AD NUMBER

AD038250

CLASSIFICATION CHANGES

<table>
<thead>
<tr>
<th>TO:</th>
<th>unclassified</th>
</tr>
</thead>
<tbody>
<tr>
<td>FROM:</td>
<td>confidential</td>
</tr>
</tbody>
</table>

LIMITATION CHANGES

TO:
Approved for public release; distribution is unlimited.

FROM:
Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; JUL 1945. Other requests shall be referred to Army Harry Diamond Laboratory, Adelphi, MD.

AUTHORITY

31 jul 1957, DoDD 5200.10; hd1, d/a ltr, 15 aug 1979
THIS REPORT HAS BEEN DELIMITED
AND CLEARED FOR PUBLIC RELEASE
UNDER DOD DIRECTIVE 5200.20 AND
NO RESTRICTIONS ARE IMPOSED UPON
ITS USE AND DISCLOSURE.

DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.
Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by DOCUMENT SERVICE CENTER
K-10, DAYTON, 2, OHIO
NOTICE: THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE
NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING
THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN
ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW.
Address to: 
Photoduplication Service 
The Library of Congress 
Washington 25, D. C. 

Enclosed find $ for which please duplicate the item listed below. I hereby accept the conditions set forth on the reverse of this page.

Signature: 
Rowena Wise

Name: 

Address: 
Technical Information Division 
ASTIA Reference Center

Please make one microfilm of the following: Ordnance Development Lab., National Bureau of Standards. THE EFFECT OF SIGHT MISALIGNMENT AND ANGLE OF ATTACK, by S. H. Lachenbruch. 23 July 45. (Rept. no. OD-SP-131)

<table>
<thead>
<tr>
<th>Facsimile prints</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negatives [White on black]</td>
<td>@ $</td>
</tr>
<tr>
<td>Positives [Black on white]</td>
<td>@ $</td>
</tr>
<tr>
<td>Enlargements or reductions</td>
<td>@ $0.10</td>
</tr>
<tr>
<td>Combinations</td>
<td>@ $0.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Microfilms</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative exposures</td>
<td>@ $</td>
</tr>
<tr>
<td>Positives, per foot</td>
<td>@ $0.07</td>
</tr>
<tr>
<td>Spools and boxes</td>
<td>@ $0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Enlargements from 35-mm. film</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;x&quot;</td>
<td>@ $</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Photographs</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negatives</td>
<td>@ $</td>
</tr>
<tr>
<td>Contact prints</td>
<td>@ $</td>
</tr>
<tr>
<td>Enlargements</td>
<td>@ $</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lantern slides</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slide, including negative</td>
<td>@ $1.75</td>
</tr>
<tr>
<td>Slide only</td>
<td>@ $0.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Postal registry fee</th>
<th>Amount</th>
</tr>
</thead>
</table>

Special instructions: 

Total cost

Deposit

Balance due

Refund
THE EFFECT OF SIGHT MISALIGNMENT
AND ANGLE OF ATTACK VARIATION.

S. H. Lechenbruch

ABSTRACT

In any toss bombing maneuver in which sight line and flight line prior to pull-up do not coincide, an impact error results. The amount of this error per mil of sight error is given by Equation (2) or Figure 2. The error may be corrected with the KPI adjustment dial by an amount determined from Equation (9) or Figure 3.

Sight misalignments depend on (a) the fixed orientation of the sight with respect to the airplane, and (b) the orientation of the airplane with respect to its flight line. The latter is the angle of attack, which is increased by a decrease in either dive angle or air speed, and therefore varies considerably since decreases in dive angle are usually accompanied by decreases in air speed. Differences in angle of attack may be determined from Equation (13) or from the nomogram of Figure 5.

If the effect of a sight misalignment is counteracted at some mean value of altitude, dive angle, and air speed, by an KPI adjustment $\Delta TC/TC$, the impact error at another altitude, dive angle, and air speed is the algebraic sum of that due to (a) the difference between the angles of attack at the mean value and second value of dive angle and air speed, (b) the sight error for which adjustment was made, and (c) the $\Delta TC/TC$ adjustment itself. Errors (b) and (c) are in opposite directions and partially offset each other, whereas (a) may affect the result considerably in either direction.

