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UNCLASSIFIED
THE RADIO DIRECTION FINDING
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SUMMARY TECHNICAL REPORT
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ABSTRACT

The various phases of the research program on Radio Direction Finding at the University of Illinois are outlined.

The first part of the program, now essentially completed, consisted of a thorough survey of the literature on radio direction finding. This part resulted in Technical Reports Nos. 1, 2 and 3, which are bibliographies and abstracts of articles and reports on all aspects of the radio direction finding problem. As a result of this survey several general conclusions have been drawn.

1. A large proportion of the effort already expended on direction finding has been in developing and improving conventional systems. The probability of radical improvements resulting from further work along these same lines is not too great.

2. This does not exclude the very good possibility that greatly improved systems may result from the application of new electronic techniques, which make feasible certain schemes once considered impractical.

3. It is noted that, with the exception of techniques like dotlock, which make use of the element of time, most present-day systems extract no more actual information from the incoming signal than did the original rotatable loop system.

The second part of the research program, now in progress, consists of several theoretical and experimental projects, the need for which was indicated by the survey. Under actual operating conditions where several waves arrive at the receiver simultaneously (due to scattering and multipath transmission phenomena) the conventional direction-finder
is incapable of giving a correct bearing, because it does not obtain sufficient information to solve for all the unknowns. A wave interference study is in progress to determine what information is available, and how much information is necessary, in order to resolve the component waves. The experimental part of this program is concerned with the effects of the pick-up antennas on the interference pattern. Preliminary results indicate that wide-aperture antenna systems (extending over several wavelengths) are necessary for accurate separation of the waves. One such wide-aperture system is being investigated. Detailed analysis has also been made of information obtainable from ordinary crossed-Adcock (small aperture) systems. It is concluded that conventional small aperture systems do not utilize all the information available. It is shown that with suitable connections to the antenna elements, it is theoretically possible to obtain sufficient information from a small aperture system to separate out two waves. Work continues to determine how practical this method might be. To facilitate this latter work an antenna simulator is being built.

Because the "super-gain" antenna array is one type of small aperture system, theoretically capable of sufficient directivity to resolve the separate waves, such arrays have been investigated. It is shown that due to their extremely low effective radiation resistance such arrays are not practical for direction-finder use.

Where rapidly the operation of different types of direction-finder systems under conditions where more than one wave is present, the D-F System Analyzer has been developed. By feeding in voltages corresponding to the arrival of one or several waves this device gives direct answers as to the relative accuracy of different systems under such conditions.

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The effects of changing antenna aperture, number of antenna elements, method of detection, etc., can be explored rapidly and thoroughly, so that time-consuming analyses or costly experimental-installations are avoided.

One of the "new" d-f systems being investigated is the Doppler-effect system. This shows promise as a wide-aperture system. Whether it has definite advantages over other wide-aperture systems is as yet undetermined. Its possibilities are to be explored with the D-F System Analyzer.

A preliminary study of an instantaneous, wide-aperture system is being started. In order to adequately display the information received by such systems, an eight-beam cathode-ray tube presentation system is under construction.
II
INTRODUCTION

E. C. Jordan

The purpose of this report is to summarize briefly some of the objectives and present activities of the radio direction-finding research program at the University of Illinois. The research program was undertaken to investigate the basic problems of direction-finding, with the ultimate objective of evolving a direction-finder system which would approach more closely to the ideal. The "ideal" direction-finding system is considered to be one capable of giving an accurate, instantaneous bearing on a signal of any frequency coming from any direction.

In order to acquire the necessary background, and to avoid repeating work already done elsewhere, a thorough survey was made of the literature on direction-finding. The great extent of this literature (some 4,000 papers and reports have been recorded) emphasizes the fact that many capable individuals and organizations have already worked on the problems of direction finding. However, the major portion of this effort has been applied to the development and improvement of what might be called conventional systems. Because of this, it is felt that the probability of radical improvements arising from further work along these lines is small. The best possibilities for major advances seem to lie in applying new electronic techniques to the problems, or in determining and overcoming certain fundamental failings common to all conventional systems. In this connection it is noted that most systems, however complicated they may be, extract no more information from the electromagnetic field than do the simple crossed-loop or crossed-Adcock systems.

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The several theoretical and experimental projects now being worked on are consistent with the above conclusions. In actual operation, because of scattering, reradiation and multipath propagation phenomena, there are usually several waves arriving at the collector system. Under these conditions the conventional radio direction finding system is incapable of giving the correct bearing because it does not obtain enough information to solve for all the unknowns. The problem then is to determine how much information is necessary, the type of antenna system required to extract it, and the best methods of operating on and presenting the information obtained. The problem divides naturally into two parts: the first is an electromagnetic field and antenna problem; the second is a circuit and systems problem.

The wave interference study is concerned with the problems of how much information is required to separate the several waves, how much can be obtained, and what is the optimum configuration of probes to obtain this information. The results obtained so far indicate that in the presence of more than one wave wide-aperture systems (that is, antenna systems large in wavelengths) are necessary for an accurate indication of direction of arrival. For this reason the major emphasis is being placed upon such systems. However, because in many applications wide-aperture systems are entirely out of the question, the possibility of obtaining sufficient information from small aperture systems is also being investigated. In this connection the analysis of conventional systems has shown that, if the proper steps are taken, additional information can be obtained from small aperture systems, sufficient (in theory at least) to separate out two waves and indicate their respective directions of arrival. Although the method may prove impractical in operation it is to be tried out in a laboratory experimental set-up.
One type of small aperture system which is known to be capable of resolving waves from different directions is a super-gain antenna array. With such arrays it is possible, theoretically, to obtain arbitrarily sharp directivity with an array of given length. A study has been made of the directivity and impedance characteristics of several of these arrays. It has been found that as soon as an attempt is made to obtain high-gain with arrays of small aperture, the effective radiation resistance of the array decreases rapidly to the point where it becomes impracticable. In a recent paper*, Chu has considered this problem much more generally and has shown that the bandwidth of such systems decreases rapidly to zero as the aperture decreases.

After collecting the information with a large or small aperture antenna system it is necessary to operate on it in order to put the information in a form suitable for presentation. There are a great many different ways, both tried and untried, for doing this, and the various methods give rise to different systems of direction finding. Although a separate analysis could be carried through on each system for each of the many combinations of arriving waves, such analyses are laborious and time-consuming, and the comparison of different systems is difficult. On the other hand the purely experimental approach in which each system complete with antennas is set up and tried out in the field is costly and subject to the difficulties of unknown or uncontrollable variables. To avoid the difficulties of both of these approaches, the direction finding system analyzer was devised. In this device the voltages which would exist at the terminals of the antenna system in the presence of one or more waves are calculated, and these voltages,

obtained from an oscillator through appropriate attenuation and phase-shift networks, are fed to the system (or a suitable model of it) under test. This arrangement makes possible the rapid exploration of the effect of changes in any of the many variables, such as antenna aperture, direction, magnitude and phase of arriving waves, different methods of detection and of presentation. In its initial application the system analyzer will be used to explore the operation of the Doppler Effect method of direction finding, upon which a considerable amount of analytical work has already been done by the group. In particular, the performance of the Doppler Effect method under conditions of more than one wave arriving is to be investigated, and compared with that of a common crossed-Adcock system.
The first task undertaken by the Direction Finding Research Group was a literature survey, intended to acquaint the members of the group with the radio direction finding problem and incidentally to gather together a ready reference of all published material. The first published result of this survey was Technical Report No. 1, *Bibliography of Published Articles on Radio Direction Finding*, dated June 1, 1947. This report contained a comprehensive listing of radio direction finding articles published in this country and throughout the world from the early beginning of direction finding to the present.

