volts/mrad/sec., the linear region of the servo system will be approximately ± .35 degrees.

(3) Servo System Dynamics

To provide a system that will drive to its azimuth reference or null position with a minimum of settling time it is necessary to analyze the servo loop dynamics. A system which would drive to the null position and settle with no more than 2 to 3 overshoots would be satisfactory. This indicates that the damping ratio (\( \zeta \)) of the closed loop system should be approximately 0.5.

The dynamics of the individual portions of the loop are defined by the following equations:

\( a. \) Gyro

The characteristic equation of the gyro loop dynamics is:

\[
\frac{\theta_o}{\theta_{Earth}} = \frac{p}{p^2/W_n^2 + 2\zeta p/W_n + 1}
\]

where \( \theta_o \) is the angle of rotation of the gyro gimbal about its output axis

\( \theta_{Earth} \) is the integral of Earth's rate input

\( p \) is used to denote the function \( d/dt \)

\( W_n \) is the gyro undamped natural frequency

\( \zeta \) is the gyro damping ratio

\( b. \) Motor and inertia of gearing, inner gimbal, and gyro

The characteristic equation of the motor and reflected inertia is:

\[
\frac{\theta_{out}}{\theta_{in}} = \frac{i}{p(Tp + 1)} \quad \text{where} \quad T = 1/f
\]

where \( I \) is the reflected inertia of the gear train, inner gimbal, and gyro, measured at the motor shaft, and \( f \) is the viscous damping of the motor derived from the equation

\[
f = \frac{\text{Torque}}{\text{Speed}}
\]

Since the torque-speed curve of the servo motor is very nearly a straight line, this equation is correct for all practical purposes.
The dynamics of the open-cycle system are the product of the dynamics of the individual components in the loop.

\[
\frac{\theta_{\text{Earth}}}{\theta_{\text{Earth}}} \text{ (open cycle)} = \frac{1}{(T_p + 1) \left( \frac{p^2}{\omega_n^2} + 2 \zeta \frac{p}{\omega_n} + 1 \right)}
\]

A servo analysis (based on a linear system) of the system showed that with the necessary amplifier and gyro sensitivities (based on the required static accuracy) a system defined by this equation would be unstable.

c. Network Compensation

To provide a stable system it will be necessary to insert a passive compensating lead network of the form:

\[
\frac{E_{\text{out}}}{E_{\text{in}}} = \frac{1}{\frac{\zeta}{T_p} \frac{T_p}{1} + 1}
\]

where \(\zeta\) is the attenuation at zero frequency and will be approximately 20.0, and \(T\) is the time constant which is approximately 1 millisecond. This will result in a system which has a natural frequency of approximately 40 cycles/second and a damping ratio of approximately 0.4.

C. Bearing Indication

As shown in Figure 6, there will be a dual speed synchro transmission system to measure the angular displacement between the support gimbal and the inner gimbal; thereby measuring bearing angle. These synchro transmitters, operating at 1:1 and 36:1 ratios with the output angle will transmit bearing information to the weapon operator on dual speed synchro receivers. The 1:1 synchro will provide coarse data and the 36:1 synchos will provide the vernier, one revolution being equal to 10 degrees of bearing. The dial will be marked in one minute divisions.

If a meter indication is preferred, control transformer synchros may be utilized. In this case bearing angle will be preset and null indication of the meter will indicate proper alignment.

D. Azimuth Compensating Mount

As previously stated, the present azimuth compensating mount M18A1, with structural modifications, will be used for mounting the gyroscopic aiming device gimbals (figure 6).

It is estimated that the gimbaling system which includes the support gimbal, inner gimbal, gyro, servo motor, and synchros, will weigh 40 pounds. Assuming that the gimbaling system will be subjected to a maximum shock of 20g when the weapon is fired, the following modifications must be incorporated in the present compensating mount to insure proper operation when supporting the increased mass of the gimbaling system.
Figure 6. Mechanical Schematic - Gimbaling System, Gyroscopic Aiming Device
(1) **Housing (D6528636)**

The cross-sectional area at the point of attachment to the gun mount will have to be increased by a factor of approximately 50 percent and the depth of the channel will have to be increased by approximately 25 percent.

(2) **Cross Leveling Assembly**

The present cross-leveling mechanism will not withstand the shock of 20g with the added weight of the gyroscopic aiming device. It will be necessary to increase the diameter of the worm (A5034750) to approximately 2 inches, and increase the width of the segment gear (part of assembly B6177107) to approximately 7/8 inch. The entire segment assembly should be redesigned to allow for twice the present strength.