When the sight line is below the flight line, much greater bombing accuracy is achieved by a fixed readjustment of the sight itself than by an KPI adjustment. Under certain conditions it may be advantageous to overadjust the sight somewhat, bringing sight line above flight line, and correcting the resulting misalignment by a $\Delta TC/TC$ adjustment.

INTRODUCTION

In toss bombing, the flight path of the airplane before pull-up is assumed to be a collision course, i.e., a straight path directed toward the target. Since the flight path is determined in practice by means of a sight which is kept fixed on the target, the actual path can be a collision course only if the plane constantly flies along the sight line.

Sight misalignments may arise from the fact that for any aircraft the sight is usually adjusted to be correct for shooting of the machine guns. This adjustment is not suitable for bombs since bombs leave the aircraft in a direct
tion tangent to the flight path, while bullets, because of their high muzzle velocity, leave in the direction in which the guns are pointing. Differences between the sight and flight lines may also occur as a result of variations in the angle of attack, which for any type of plane depends on its weight, velocity, and dive angle. The errors resulting from such misalignments, and means of correcting them, form the subject of this paper.

Figure 1 represents the flight of plane and bomb for the case in which the line of flight makes a constant angle \( \phi \) with the sight line, the sight being kept fixed on the target \( K' \) until pull-up. The vectors marked \( \mathbf{q} \) represent instantaneous directions of the sight line, directed toward the target; those marked \( \mathbf{s} \), making an angle \( \phi \) with the \( \mathbf{q} \)-vectors, represent the corresponding instantaneous directions of flight. It will be noted that the path \( \overline{AO} \) is slightly concave upward; it would be concave downward if \( \phi \) had the opposite sense. For small angles \( \phi \), however, very little sacrifice of accuracy results from assuming that arc \( \overline{AO} \) is straight.

If\( A \) is the first altitude point, the values of dive angle \( \alpha \), time-to-target \( T_t \), and airspeed \( V \), which are fed into the computer are determined by conditions along \( \overline{AO} \), assuming that the dive angle gyro is aligned with respect to flight line. Barring other errors, therefore, the release time \( T_p = \frac{L}{V} \) will be so determined as to cause a hit at \( H \), on the extension of \( \overline{AO} \), the corresponding slant range \( \overline{OH} \) being \( S = V \cdot T_p \). The horizontal impact error due to the misalignment is \( S = K' \overline{OH} \).

The angle \( \phi \) is considered positive when the sight line is above the flight line, and the error \( S \) positive when it represents an impact beyond the target. In Figure 1, \( \phi \) is positive and \( S \) negative.

### Impact Error Resulting from Sight Misalignment

The exact relation between \( \phi \) and \( S \) is obtained at once by application of the sine law to triangle \( \overline{OHK} \):

\[
-S = S \sin \phi / \sin(\alpha - \phi) = S / \sin \alpha (\cot \phi - \cot \alpha).
\]

When \( |\phi| \) is small compared with \( \alpha \), a close approximation is

\[
S = -S \phi / \sin \alpha, \quad \alpha - \phi = S \sec \alpha.
\]

\( \phi \) being in radians. The amount by which (1) is in error as compared to the exact expression (i), is given approximately by the ratio \( \phi / \alpha \); thus for \( \phi \) values \( \pm 10^\circ \) or 17 miles \( \alpha \) is accurate to within \( 5\% \) for \( \alpha \) as small as \( 20^\circ \), and to within \( 10\% \) for \( \alpha = 10^\circ \).

Figure 2 is a contour map, in polar co-ordinates, of the relation between \( S \) and \( \phi \) corresponding to several fixed values of the ratio \(-S / \phi\), \( \phi \) being the impact error caused by sight error \( \phi \). The corresponding rectangular coordinates of points on these curves are the horizontal range \( (S \cos \alpha) \) and second altitude \( (R_2 = S \sin \alpha) \); hence the curves may be considered pictorially as the loci of spatial release points corresponding to given \(-S / \phi\), the
target being at the origin. The value of $-\frac{S_p}{\phi}$ for a given release point may be obtained by interpolation between consecutive curves, so that Figure 2 is in one sense a graph of $-\frac{S_p}{\phi}$ as a function of release point. (Several other contour maps of this type are included in this paper.)