Technical Report No. 2, *Abstracts of Published Articles on Radio Direction Finding*, dated September 1, 1947, contained abstracts of about 450 of the articles listed in the bibliography; only the articles considered most important and relevant to the group's work were abstracted. This report of abstracts was prepared initially for the group's use, but it was decided that the results of the survey should be made available to others working in the field.

Inasmuch as the wartime work in the field of radio direction finding was generally unavailable in the literature, and because much of the work will never be published, a thorough search of research files of the government and commercial concerns was undertaken in order to list as completely as possible all written reports. This search culminated in the group's Technical Report No. 3, a bibliography of reports on radio direction finding, dated February 1, 1948.
In order to make complete the literature survey, a **Bibliography and Abstracts of Patents on Radio Direction Finding** is being prepared and will be published in the near future. The publication of this report will complete the work on this phase of the group's program, but it is intended that the group will keep an up-to-date listing of all patents and papers written in the field. When the number so warrants, supplements to the bibliographies will be published.
IV

DIRECTION FINDING ON SIGNALS CONSISTING OF MORE THAN ONE RAY

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In a simple linearly-polarized plane wave, the wave-fronts, or equiphase surfaces, are perpendicular to the direction of propagation. Most of the direction finding systems in use at present are designed to determine the direction of arrival of such a wave; in most cases the wave should be vertically polarized. The direction finder usually locates, either directly or indirectly, an equiphase surface. Then the direction of arrival is assumed to be perpendicular to this surface. If the arriving wave departs from the ideal condition, the measured bearings are subject to error, the amount and type of error depending on the particular system. Such departures from the ideal condition may, for example, take the form of; 1) arrival of the wave from an appreciable vertical angle, 2) rotation or ellipticity of polarization, 3) multipath propagation, or 4) combinations of these.

In this study we are particularly interested in the multipath phenomena. There are two aspects to the problem; 1) determination of the directions of arrival of the rays at the direction finder location (the resolution of the total field at the direction finder into component rays) and 2) location of the transmitter emitting the signal, taking into account the irregular paths traveled by the rays. For the present we are concerned with the first portion of the problem.

When a signal arrives at the receiving location over a single path it usually approximates a plane wave. If, however, the same signal
is received over two paths having different directions of approach to the receiving site, the equiphase surfaces of the combined field are in general neither plane nor perpendicular to the direction of arrival of either ray. The presence of more than two rays of course further complicates the picture.

The basic problem in unscrambling the rays is to discover, for a given situation, what information is available in the field, what portions of this information are needed to determine the direction of arrival of the rays, and how to obtain this information by actual measurements. The situation illustrated in Plate 1 will serve as an example. This situation is comparable to that which exists when two ground-wave rays arrive at a receiving location. If the two paths are stable (no fading) the combination of the two rays produces a standing wave pattern. The configuration of the pattern depends on the directions of arrival of the rays, their relative amplitudes, their polarizations, and their phase difference.

In Figure 1 of Plate 1, the rays are separated by an angle, \( 2\alpha \), of 30°. The x and y axes are chosen so that the x axis bisects the angle between the rays and so that both rays have the same phase at the origin. The electric vector is assumed to be perpendicular to the page. The loci of the maxima and minima of the standing wave pattern are shown. Their locations are given by the following equations:

Maxima: \( y = \frac{n \lambda}{2 \sin \alpha} \)

Minima: \( y = \frac{(2n+1) \lambda}{4 \sin \alpha} \)

Note that the locations of the maxima and minima depend only on \( \lambda \) and \( \alpha \); they are independent of the strengths of the rays. Also plotted in Figure 1 are the equiphase surfaces for three values of \( K \) (\( K \) is defined to be the...
ratio of the strength of ray #2 to the strength of ray #1. The equiphase surfaces shown in this plate have the same phase as that of the two rays at the origin. These surfaces may be plotted from the following expression:

\[ y = \frac{\lambda}{2 \pi \sin \alpha} \tan \left[ \frac{1}{2k} \tan \left( \frac{2 \pi x}{\lambda} \right) \right] - \frac{n \lambda}{\sin \alpha} \quad n = 0, 1, 2, \ldots \]

Figure 2 of Plate 1 is similar to figure 1, except that \( \alpha = 22.5^\circ \).

There are several distinctive features about the field patterns illustrated in this plate: 1) the loci of the minima (or maxima) in the standing wave pattern are parallel to the bisector of the angle between the rays; this is true regardless of the size of the angle and the relative amplitudes or phases of the rays; 2) the distance between adjacent minima (or maxima), measured in a direction perpendicular to the bisector of the angle between the rays, depends on the wavelength of the signal in space and on the angle between the rays; this distance is equal to \( \lambda / (2 \sin \alpha) \), and varies from \( \lambda/2 \) when the rays are separated by an angle of 180° toward infinity when the rays both arrive from the same direction; 3) "on the average", the equiphase surfaces are perpendicular to the stronger ray; thus the direction of the stronger ray could be approximated by use of an equiphase-surface-locating device having a wide enough aperture so that it would give the "average" direction of the surface, rather than the direction at a particular point.

The directions of both rays can be calculated from a knowledge of the wavelength of the signal, the distance between adjacent minima, and the fact that the minima are parallel to the bisector of the angle between the rays. This is not the only set of information that can be used but is one possibility. There of course remains to be solved the problem of obtaining and presenting the information quickly, accurately, and conveniently.
Most direction finders do not obtain, or at least do not use, enough information from the field to permit unscrambling the rays. A large part of our problem is to discover how much and what information must be obtained from the field in order to accomplish this end. Thus, of course, means of obtaining this information by actual measurement must be worked out.

One very important aspect of this problem is the question of the minimum possible size of a system which will obtain sufficient, and sufficiently accurate, information to separate the rays. In other words, over how great an area must measurements be made to get the necessary data? What is the smallest aperture that can be used?

Microwave systems can distinguish between two rays if they are separated by a large enough angle; the size of the angle required depends on the beam-width of the antenna array. However, this technique becomes impractical, or at least difficult and costly, at lower frequencies because of the large size of the antenna system required to give sufficient directivity. It would be very helpful to know whether it is possible to build a system which does not require as much space as the type just mentioned, and if so, the size to which the system can be reduced.

Work on super-gain antenna arrays discussed elsewhere in this report indicates that it is theoretically possible to get any desired directivity from an antenna array as small as one wishes. This seems to indicate that it is possible, theoretically at least, to build a system of high resolving power in a very small space. However, practical considerations place a lower limit on the size of such a system. Thus, while it seems to be theoretically possible to build a system that is as small as desired, practical reasons (sensitivity, bandwidth, limitations on the accuracy with which quantities can be measured, etc.) seem to indicate that a fairly large
aperture may be necessary. One of the problems to be solved then, is that of determining (in general if possible, but at least for certain specific systems) just how large the aperture needs to be, considering both the theoretical and practical aspects of the situation.