**E. Auxiliary Equipment**

The system will contain the following equipment:

1. Gimbaling system which supports the servo motor, synchros, gyro, gyro temperature control, and slip rings.
2. Amplifier assembly which will include the control switches, servo amplifier, and power supply for the amplifier.
3. Inverter. Inverter will operate from 28 v.d.c. and supply 400 cycle power to the system at a level of approximately 100 watts. 28 v.d.c. will also be supplied directly to the gyro heaters at a level of 200 watts.
4. Bearing indicator. Additional synchros and dials for the operator and a meter for azimuth indication if this method is preferred to the vernier dials.
IV. ACCURACIES UNDER FIELD CONDITIONS

A. Gyro

The values used in the computation of parameters for the gyro having an angular momentum of 45 x (10)^6 dyne-cm·sec/rad are necessarily based on data interpolated from gyros which are as yet being made only in prototype quantities. These gyros have been tested under laboratory conditions and results are based on conditions not compatible with field conditions, i.e. the gyro is allowed to stabilize with respect to temperature for a period of 6—8 hours, and the wheel is running continuously. The following estimates of accuracy under field conditions, based on specifications of F.C.P.D. No. 157, are therefore extrapolated from the laboratory gyro. The sources of uncertainty are listed below. Appropriate columns show the measured laboratory results and the estimated results under the specified field conditions.

<table>
<thead>
<tr>
<th>Uncertainties</th>
<th>Laboratory Conditions</th>
<th>Field Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cause</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Flex lead torque</td>
<td>0.7 dyne-cm</td>
<td>3.0 to 5.0 dyne-cm</td>
</tr>
<tr>
<td>2. Friction</td>
<td>0.2 dyne-cm</td>
<td>0.2 dyne-cm</td>
</tr>
<tr>
<td>3. Signal Generator Reaction Torque</td>
<td>0.1 dyne-cm</td>
<td>0.7 dyne-cm</td>
</tr>
<tr>
<td>4. Torque Generator Reaction Torque</td>
<td>0.1 dyne-cm</td>
<td>0.7 dyne-cm</td>
</tr>
<tr>
<td>5. Gyro Feedback Amplifier (K^2) Drift</td>
<td>0.1 dyne-cm</td>
<td>0.5 dyne-cm</td>
</tr>
<tr>
<td>6. Torque acting to rotate gyro about output axis due to mass unbalance along gyro input or spin reference axes</td>
<td>Negligible with gyro output axis vertical.</td>
<td></td>
</tr>
</tbody>
</table>

The increase in uncertainty level is due primarily to temperature cycling due to normal field use in turning the heaters on and off.

B. Servo Control System

Since the development, design, and manufacture of control systems, which closely duplicate the gyroscopic aiming device control system, are the primary activity of the Control Engineering Corp., an extensive source of information is available for assessing the accuracy of the system proposed in this report. The individual items of the control system and their estimated sources of inaccuracy are outlined below:

Amplifier

Gain Stability — As used in a feedback control system, the errors resulting from expected gain deviations are negligible.

Unbalance — By using A.C. amplification techniques, particularly in the high gain preamplifier stage, unbalance effect can be expected to cause not more than 0.5 minutes of error at the output shaft.
Gimbalng System —

Gearing — Total backlash throughout the specified life of the equipment will not exceed 0.3 minutes of arc.

Friction — With the use of a high gear ratio of 2000:1, friction torques reflected to the motor should be negligible even when subjected to extreme low temperatures that might be encountered in field use.

C. Indication System

Synchro errors may be expected to be 0.5 degree at the single speed shaft which would result in an error of approximately 0.6 minutes of arc at the bearing vernier indication. Since 36 speeed is merely a convenient ratio (1 revolution = 10 degrees) the vernier may be increased to where the indication error is negligible.
V. CONCLUSIONS

Based on the summation of expected performance of the various components of the gyroscopic aiming device, namely the 5 to 7 dyne-cm uncertainty level of the gyro under field conditions and the 1 to 2 minutes of arc error of the servo system, a gyroscopic aiming device capable of indicating a reliable azimuth reference to an accuracy of 0.03 degrees at 40° latitude is not feasible. It appears, however, that a reasonable target for a development program for the gyroscopic aiming device would be an accuracy in the order of 0.15 to 0.2 degree.

VI. RECAPITULATION

The scope of the feasibility study was necessarily limited to investigation of gyros which were presently either in production or in the prototype stage.

Limitations imposed on the system accuracy by the gyro were primarily due to 1) the summation of uncertainty torques limiting the reliable accuracy of the gyro and, 2) the limitations of present electromagnetic transducers with regard to signal to noise ratio.

These discrepancies are by no means insurmountable and it is the opinion of the gyro designers that a development program would result in a gyroscopic aiming device which would have an accuracy compatible with the gun laying problems of field artillery.

The scope of the feasibility study did not permit an additional important phase of the investigation which would have necessitated construction of the system. This investigation would approach the problem from the opposite viewpoint; namely, accepting the component inaccuracies but providing field methods of determining the deviations and thereby providing a means of calibration.

It is probable that uncertainties in the gyro could be determined by allowing the system to settle with gyro input axis East, then reversing the direction of rotation of the wheel (or rotating the entire gyro 180 degrees about its spin axis). This would change the phase of the system and the gyro input axis would be in the West direction. Accurate azimuth reference might then be determined by averaging the East and West errors. It would seem that this might be the most feasible approach to a system utilizing a gyro of reasonable size and still conforming to the specifications of the gyroscopic aiming device.
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