On converting (2) to rectangular coordinates, it becomes apparent that the curves of Figure 2 are arcs of circles which pass through the origin, with centers on the vertical axis and radii $\frac{S_p}{\phi}$. $\phi$ being in radians, or $-500 \frac{S_p}{\phi_{\text{mill}}}$s. It will be noted that $\frac{S_p}{\phi}$ is independent of air speed and pull-up acceleration.

According to the exact equation (1), $\frac{S_p}{\phi}$ is actually proportional, not to $\phi$, but to $1/(\cot \phi - \cot \alpha)$, which differs significantly from $\phi$ in radians when $\alpha$ and $\phi$ are of comparable magnitude, i.e., for very small $\alpha$ or very large $\phi$. The graph of Figure 2 therefore becomes accurate for all values of $\alpha$ and $\phi$ if the "feet per min" values with which the curves are labeled are considered as values, not of $-\frac{S_p}{\phi_{\text{mill}}}$s, but of $-1000 \frac{S_p}{(\cot \phi - \cot \alpha)}$.

**CORRECTION FOR RIGHT MISALIGNMENT BY KPJ ADJUSTMENT $\Delta T_c / T_c$**

The error due to non-coincidence of sight and flight lines would be eliminated, causing a hit on the target $K'$ (Fig. 1), if the $T_c$ value fed into the computer were adjusted to correspond to a target at $H'$ instead of $H$. This would mean a relative increase $\epsilon_c = \Delta T_c / T_c$ (or a relative decrease $-\Delta T_c / T_c$), where $V \Delta T_c = H H'$ and $\Delta T_c / T_c = H H' / \sigma H$.

Application of the sine law to the triangle $H H' K'$, on the usual assumption that the parabolic arc $K' H'$ has negligible curvature, yields at once

$$V \Delta T_c = -\frac{S}{\sin \alpha} \sin (\alpha + \theta') / \sin \theta' = -S \overline{\theta} \sin \alpha,$$

i.e.,

$$\epsilon_c = \frac{V \Delta T_c}{S} = -\frac{S \overline{\theta} \sin \alpha}{S},$$

or

$$S / \epsilon_c = -S / \overline{\theta} \sin \alpha,$$

(4)

where $\theta'$ is the angle between trajectory and collision course at their intersection, and

$$\overline{\theta} = \frac{\sin (\alpha + \theta')}{\sin \alpha \sin \theta'} = \cot \theta' + \cot \alpha$$

(5)

Formula (4) or its equivalent has been encountered in every problem involving correction of an impact error. The value of $\overline{\theta}$ in terms of $S$, $V$, $\alpha$, and $K'$ (where $K' \overline{g}$ is pull-up acceleration) is implied in Report OD-SP-105. Equa-

---

* The negative sign in (4) holds only when $\epsilon_c$ represents the adjustment required to offset the error $S$. When it represents the adjustment required to produce the error $S$, the sign of the right member of (4) is positive.
tions (9), (3), and (8):

\[ \Theta = \left[ \sqrt{\frac{1}{K}} + \sqrt{1 + \frac{1}{K} \sin^2 \alpha} \right] \left( 1 + 2 \beta \right) \tan \alpha / 2 \beta \]  

\[ = \frac{\sqrt{\frac{1}{K}} \tan \alpha}{1 + 2 \beta} \left( \sqrt{\frac{1}{K}} + \sqrt{1 + 2 \beta} \right) + \sec \alpha \csc \alpha, \]  

where \( \alpha = K \cdot \cos \alpha \), and \( \beta = \frac{S}{\sqrt{2}} \cdot \mu \cdot \sin \alpha \)  

Substitution of (1) into (4) gives the correction required to effect a given \( \phi \):

\[ E_c = \frac{V \Delta T_c}{S} = \frac{\Theta}{\cot \phi - \cot \phi} = \frac{\cot \Theta \phi + \cot \phi}{\cot \phi - \cot \phi} \]  

Again, when \( |\phi| \) is small compared with \( \alpha \), a close approximation is

\[ E_c \approx \Theta \phi, \quad \text{or} \quad \frac{E_c}{\phi} = \Theta \]  

The accuracy of (9) relative to (6) is similar to that given in the statement following equation (3).