In order to confirm theoretical conclusions and to test the feasibility of various systems of obtaining information from the electromagnetic field, an experimental measuring system is being used.

One of the difficulties encountered in putting a proposed system into practice is that of the effect of the antenna or collector system on the field. Current flowing in one portion of an antenna system affects the field seen by other parts of the system. Some idea of the magnitude of such effects can be obtained by calculation, but the final proof must come from actual operation and testing of the system.

Liberal use is being made of the modeling techniques developed in recent years. A block diagram of the equipment is shown in Plate 2. At present a frequency of about 3000 mcs is being used. If only one horn is excited a simple plane wave can be obtained at a sufficiently large distance from the horn. If both horns are excited, an interference field is obtained. The relative amplitudes, phases, polarizations, and directions of arrival of the two rays can be controlled. This type of setup permits making measurements on fields of known configuration, as well as providing a method of obtaining a wide variety of configurations.
The Direction Finding System Analyzer is a device designed to facilitate the rapid investigation and comparison of different types of direction-finding systems, especially under conditions of operation where more than one wave is present (e.g., reradiation, multi-path transmission, etc.). With the analyzer the electromagnetic field and antenna system are replaced by a set of terminals equal in number to the antenna terminals. To these terminals are fed voltages of appropriate magnitude and phase to represent the arrival of one or several waves at the antenna system. Connections are made from these terminals to the direction finding system (or a suitable model of it) under test.

This method possesses several very real advantages over both the straight analytical and the straight experimental approaches. The analytical approach can be carried through for certain idealized cases but the analyses are involved and time-consuming so that a complete exploration of all the variables is not possible. The purely experimental approach in which the system (complete with antennas) is set up in the field is also a long job, and for the larger systems, a costly one as well. In addition, there are unknown variables, such as reradiation and stray reflections, over which the experimenter has little or no control. This is a serious disadvantage when the objective is the comparison of systems under prescribed conditions. Because of the rapidity with which measurements can be made under controlled conditions it is believed that the Analyzer will prove a powerful tool in
appraising the value of new systems and comparing them with conventional systems.

In the Direction Finding System Analyzer the voltages at the antenna terminals of any arbitrary direction finding system are simulated by generating appropriate voltages to correspond to any number of arriving waves and any number of pickup points, and mixing them in a suitable manner. These resultant voltages are then available for introduction into any radio direction finding indicating system for presentation. Provisions have been made for introducing the voltages into the DAK direction finding system for presentation in the "propeller pattern"; also, an electronic switching circuit has been constructed for analyzing the data obtained in a Doppler Effect direction finding system. The conceivable methods for analyzing the data are many and are not limited by the presentation methods herein described.

In applying the Analyzer to the analysis of a single electromagnetic wave, it is necessary only to compute the phase of the voltages at the various pickup points. This is a simple set of calculations. This information is then introduced into the Analyzer by setting a calibrated dial to the required phase and another dial to the required amplitude. By making settings corresponding to many directions-of-arrival of the arriving waves, and further settings corresponding to many time phases of the arriving waves, complete data on the effects of more than one signal at the collector system of the direction finding may be gathered. This will show the effects of known changes in the arriving waves, changes which in experimental measurements are sometimes difficult to obtain and to isolate from undesired changes. It is expected that the time required for analysis of any given direction finding system will be reduced by a large factor.
The Analyzer as being built operates at 175 kc., where it is relatively a simple matter to obtain known phase shifts. By scaling down to this frequency, the Analyzer in effect is a model on which measurements are made at frequencies with which it is easy to work. In a Doppler radio direction finding system, for example, the switching (or rotating) rate of the antenna elements would be scaled down by the same factor as the carrier frequency in order to retain in the Analyzer the same ratio between these frequencies.

By introducing suitable dial settings, it will be possible to investigate quantitatively the effect of collector aperture on error. Present evidence seems to indicate that collector arrays large in wavelength are better able to take accurate bearings than those of small aperture; the Analyzer offers an easy way to investigate this variable.

In Plate 4 there is shown pictorially the phase difference of received voltages at the terminals of a 4-element antenna array in the field of an e.m. wave; the charts beneath the figure give the phase of the impinging wave for different angles-of-arrival and different apertures, where phases are referred to the center of the array.

A rear view of the Analyzer is shown in Plate 3. Plates 5, 6, 7, and 8 are block diagrams of the Direction Finding System Analyzer as being constructed. In Plate 5 is shown the 175 kc. signal generators, which originate the signal operated on by the phase shifters. Provisions have been made to use either a crystal-controlled frequency source, necessary for Doppler measurements, or a variable frequency oscillator, which is convenient if the simulated voltages are to be applied to the indicator of an existing direction finder. The DAK direction finding system has an i.f. frequency of 175 kc; so it will be possible to tune the v.f.o. to exact resonance with
The phase shifters and amplitude controls immediately following the oscillators give the operator control over the apparent time-phase of the arriving signals. If the arriving waves take different paths in arriving at the collector, in general there will be an arbitrary phase difference between them corresponding to the difference in path length. There are shown two branches, with outputs A and B; these provide two separate channels simulating the arrival of two signals from the same source. There is no reason why there could not be an indefinite number of branches simulating the arrival of a like number of signals; but it was considered sufficient to provide for the simulation of two arriving waves, at least for the first model of the Analyzer. The mechanical construction and electrical connections are such that additional channels may be added as desired.

The outputs A and B are fed into two identical channels, shown in Plate 6. These phase shifters, labeled Space Phase Shifters, allow the operator to adjust individually the phases of the voltages corresponding to eight separate pick-up points receiving simultaneously two waves. For an Adcock system, only four of the available pick-up points will be used; for the Doppler system, however, all eight will be used, and it is likely that eight additional points will be provided. The bank of phase shifters fed by channel A are identical to those for channel B. The outputs of the individual phase shifters are then combined in pairs (one signal from A and one from B) and the resultant outputs supplied to terminals labeled D₁, D₂, ..., D₈. At these terminals there appear the voltages which simulate the voltages appearing at eight antennas in an e.m. field consisting of two arriving signals of arbitrary direction, phase, and amplitude, with arbitrary location.
of the antenna elements.

Plate 7 shows the block diagram of the circuits used for applying the voltages to the DAK direction finding system. The inputs $D_1$, $D_2$, $D_3$, and $D_4$ coming from the phase shifters in Plate 6 are combined differentially to give a pair of output voltages equivalent to the voltages appearing at the output terminals of the crossed-loops of the DAK. This pair of equivalent voltages is then fed into the goniometer of the DAK and indication accomplished in the conventional manner. Provisions are made to provide by means of a switch any of three pairs of voltage to the indicator; Position 1 of the switch, shown in Plate 7, allows the operator to see the bearing of one of the arriving signals, say, the desired or main signal; Position 2 allows the operator to display the combination or apparent bearing caused by the presence of two arriving signals; and Position 3 provides for displaying the bearings of the undesired or interfering signal.