A formula equivalent to (6) was already derived in Report OD-57669 in the course of a general discussion of angles.

Equation (9) may be solved for \( S \) by solving (6b) for \( \sqrt{1 + 2 \beta} \) and using the relations \( \beta = \frac{1}{2} \left( \left( 1 + 2 \beta \right)^2 - 1 \right), \ S = \beta \cdot V^2 K/Mg \cdot \sin \alpha \)

The result is

\[ S = \frac{V^2}{g \cdot \mu}, \quad \frac{K}{N}, \quad \frac{N}{D} \left( 1 + \frac{1}{2} \frac{N}{D} \right) \]  

where

\[ N = \tan \alpha \sqrt{\frac{1}{K} \left( 1 + \sqrt{1/K} \right)}, \ D = \Theta / \phi - \sqrt{K} \tan \alpha - \sec \alpha \csc \alpha \]  

Figure 2 is a contour map, based on (10), of \( E_c / \phi \), as percent-change in \( T_c \) per unit of flight error, as a function of release point. \( K \) is held constant at the values \( \phi \), the variation of \( E_c / \phi \) with \( K \) being quite small. The range and altitude axes involve a factor \( \phi \) which is unity when \( V = 250 \) knots. In accordance with (10), which implies that \( S \) varies as \( V^2 \) for fixed \( E_c / \phi \) and \( \alpha \), the graph may be used for any \( a \) speed \( V \) by substituting \( n = \left( V/250 \text{ knots} \right)^2 \). At 250 and 300 knots, \( n \approx 1/2 \) and 2, respectively.

* actually 260 \( \pm 2 \) and 393.5 knots.
As in the case of Figure 2, Figure 3 as such becomes inaccurate for very small $\phi$ or very large $\phi$, but becomes accurate for all $\phi$ and $\phi$ if the "$\%\text{ per mile}$" figures given are considered as values, not of $\phi$ but of $1000\phi/c$ (cot $\phi$ - cot $\phi$).

According to (9), Figure 3 may also be interpreted as a contour map of the much-used function $\phi$. When used as such, the "$\%\text{ per mile}$" figures must be reconverted into "units per radian" by multiplication by .01/.001 = 10. The curve marked 0.8% per mile, for example, is as well the locus of $\phi = 8$.

Figure 4 is a contour map, based on (4), of $|\delta_c|/\phi$ as a function of release point, $\delta_c$ being the impact error corresponding to the change $E_c$. From Figure 4 may be read the amount of displacement of the impact point per percent change in $\phi$, both changes having the same algebraic sign. Furthermore, the reciprocals of the $|\delta_c|/\phi$ figures on the curves represent the percent change in $\phi_c$ required to offset such foot of impact error; and when so interpreted, $\delta_c$ and $E_c$ have opposite algebraic signs. The graph is of a general nature, applicable not only to the sight error problem but to all problems involving correction of an impact error by adjustment of $\phi$.

Figure 4 has been constructed, not by solving (4) for $\delta$ directly — this would have yielded a set of cubic equations — but rather from Figures 2 and 3, by determining points of intersection of curves in Figure 4 with circles of appropriate radii similar to those in Figure 2.

**VARIATION OF ANGLE OF ATTACK**

Application of the preceding section, and of Figures 2 and 3, requires that the value of the sight error $\phi$ for any specific case, or at least the difference between $\phi$ values for two specific cases, be known. The effective sight error, however, depends not only on the fixed orientation of the sight with respect to the airplane, but also on the angle of attack, or orientation of the plane with respect to its flight line. The variation of the latter with air speed and dive angle is appreciable and must be taken into account.

The angle of attack $\phi_a$ is given by the formula

$$\phi_a = \frac{C W \cos \alpha}{V^2} - \kappa,$$

where $W$ is the gross weight of the plane, and $C$ and $\kappa$ are constants for any one plane. The value of $\kappa$ also depends on the choice of a reference line in the airplane, and for purposes of calculating differences in angles of attack.
one may set \( k = 0 \) with no loss of generality:

\[
\phi_a = \frac{C \omega \cos \alpha}{\sqrt{a}}
\]  

(13)

It is found generally that \( C \) has the same value for all planes of a series, as

\( \text{TRM-1 and TRM-10} \).

The nomogram of Figure 5 gives this relative angle of attack \( \phi_a \), as in (13), as a function of \( C, \omega, \sqrt{a} \) and \( \alpha \). The calibrations on lines I, II, IV, and VI, have been determined from values of \( \log C, \log \omega, \log \sqrt{a}, \) and \( \log \cos \alpha \), respectively. Points on line III have been pre-determined for each plane at its nominal weight \( g \), thus obviating the use of lines I and II whenever the gross weight is approximately nominal.

Lines III through VII have been so spaced that \( \sqrt{a} \) and \( \alpha \) may be entered in either order. This property is illustrated by the example appearing on the nomogram. The point on line V marked "750, 550 knots" may be joined with several points in turn on line VI to obtain \( \phi_a \) on line VII for different dive angles at the fixed air speed; or the point marked "750, 40°" may be joined with points on line IV to obtain \( \phi_a \) for different air speeds at the fixed dive angle. The relative convenience of the two methods depends on whether one is concerned more with variation of \( \phi_a \) with dive angle or with air speed.

According to (13), the angle of attack, while independent of range, is increased by a decrease in either air speed or dive angle. Hence the range of variation in \( \phi_a \) for any given plane is widened by the fact that decreases in dive angle are in practice usually accompanied by decreases in air speed.

**APPLICATION OF METHOD**

The preceding sections suggest two alternative methods of correcting for eight misalignment at a given range, dive angle, and air speed: (a) direct adjustment of the sight until flight and sight lines coincide; or (b) integrator adjustment, i.e., changing \( T_c \) by a percentage \( 

\% \) determined from Figure 3.

The adjustment required, however, depends on either case on the values of \( \phi_a \) and \( \sqrt{a} \), and in case (b) on \( S \) as well. Since it is not feasible to make frequent readjustments of either type in any one plane, in practice a fixed adjustment must be made, such as would completely offset the sight error at some chosen intermediate or normal value of \( S, \phi_a, \) and \( \sqrt{a} \). At other values an impact error will generally result, the magnitude of which depends on the method used -- a fixed sight adjustment, a fixed integrator adjustment, or some combination of the two. The most efficient method is of course the one which minimizes this impact dispersion.

The following example illustrates the results of a fixed integrator adjustment: The sight installed in an FV plane at nominal weight (12,400 lbs.) is misaligned by -20 miles at slant range \( S = 7500 \) feet, dive angle \( \phi = 40° \), and air speed \( \sqrt{a} = 350 \) knots. A fixed percent-adjustment in \( T_c \) is made in an amount of 908.5

* As obtained from OARD Reports 226-4 (CIT/UNC 3), 2271-2775 (CIT/UNC 48), 2563 (CIT/UNC 28), and 2547 (CIT/UNC 28).
...sufficient to offset the -20 mil sight error at \((\vec{S}_i, \vec{d}, \vec{V})\), causing a hit. It is desired to determine the resulting impact error at slant range \(S_i = 12,000 \) feet, dive angle \(d_i = 60^\circ\), and air speed \(V_i = 375 \) knots.

From Figure 3 the required percent-adjustment in \(T_c\) per mil at 7000 feet, 40\(^\circ\), and 300 knots (Point \(P\)) is \(E_c/\phi = 0.78\)% per mil, so that for \(\phi = -20\) alls the fixed adjustment is \(E_c = -15.0\)%.

In other words, under these conditions the adjustment causes a 15% decrease in \(T_c\), by itself, would displace the impact point by an amount equal and opposite to the displacement which would result from a -20 mil sight error alone.

At another range, dive angle, and air speed these two displacements, while oppositely directed, will generally be unequal in magnitude; and in addition, the angle of attack will generally be different. Thus the impact error at \((S_i, d_i, V_i)\) will be the algebraic sum of three displacement components: (a) the difference \(\delta\phi_a\) between the angles of attack at \((d, V)\) and \((d_i, V_i)\) is a change in sight error and therefore causes an impact displacement \(\delta a\); (b) the original sight error \(\phi = -20\) mile, by itself, would result in a positive (beyond-target) impact error \(\delta\phi\); (c) the adjustment \(E_c = -15\)% by itself, would give a negative (short-of-target) error \(\delta c\).