Display of the Doppler direction finding system, described elsewhere in this report, is shown in block form in Plate 8. The outputs $D_1$, $D_2$, ..., $D_8$ are applied to the grids of mixer tubes which are gated by a suitable ring counter, and the output is taken off of the common plate impedance of the mixer tubes. The ring counter is fired cyclically by an 8000 cycle audio voltage, peaked suitably for firing the gas tubes of the counter. Thus, voltages corresponding to the eight individual pick-up points are channeled sequentially, with a repetition rate of 1000 cycles, into a common channel; each of the eight signals is gated for only one-eighth of the repetition interval inasmuch as the eight share equally in time the repetition interval. After being clipped to remove switching transients, this complex waveform is fed into a phase detector, described elsewhere in this report.

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The output of the phase detector is an audio wave with a fundamental frequency equal to the repetition frequency (1000 cycles) of the ring counter. In general, this wave will be a distorted sine wave; so it is applied to a 1000 cycle band-pass filter, the output of which is then a pure 1000 cycle sine wave. This sine wave is peaked and used to drive a delay multivibrator whose delay is adjustable over about 1-1/4 of the repetition period. The output of the multivibrator is differentiated and clipped, and the resultant voltage is a positive pulse of short duration, say, 10 microseconds. This pulse is used to intensify a cathode-ray tube, which normally is adjusted so that the circular sweep is black.

Four of the pulses from the ring counter output are mixed together to form one square wave of a fundamental frequency of 1000 cycles; this square wave is applied to a 1000 ke. band-pass filter to extract the fundamental component. The pure 1000 cycle wave is put through a 90° phase shifter; there then is available a pair of 1000 cycle quadrature voltages which are used to generate a circular sweep on the cathode-ray tube mentioned above. The screen of the cathode-ray tube is black except when the tube is intensified by the pulse generated as explained above. The position of the spot on the tube is an indication of the phase difference between the 1000 cycle wave from the phase detector and the 1000 cycle voltage which generates the sweep. This phase difference is a direct measure of the angle-of-arrival of the e.m. wave at the antenna collector. The delay multivibrator previously mentioned allows the spot to be zeroed to correspond to zero direction-of-arrival. A movable cursor will allow the apparent direction-of-arrival to be read off directly in degrees.

In applying the Analyzer to the analysis of a given direction...
finding system, a chart of phase shifter dial settings will be required for setting up the system to simulate the desired antenna configuration and arriving wave directions. However, as indicated before, the calculations are quite simple; calculations for any direction-of-arrival and any antenna aperture may be made rapidly and will serve as permanent data for setting up the Analyzer. Compared to a possible time of, say, weeks for obtaining experimental or analytical data from an actual direction finding system, the Analyzer should reduce the amount of time and work required for a given analysis by possibly a factor of 50. And further, any proposed direction finding system may be set up simply by setting dials; no costly construction of a complete direction finding system should be necessary. The Analyzer is potentially a valuable instrument for the study of direction finding errors.
In order to display adequately the large amount of information that it is necessary to extract from the various systems to be investigated, an eight-gun cathode-ray presentation system is being constructed. The deflection amplifiers have been designed to have a band width of about two megacycles. Because of the size of the tube and the many terminals around the periphery at the base (see Plate 9), the amplifiers are being constructed in a novel manner: the amplifiers for each of the eight sets of deflection plates and grids are identical, electrically and mechanically, and are built in a turret arrangement around the neck of the tube.

It is expected that this multi-gun tube will be of considerable use in an investigation of instantaneous presentation methods because the eight-guns will allow the simultaneous comparison of the phases of eight or more voltages.
To determine direction of arrival of a radio wave the Doppler direction finder makes use of the change in received frequency which occurs when an antenna moves in some prescribed path. When the antenna moves toward the transmitting source, the received frequency increases, and it decreases when it moves away from the transmitting source; the change in frequency is \( \Delta f = \frac{v}{c} \), where \( v \) is the component of velocity of the antenna in the direction of propagation of the wave, \( f \) is the carrier frequency, and \( c \) is the velocity of light.

For a single vertical antenna rotating about a vertical axis, the received wave has the form,

\[
C = A \cos \left[ \omega_c t + \frac{\pi d}{\lambda} \cos (\Theta - \Omega_{\omega} t) \right]
\]

as illustrated in Plate 10, Figure 1, where

- \( \omega_c \) = angular velocity of the received wave.
- \( \lambda \) = wavelength of the same wave.
- \( \pi d \) = distance between antenna and the axis of rotation expressed in radians.
- \( \Omega_{\omega} \) = angular velocity of the rotating antenna.
- \( \Theta \) = angle of arrival of the radio wave.

With a mechanically rotated element the frequency deviation of the received wave is too small at any practical rotational frequency and value of \( d \). Therefore a set of fixed antennas with high-speed electronic switching from antenna to antenna must be considered.
The case of a linear array of fixed antennas commutated in succession was considered (see Plate 10, Figure 2). The resulting magnitude of Doppler frequency change is an indication of the angle-of-arrival of the wave. Because the frequency change is the same for waves arriving at the same angle to, but on opposite sides of the axis of the array, there is an ambiguity of the bearing. This ambiguity can be eliminated by using another line of antenna perpendicular to the first.

An alternative arrangement is a circular array of fixed antennas (Plate 11). The abrupt switching from one antenna to another presents problems in detection inasmuch as discrete jumps in phase cause accompanying infinite frequency changes. To alleviate this, a law of coupling to the antennas was studied. Though it seems impossible in the light of recent work to find a law that will not depend on the received frequency and aperture of the system, a linear law of coupling was found to give a close approximation to the sinusoidal variation of phase and frequency obtained in the case of the single rotating antenna.

Law of Coupling

Consider two elements in a circular array of fixed antennas. Call one element the "nth" element and the next, the "(n + l)th" element. It is desired to find the law of coupling as switching is completed between the two antennas such that the voltage output will have a sinusoidal variation of phase. Let the voltage received by the "nth" antenna be:

\[ \cos(\omega t + \frac{2\pi}{\lambda} \cos(\Theta - \frac{2\pi}{\lambda}k)) \]

and by the "(n + l)th" antenna be:

\[ \cos(\omega t + \frac{2\pi}{\lambda} \cos(\Theta - \frac{2\pi}{\lambda}k)) \]

where the notation is the same as for Plate 12, Figure 1. The addition of these two voltages, with the appropriate law of coupling, would be required to take the form:

\[ A \cos(\omega t + \frac{2\pi}{\lambda} \cos(\Theta - 2\pi at)) \]
That is,
\[ G(t) \cos \left( \frac{2\pi t}{\lambda} \cos \left[ \theta - \frac{2\pi (n-1)}{k} \right] \right) + H(t) \cos \left( \frac{2\pi t}{\lambda} \cos \left[ \theta - \frac{2\pi n}{k} \right] \right) = A \cos \left( \frac{2\pi t}{\lambda} \cos (\theta - \frac{2\pi n}{k}) \right) \]

Analyzing \( G(t) \) and \( H(t) \), using calculus of variations, we arrive at the conclusion that there can be no general law of coupling to give the desired phase variation which does not specify a change in the angle \( \frac{2\pi}{k} \) between antennas which was assumed fixed. However, linear coupling, provided there are not less than eight antennas in the array, gives a close approximation to a sinusoidal variation.

**Interference Phase Pattern**

A detailed analysis of the effects of an interfering wave on the envelope of desired phase variations is to be presented in a later report. The results for a single rotating antenna or its electronic equivalent can be stated briefly.

Designating the phase variation (see Plate 12, Figure 2) by \( \psi \), it is found to be composed of an original desired function and a series of error functions, taking the form:

\[ \psi = \omega t + \phi \cos \omega_1 t + \lambda \sin \phi - \frac{\phi^2}{2} \sin 2 \phi + \frac{\phi^3}{3} \sin 3 \phi + \text{etc.} \]

where \( \lambda = \frac{e_2}{e_1} \) is the ratio of amplitudes of the undesired and desired signals

\[ \phi = \frac{2\pi r}{\lambda} \]

\( \omega_1 = \) rotational angular velocity

\[ \phi = \phi \left[ \cos (\omega_1 t - \theta) - \cos (\omega_2 t) \right] + \delta \]

\( \theta = \) angle between line of arrival of \( e_1 \) and \( e_2 \)

and \( \delta = \) random phase between \( e_2 \) and \( e_1 \)
If $\alpha = 0$, then $\psi = \omega t + \beta \cos \omega t$ as would be expected. If $\Theta$ is small, the error, designated $\Delta e$, depends only upon $\gamma$ and $\alpha$.

$$\Delta e = \Delta \sin \gamma - \frac{\Delta^2}{2} \sin 2\gamma + \text{etc.}$$

Curves showing the effect of varying the quantities that characterize the interfering signal have been plotted, and a cross section of these curves has been included in this report. Plate 21 gives a plot of the error function,

$$\Delta e = \Delta \sin \gamma - \frac{\Delta^2}{2} \sin 2\gamma + \frac{\Delta^3}{3} \sin 3\gamma - \text{etc.}$$

as $\gamma$ varies from 0° to 360°. It shows the maximum phase error (not to be confused with bearing error) possible with different values of $\Delta$, and provides a working curve used in plotting the remaining curves. Plates 22 through 25 give the actual phase variation of the signal induced in the rotating antenna, plotted against position around the circle. The parameters $\Theta$ and $\gamma$ and $\beta$, are chosen to show clearly the effect of different size apertures on the phase variation. In each case the phase variation in absence of the interfering signal is shown by the dotted curve. It will be observed that for an interfering wave having an amplitude equal to one-half that of the desired wave ($\Delta = 0.5$) the phase error can be as much as 30°. For the narrow aperture system, that is for $\Theta = 45°$ (radius $= \lambda/8$), the maximum phase deviation is only 90° so a phase error of 30° can and will produce a fairly large bearing error as shown in plates 22 through 25 (curves C). For a wide aperture system, say radius $= 5\lambda$, the maximum phase error remains at 30° for $\Delta = 0.5$ but the maximum deviation is now 180° so the effect of the interference (that is, the error) is negligible in this case. This discrimination against the undesired wave is exactly analogous to the discrimination against an interfering signal which occurs in wide-
frequency modulation systems. It should be noted that the improvement is even better than is indicated by the ratio of phase error to maximum phase deviation, because the phase error shows up as a high-frequency ripple superimposed upon the sinusoidal variation, and this ripple can be filtered out after detection. Plate 26 shows how the frequency of $\Delta e$ increases with aperture, but its amplitude remains constant.

Equipment is under construction to verify these results experimentally, and to explore the capabilities of the system in operation.

Experimental

Described elsewhere in this report is equipment used for antenna simulation by electronic means. With the aid of this equipment, it is proposed to simulate a fixed-antenna, Doppler type direction finder, making possible a test of the theoretical conclusions obtained.

Large aperture systems, which reduce site error, are of main interest, and this experimental method seems to offer a means for fast and accurate verification. The reception and detection of this phase-modulated wave will give a chance to study experimentally different types of phase discrimination and methods of presentation.

Bibliography

3. Patents by Paul G. Hansol
   a.) Dealing with a method of increasing the electrical length between two antennas without increasing the physical spacing.
   b.) Direction Finder of the Doppler type utilizing revolving vertical antennas and direct reading phasemeter. The possibility of using a circular array of fixed antennas to replace the one rotating antenna is suggested.
One of the immediate results of the literature survey was the realization of the existence of a large number of radio direction finder systems. Although no consistent classification of these systems has been found, some of the more commonly used radio direction finders are the following:

(a) Automatic bearing indicator system  
(b) Manually rotated loop system  
(c) Manually rotated goniometer system  
(d) Elevated-H system  
(e) Watson-Watt or Twin-Channel, Cathode-Ray Indicator system  
(f) Spaced-loop system  
(g) Null-seeking system  
(h) Visual Matched-line system

Under the ideal conditions of a single, vertically polarized wave arriving at the collector of the radio direction finder, each of the above types will give the true bearing of the wave. Under actual conditions, where multipath transmission may result in more than one wave component arriving with random polarization at the antenna system, each of the above systems is subject to bearing error and blur. The merits and demerits of each system are known and have been covered in the literature; however, in this connection it was noted that very little analytical study had been made of the behavior of the above systems under conditions of more than one wave arriving at the antenna. It seemed in order, therefore, to investigate the behavior of existing radio direction finders under this condition.

A mathematical analysis is being made of the class of radio direc-
tion finders which functions by virtue of the rotation of the antenna pattern. This rotation may be affected either by the mechanical rotation of a loop, Adcock, crossed loop, crossed Adcock, or crossed-H antenna system respectively, or by the mechanical rotation of the search coil of a goniometer suitably connected to a crossed antenna system of one of the above systems. The automatic bearing indicator type of radio direction finder is of this class.

The initial analysis is based upon the idealized condition of radio direction finding, i.e., a single, vertically polarized, wave arriving at the antenna system.

Since, for this class of radio direction finder, the electrical output of the antenna system or goniometer is eventually demodulated before extracting the directional information, the results of the analysis are presented in terms of amplitude, phase and frequency detection respectively. Generalized families of curves show bearing error, blur, effect of sense, and balance voltage, respectively, as a function of angular displacement of the rotating elements for each type of detection. Approximately 50 pages of curves constitute the body of this analysis. The curves have a very broad application in that they provide a method of evaluating the above class of radio direction finders.

The second phase of this study is an extension of the analysis to include the case of the simultaneous arrival of two signals of the same frequency but with differing amplitude, time phase and space phase (azimuth).

Approximately 25 families of generalized curves have been calculated for this case. The curves show relative amplitude, blur, and bearing error for varying amounts of time phase, space phase, and amplitude difference.

Certain conclusions (some already known) have been obtained from these analyses. A summary of the more important conclusions follows:
1. The electrical output of the rotating antenna system or rotating goniometer consists of two side frequencies with the carrier suppressed.

2. The side-frequency (carrier frequency ± mechanical rotational frequency) components can have any relative amplitude.

3. Addition of the sense or balance voltage to the above electrical output corresponds to a reinsertion of the carrier frequency voltage component. It is shown that the insertion of a sense voltage gives an amplitude modulated resultant, and the insertion of a balance voltage gives a phase modulated resultant.