(a) The difference in angle of attack is determined from Figure 5. For an F6F plane at nominal weight, this nomogram gives \(\phi_a = 14.9\) mile at \(d = 40^\circ\), \(V = 300\) knots, and 8.5 mile at \(d_i = 60^\circ\), \(V_i = 375\) knots. Hence \(\delta\phi_a = -6.4\) mile.

But from Figure 2, at \(d_i = 60^\circ\) and \(S_i = 12,000\) feet (Point \(Q\)), \(-\phi/\phi = \) about 14 feet per mil, so that the impact displacement due to change in angle of attack is \(\delta a = 490\) feet.

(b) From this same ratio \(-\phi/\phi = 14\) feet per mil, it follows that the impact error in itself causes a target displacement \(\delta\phi = -490\) feet.

(c) Finally, at 12,000 feet, 60\(^\circ\), and 375 knots, the value of \(\delta c/\delta c\) may be read from Figure 4. As there defined, \(\gamma = (375/350)^2 = 1.14\), so that at 375 knots, 12,000 feet = 12,000 \(\gamma = 10,450\) n feet. The point \(Q\) determined by this slant range and 60\(^\circ\) dive angle lies between the curves marked 10a and 10b, slightly nearer the latter, so that approximately \(\delta c/\delta c = 15a = 14.9\) ft. per \(\%\). The -15.0% adjustment in \(T_c\) therefore displaces the impact point by about \(\delta c = -220\) feet.

The resultant impact error at \((S_i, d_i, V_i)\) is then

\[
\delta a + \delta\phi + \delta c = 490 + 1490 -220 \approx 1850\ 	ext{feet};
\]

i.e., the bomb may be expected to fall 1850 feet beyond the target under the given conditions.

The effect of angle of attack variation may be seen to be of considerable importance. If \(\delta a\) were neglected in this example, (b) and (c) alone would yield an impact error of 180 feet, or only 40% of that given by (14). Quite often even the algebraic sign of the resultant error is changed by this effect.

* Determined by extending line VII and using lower extension scale.
According to the wording of the hypothesis, this example may at first appear to typify only the integrator adjustment method. But if the original misalignment were given not as -30° but as -30 miles, of which 10 miles were corrected by a sight adjustment and the balance by an integrator adjustment, the solution would be identical, yet the problem would appear more general, involving a combination of sight and integrator adjustments. The above example illustrates the method of determination of the extent to which a fixed adjustment fails to offset a sight misalignment at ranges, dive angles, and air speeds other than the central value for which the adjustment is made. The mode and extent of variation of this residual impact error with range, dive angle, and air speed is illustrated in Figures 6, 7, and 8, for both the sight and integrator adjustment methods and various combinations of the two. In these graphs are plotted the results of fixed adjustments as determined as to eliminate completely the effect of the sight error at the modal values \( R = 7500 \) feet, \( \alpha = 40^\circ \), \( V = 350 \) knots, as in the example above. They are based on Figures 2 - 5 and on the formulas underlying them. Explanation of the derivation, interpretation, and use of Figures 6, 7, and 8 follows:

At \( \alpha = \bar{\alpha} \) and \( V = \bar{V} \), let \( \phi \) be that portion of the original sight error which remains after any direct sight adjustment is made; \( \phi \) is then the error to be offset by an integrator adjustment. If this latter adjustment consists of a change in \( T \) in the ratio \( \varepsilon \), then by (9),

\[
\varepsilon \phi = \phi \otimes \phi,
\]

where \( \otimes \) is the value of \( \otimes \) when \( \phi = \bar{\phi} \) and \( V = \bar{V} \). As determined in the example, \( \otimes = 0.75\% \) per mil.