4. If the spacing of the elements in the antenna array or collector system is any appreciable part of a wavelength, integral harmonics of the rotational frequency are also present. Using a Bessel's function expansion, it is shown that the spacing or octantal error is produced by the odd harmonics of the rotational frequency (any antenna pattern satisfying the Dirichlet conditions can be represented by a Fourier Series expansion of sine and cosine terms having fundamental and harmonic arguments of the rotation frequency).

5. Breuninger* has shown that the spacing error of the conventional Adcock System may be reduced by the use of more antenna elements in the array—8 elements were given as a practical compromise.

6. Under the condition of the simultaneous arrival of two waves of arbitrary amplitude, azimuth, and time phase, this class of radio direction finders is subject to bearing errors of ± 90° and blur up to 100%. A reconnection of the antenna elements to give a different antenna pattern in general yields different amounts of blur and bearing error for the same condition of

7. This last fact suggests and provides a means of separating two interfering signals. Thus by using an eight mast Adcock connection and then a suitable reconnection of antenna elements to obtain the second harmonic antenna pattern, there is obtained a set of readings consisting of two bearing indications and two blur indications. It is then possible through the medium of the families of generalized curves to determine the bearing of each of the interfering signals. Under some conditions the resolution may be poor and if there are more than two waves, it becomes necessary to use additional harmonic antenna patterns to separate the signals. However, by comparing the bearings obtained with the several possible antenna connections, some indication of the probable magnitude of error is obtained.

In order to investigate the feasibility of the operation of a practical direction finder in manner of the above, an antenna pattern simulator has been designed and is currently being constructed. Initially, this device will function in the audio frequency range.

In this frequency range there are readily available simple and accurate means for the generation, control, and measurement of the amplitude and phase, respectively, of several voltages of a desired frequency. It is hoped this device will indicate in a quantitative manner what can be done to improve the bearing accuracy of existing radio direction finding systems.

Plate 13 shows the block diagram of the antenna pattern simulator, assembly of which has been completed. Plates 14, 15 and 16 show block diagrams of several existing radio direction finding systems which will be evaluated by this device.

To illustrate more concretely the procedure for determining the
respective directions of arrival of two interfering signals, the method outlined under conclusion seven above will be indicated. For the case to be considered, each of the interfering signals is assumed to be vertically polarized with respect to the ground plane of the radio direction finder. Each signal may have arbitrary amplitude, space phase, and time phase. The collector system is composed of eight vertical antennas equally spaced around the periphery of a circle. A sense antenna may be placed at the center of the array. Provision is made for the connection of the antenna elements as a crossed-Adcock array and again as an array producing a quadrifolium antenna pattern. The former connection will be indicated as the fundamental antenna array, the latter connection will be indicated as the second-harmonic antenna array.

Certain symbols are defined as follows:

\[ K_0 = \text{gain of the sense antenna and associated circuit.} \]
\[ K_1 = \text{gain of the Adcock-connected antenna array and associated circuit.} \]
\[ K_2 = \text{gain of the second-harmonic antenna array and associated circuit.} \]
\[ B_1 = \text{field strength of stronger signal.} \]
\[ E_2 = \text{field strength of weaker signal.} \]
\[ s_0 = \text{resultant instantaneous voltage output of the sense antenna and associated circuit.} \]
\[ s_1 = \text{resultant instantaneous voltage output of the fundamental antenna array and associated circuit.} \]
\[ s_2 = \text{resultant instantaneous voltage output of the second harmonic array and associated circuit.} \]
\[ h = \frac{E_2}{B_1}, \text{a numeric ratio equal to or less than unity.} \]
\[ \Theta = \text{angular setting of the goniometer search coil.} \]
\[ \alpha = \text{space phase angle between the reference axis and the direction of arrival of the stronger signal.} \]
\[ \beta = \text{space phase angle between the stronger and the weaker signal.} \]
\[ \varphi = \text{time phase angle between the stronger and the weaker signal.} \]

Expressions for the resultant instantaneous voltage output of the sense antenna, fundamental antenna array, and second harmonic array, respectively, have been developed as follows:

\[ e_0 = K_0 E_1 \sqrt{1 + h^2 + 2h \cos \varphi} \cos (\omega t + \tan^{-1} \frac{h \sin \varphi}{1 + h \cos \varphi}) \]
\[ e_1 = K_1 E_1 \sqrt{1 + h^2 + 2h \cos (-\beta - \varphi)} \cos \left[ \omega t + \tan^{-1} \frac{h \sin (\beta + \varphi)}{1 + h \cos (\beta + \varphi)} \right] \]
\[ + K_2 E_1 \sqrt{1 + h^2 + 2h \cos (-\beta + \varphi)} \cos \left[ \omega t - \tan^{-1} \frac{h \sin (\beta - \varphi)}{1 + h \cos (\beta - \varphi)} \right] \]
\[ e_2 = K_2 E_1 \sqrt{1 + h^2 + 2h \cos (-2\beta - \varphi)} \cos \left[ \omega t + 2\omega t + \tan^{-1} \frac{h \sin (2\beta + \varphi)}{1 + h \cos (2\beta + \varphi)} \right] \]
\[ + K_2 E_1 \sqrt{1 + h^2 + 2h \cos (-2\beta + \varphi)} \cos \left[ \omega t - 2\omega t - \tan^{-1} \frac{h \sin (2\beta - \varphi)}{1 + h \cos (2\beta - \varphi)} \right] \]

An expression for the relative blurring of the nulls has been developed for each type of antenna connection as follows: The blur for the fundamental antenna array is

\[ \text{(Blur)}_1 = \frac{1 + h^2 + 2h \cos (\beta - \varphi)}{\sqrt{1 + h^2 + 2h \cos (\beta - \varphi) + 1 + h^2 + 2h \cos (\beta + \varphi)}} \]

The blur for the second harmonic antenna array is

\[ \text{(Blur)}_2 = \frac{1 + h^2 + 2h \cos (-\beta - \varphi)}{\sqrt{1 + h^2 + 2h \cos (-\beta - \varphi) + 1 + h^2 + 2h \cos (-\beta + \varphi)}} \]

In plate 17, the \( \text{(Blur)}_1 \) is assumed to be 40%. The loci are for fixed values of \( h \) in terms of \( \beta \) and \( \varphi \). The \( \text{(Blur)}_2 \) is assumed to be 70%. The loci are plotted for fixed values of \( h \) in terms of \( \beta \) and \( \varphi \).

The two sets of loci are solved simultaneously to give the possible solutions indicated by the heavy solid lines. To reduce the number of possible solutions, \( K_1 \) is made identically equal to \( K_2 \) and the ratio of \( e_2 \) to \( e_1 \) is then measured. By expressions given above it is seen that the ratio becomes
\[
\begin{align*}
|E_1| &= \sqrt{1+h^2 + 2h \cos(-2\beta - \varphi)} + \sqrt{1+h^2 + 2h \cos(-2\beta + \varphi)} \\
|E_2| &= \sqrt{1+h^2 + 2h \cos(-\beta - \varphi)} + \sqrt{1+h^2 + 2h \cos(-\beta + \varphi)}
\end{align*}
\]

The loci for different values of \(h\) as a function of \(\beta\) and \(\varphi\) have been plotted for an assumed ratio of \(e_2/e_1 = 0.7\). When these loci are solved simultaneously with the possible solutions indicated above, only four possible solutions remain (indicated by heavy black arrows on plate 17). If now the polarity of the fundamental and the second harmonic antenna pattern, respectively, be determined, three of the four possible solutions are incompatible. A unique solution is thus obtained which gives \(h\), \(\beta\), and \(\varphi\) for two interfering signals of arbitrary relative amplitudes, relative space phase, relative time phase, respectively. There still remains the problem of determining the absolute space phase of the stronger signal. This is obtained through the medium of generalized bearing error curves which can be calculated.

From the cursory outline given above, it is apparent that the accurate determination of the direction of arrival is made considerably more complex by the presence of one or more interfering signals.

The interesting aspect of this study is the evident fact that there is theoretically available from modified existing radio direction finders enough information to completely separate and identify the directions of arrival of two interfering signals.

The practicability of this method will be evaluated by means of the antenna pattern simulator.

At the completion of these studies, a comprehensive report is to be issued.
SMALL HIGH-GAIN ARRAYS FOR DIRECTION FINDING
Nicholas Yaru

A study has been made of the applicability of small, high-gain arrays ("super-gain" antennas) to the direction finding problem. Most conventional direction finding systems in the medium and high-frequency bands operate on the null of the antenna pattern rather than the maximum because of the broadness of the lobe obtained with ordinary directive antenna systems which are small in wavelength. A directive array which is capable of producing a narrow beam and which is also small in wavelength would be of great value in direction finding. To determine how practical such arrays might be, calculations were made of the directivity, driving-point impedances, and probable efficiencies of several small, high-gain arrays. The results are indicated by the typical examples shown.

Plates 16 and 19 show calculated radiation patterns for linear end-fire arrays having overall lengths of three-quarter wavelength and one-quarter wavelength respectively. Plate 19 compares a 3-element array having 1/8 wavelength spacing with a 5-element array having the same overall length of 1/4 wavelength. Plate 18 shows the patterns of arrays of 4, 7, 13 and 25 elements, each array having an over-all length of 3/4 wave length. Plate 18 especially indicates the theoretical possibility of obtaining arbitrarily great directivity with arrays of given lengths, if the elements are fed with currents of proper magnitude and phase. The relative currents required to obtain the patterns of Plates 18 and 19 are shown in Table I.

The driving point impedances, which result for the arrays of Plate 19 for 1/4 wavelength elements and 1/6 wavelength elements, are tabulated.
in Table II. (These are for vertical elements fed against ground.)

Table I

Relative Currents in Endfire Arrays

<table>
<thead>
<tr>
<th>Three element array of Λ antennas spaced Λ apart</th>
<th>Five element array of Λ antennas spaced Λ apart</th>
<th>Five element array of Λ antennas spaced Λ apart</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna 1</td>
<td>I₁=1.0 Ω</td>
<td>I₁=1.0 Ω</td>
</tr>
<tr>
<td>antenna 2</td>
<td>I₂=1.85 Ω=157.5°</td>
<td>I₂=3.9 Ω=174.5°</td>
</tr>
<tr>
<td>antenna 3</td>
<td>I₃=1.0 Ω=215°</td>
<td>I₃=5.8 Ω=348.8°</td>
</tr>
<tr>
<td>antenna 4</td>
<td>I₄=3.9 Ω=523.1°</td>
<td>I₄=3.9 Ω=523.1°</td>
</tr>
<tr>
<td>antenna 5</td>
<td>I₅=1.0 Ω=697.5°</td>
<td>I₅=1.0 Ω=697.5°</td>
</tr>
</tbody>
</table>

Table II

Driving Point Impedances - Endfire Arrays

<table>
<thead>
<tr>
<th>Three element array of Λ antennas spaced Λ apart</th>
<th>Five element array of Λ antennas spaced Λ apart</th>
<th>Five element array of Λ antennas spaced Λ apart</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna 1</td>
<td>Z₁=3.7 +j11.8</td>
<td>Z₁=2.75 - j2</td>
</tr>
<tr>
<td>antenna 2</td>
<td>Z₂=4.7 +j28</td>
<td>Z₂= -75 + j4.4</td>
</tr>
<tr>
<td>antenna 3</td>
<td>Z₃=-11.3+j29</td>
<td>Z₃= -1.14 + j6.7</td>
</tr>
<tr>
<td>antenna 4</td>
<td>Z₄= -6.2 + j4.4</td>
<td>Z₄= -1.4 - j106.3</td>
</tr>
<tr>
<td>antenna 5</td>
<td>Z₅= 2.4 - j3.6</td>
<td>Z₅= 2.1 - j39.6</td>
</tr>
</tbody>
</table>

It is seen that with the close spacings and phase relations required to obtain sharp directivity, the resistive components of the driving-point impedances are reduced to extremely low values, being negative in some cases. Using these results, the theoretical array efficiencies were computed. These array efficiencies are shown in Table III, and have been obtained on the basis of assumed efficiencies of 90% and 70%, respectively, for a single element radiating by itself.

- 36 -
Table III

Endfire Array Efficiencies

<table>
<thead>
<tr>
<th>Assumed efficiency of single antenna</th>
<th>Calculated array efficiency</th>
<th>Assumed efficiency of single antenna</th>
<th>Calculated array efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>0.84%</td>
<td>90%</td>
<td>11%</td>
</tr>
<tr>
<td>70%</td>
<td>0.22%</td>
<td>70%</td>
<td>3.1%</td>
</tr>
</tbody>
</table>

It is seen from this table that the array efficiency becomes very low due to the low values of driving-point resistances (and hence low effective radiation resistance of the array). With closer spacings this effect becomes even more pronounced and the efficiency becomes vanishingly small. The apparent higher efficiency of the 1/6 wavelength antennas over the 1/4 wavelength antennas would be unobtainable in practice because the efficiency of a single element would be lower to start with.

Similar sets of calculations made for super-gain broadside arrays show essentially the same results. Plate 20 depicts the directive properties of two, linear, broadside arrays of five and nine elements each having an overall length of 1/4 wavelength. The properties of the Tohobyscheff polynomials were applied to the problem of determining the current distribution in these equispaced arrays which results in high directivity and low side lobes of the order of 26 decibels down on the main lobe. The relative currents calculated to obtain the aforementioned patterns are shown in Table IV. These calculated values show further the impracticality of small aperture arrays.
Table IV

Relative Currents in Broadside Arrays

<table>
<thead>
<tr>
<th>Five Element Array</th>
<th>Nine Element Array</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna 1</td>
<td>$I_1 = 1.0$</td>
</tr>
<tr>
<td>antenna 2</td>
<td>$I_2 = -3.8$</td>
</tr>
<tr>
<td>antenna 3</td>
<td>$I_3 = 2.3$</td>
</tr>
<tr>
<td>antenna 4</td>
<td>$I_4 = -3.8$</td>
</tr>
<tr>
<td>antenna 5</td>
<td>$I_5 = 1.0$</td>
</tr>
<tr>
<td>antenna 6</td>
<td>$I_6 = -54.6$</td>
</tr>
<tr>
<td>antenna 7</td>
<td>$I_7 = 27.5$</td>
</tr>
<tr>
<td>antenna 8</td>
<td>$I_8 = -7.9$</td>
</tr>
<tr>
<td>antenna 9</td>
<td>$I_9 = 1.0$</td>
</tr>
</tbody>
</table>

It is concluded that, because of the rapid rate with which antenna efficiency decreases as the array dimensions are reduced, high-gain antennas of dimensions small in wavelengths are not practical for direction finding systems.
STANDING WAVE PATTERN IN THE FIELD OF TWO INTERSECTING RAYS

NOTE:

a) \( \frac{K_{ray 2}}{K_{ray 1}} \)

b) ELECTRIC VECTOR ASSUMED PERPENDICULAR TO PAGE
BLOCK DIAGRAM OF EQUIPMENT FOR EXPERIMENTAL MEASUREMENTS

PLATE 2
DIRECTION OF ARRIVAL OF E.M. WAVE

\[ \begin{align*}
E_A &= \frac{D}{2} \cos \theta \\
E_B &= \frac{D}{2} \sin \theta \\
E_C &= \frac{D}{2} \sin \theta \\
E_D &= \frac{D}{2} \cos \theta
\end{align*} \]

Antenna elements

Diagram of E.M. wave arriving at 4 antenna elements

<table>
<thead>
<tr>
<th>D/2 = ( \pi /4 )</th>
<th>( \theta )</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>80°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_A )</td>
<td>270°</td>
<td>271°</td>
<td>275°</td>
<td>282°</td>
<td>291°</td>
<td>302°</td>
<td>315°</td>
<td>329°</td>
<td>344°</td>
<td>0°</td>
<td></td>
</tr>
<tr>
<td>( \phi_B )</td>
<td>90°</td>
<td>89°</td>
<td>88°</td>
<td>87°</td>
<td>86°</td>
<td>85°</td>
<td>84°</td>
<td>83°</td>
<td>82°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>( \phi_C )</td>
<td>0°</td>
<td>16°</td>
<td>31°</td>
<td>45°</td>
<td>58°</td>
<td>69°</td>
<td>78°</td>
<td>85°</td>
<td>89°</td>
<td>90°</td>
<td>0°</td>
</tr>
<tr>
<td>( \phi_D )</td>
<td>0°</td>
<td>344°</td>
<td>329°</td>
<td>315°</td>
<td>302°</td>
<td>291°</td>
<td>282°</td>
<td>275°</td>
<td>271°</td>
<td>270°</td>
<td>0°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D/2 = ( \pi /2 )</th>
<th>( \theta )</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>80°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_A )</td>
<td>180°</td>
<td>182°</td>
<td>191°</td>
<td>204°</td>
<td>222°</td>
<td>244°</td>
<td>270°</td>
<td>298°</td>
<td>349°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>( \phi_B )</td>
<td>180°</td>
<td>179°</td>
<td>169°</td>
<td>156°</td>
<td>138°</td>
<td>116°</td>
<td>90°</td>
<td>62°</td>
<td>31°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>( \phi_C )</td>
<td>0°</td>
<td>31°</td>
<td>62°</td>
<td>90°</td>
<td>116°</td>
<td>138°</td>
<td>156°</td>
<td>169°</td>
<td>179°</td>
<td>180°</td>
<td>180°</td>
</tr>
<tr>
<td>( \phi_D )</td>
<td>0°</td>
<td>349°</td>
<td>298°</td>
<td>270°</td>
<td>244°</td>
<td>222°</td>
<td>204°</td>
<td>191°</td>
<td>182°</td>
<td>180°</td>
<td>180°</td>
</tr>
</tbody>
</table>

Phases of E.M. wave at points A, B, C, & D referred to point F

Plate 4
Tech. Rpt. 4
FIG. 1 CASE OF A SINGLE ROTATING ANTENNA

FIG. 2. CASE OF A STRAIGHT-LINE ARRAY OF ANTENNAS
BLOCK DIAGRAM OF TYPICAL DOPPLER-TYPE DIRECTION FINDER
FIG. 1 CIRCULAR ARRAY OF FIXED ANTENNAS

\[ a = f \]

\[ \phi \]

ANTENNA ELEMENTS

DIRECTION OF PROPAGATION

FIG. 2 VECTOR SUM OF TWO PHASE-MODULATED WAVES

\[ R = \left( 1 + \alpha^2 + 2 \alpha \cos \phi \right) \]

PLATE 12

TECH. RPT. C
### Fig. 1: Manually-Rotated Type Using Figure 8 Pattern

<table>
<thead>
<tr>
<th>(a) BILATERAL PATTERN</th>
<th>B</th>
<th>GONIOMETER</th>
<th>AMPL. DETECTOR &amp; INDICATOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) UNILATERAL PATTERN</th>
<th>B</th>
<th>GONIOMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Fig. 2: Automatic Bearing Indicator Type

<table>
<thead>
<tr>
<th>(a) BILATERAL PATTERN</th>
<th>B</th>
<th>GONIOMETER</th>
<th>AMPL. DETECTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
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<th>B</th>
<th>GONIOMETER</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

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**PLATE 14**

**TECH. RPT. 4**

**ANTENNA SIMULATOR APPLIED TO DIRECTION FINDING SYSTEMS**
FIG. 1 WATSON-WATT TYPE

FIG. 2 SPACED-LOOP TYPE

FIG. 3 LABELING-SYSTEM TYPE

ANTENNA SIMULATOR APPLIED TO DIRECTION FINDING SYSTEMS

PLATE 15
FIG. 1 MATCHED-LINE TYPE

FIG. 2 NULL-SEEKING TYPE

ANTENNA SIMULATOR APPLIED TO DIRECTION FINDING SYSTEMS
NOTE: Loci are for constant relative amplitude h of two interfering signals, as specified below.

- - - - - h loci for fundamental pattern and 40% blur
- - - - - h loci for and harmonic pattern and 70% blur
- - - - - h loci for (64) = 0.7

Arrows indicate the simultaneous solution of three loci
- - - - - Simultaneous solution two loci

LOCUS DIAGRAMS
OVERALL LENGTH OF ARRAY $= \frac{3}{4} \lambda$

$\psi = \beta \left( \cos \phi + 1 \right)$

LINEAR END-FIRE ARRAY WITH NULLS
EQUISPACED IN RANGE OF $\psi$

PLATE 18
TECH. RPT.
OVERALL LENGTH OF ARRAY = $\frac{\lambda}{4}$

$\psi = \beta \lambda (\cos \phi - 1)$

LINEAR END-FIRE ARRAY WITH NULLS EQUISPACED IN RANGE OF $\psi$

PLATE 19
TECH. RPT. 4
DOPPLER SYSTEM PHASE VARIATION DUE TO WAVE INTERFERENCE

\( \alpha = 0.5, \theta = 20^\circ, \xi = 45^\circ \)
DOPPLER SYSTEM PHASE VARIATION DUE TO WAVE INTERFERENCE

PLATE 24
TECH. NPT. 4
ERROR FUNCTION OF WAVE INTERFERENCE (α = 0.5, θ = 20°, γ = 90°)

PLATE 26
TECH. RPT. 4
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