At any other value of \( \phi \), \( \alpha \), and \( V \), the error \( \delta \) on the ground consists of three components analogous to those lettered (a), (b), (c), in the example above: (a) By (2) and (15), the impact displacement due to change \( \Delta \phi \) in angle of attack is

\[
\delta_a = \delta - \frac{\Delta \phi}{\sin \alpha} - \frac{\bar{C}^l}{\alpha \sin \alpha} \left( \frac{\cos \alpha - \cos \bar{\alpha}}{\bar{V}^2} \right),
\]

where \( C \), \( \bar{C} \), \( \bar{\alpha} \), and \( \bar{V} \) are known constants. (b) Again by (2), that due to the modal sight error \( \bar{\phi} \) alone is

\[
\delta_{\bar{\phi}} = -\delta \phi \csc \alpha,
\]

(c) Finally, by (4) and (15), that due to the integrator adjustment \( \varepsilon \phi \) alone is

\[
\delta_{\varepsilon \phi} = \delta \varepsilon \phi \sin \alpha = \delta \phi \csc \alpha \Theta \overline{\phi},
\]

The resultant error \( \delta \) is the sum of (16a), (16b), and (16c):

\[
\delta = \delta_a + \delta_{\bar{\phi}} + \delta_{\varepsilon \phi} = \delta \phi \csc \alpha \left[ \Delta \phi - \phi (\Theta - 1 + \overline{\phi} / \overline{\phi}) \right],
\]

where \( \Delta \phi \) is a function of \( \phi \) and \( V \), and \( \Theta \) is a constant.
When the entire correction is made through a sight adjustment, then \( \phi = 0 \) and hence \( \delta = \delta_a \) as given by (16a). Figure 6 is a contour map of \( \delta_a \) as a function of release point, for air speeds 150 - 25 knots, for an F6F plane (at nominal weight) whose sight is adjusted for \( \alpha = 40^\circ \) and \( \nu = 750 \) knots. It will be noted that the curves for any one air speed approach parallelism in the direction corresponding to the dive angle \( \alpha \) for which \( \delta_a = 0 \) at that air speed — i.e., for which \( \cos \alpha / \sqrt{2} \cos 40^\circ / (350 \text{ knot}) \). While constructed on the basis of F6F data, the errors given by Figure 6 are accurate to within 2% for the SB2C, and to within 15% for the TBF and F4U. The actual value of \( \delta_a \) for any type of plane is obtainable from Figure 6 and the relation

\[
\delta_a = \delta_a(\psi \theta) \left[ \frac{cw/c}{Ef} W_{F6F} \right],
\]

which follows from (16a).

When, on the other hand, part or all of the correction is made through an integrator adjustment (\( \psi \theta \)), \( \delta \) is the sum of this \( \delta_a \), as plotted in Figure 6, and an error \( \Delta \delta = \delta_a + \delta_c = \delta_a + \cos \psi \theta \cos \phi \), proportional to \( \phi \). which would result if angle of attack were constant. The amount of this second error per mil, i.e., \( \Delta \delta / \phi \) is plotted pictorially in Figure 7 as a function of release point, again for air speeds 350 - 25 knots. The 350-knot curve marked \( \Delta \delta / \phi = 0 \) is of course identical with that curve for the family of Figure 3 which passes through the point \( \tilde{F} = 7500 \text{ feet, } \phi = 40^\circ, \nu = 350 \text{ knot} \), and is therefore the locus of \( \Delta \delta / \phi = \) 0.75 ft per mil.

The residual error \( \delta \) for the general case is given by the sum of the errors as given in Figures 6 and 7, the latter being a function of the sight misalignment \( \psi \) to which the integrator adjustment is applied. When the sight itself is adjustable, \( \psi \) may be given any pre-determined value by means of a preliminary sight adjustment. Figure 8 is a set of contours of \( \delta \), obtained by combining Figures 6 and 7, for different values of \( \psi \) between -20 and +20 mils. The scale is the same as in Figures 6 and 7, but ranges and dive angles outside the most probable operating limits are excluded in order to facilitate analysis of the errors likely to be encountered in practice. The central graph (c), which is for \( \psi = 0 \), is identical with Figure 6, except that the contour interval used in Figure 8 is 50 feet.

The complete solution of the above example is obtainable from Figures 6, 7, and 8. The point \( \tilde{F} \) in Figures 6, 7, and 8(a) represents \( \phi = 12,000 \text{ feet, } \phi = 60^\circ \). Interpolation between consecutive error curves for \( \psi = 375 \text{ knots, with } \phi = -20 \text{ mils, gives respectively } \delta_a = 90 \text{ feet, } \Delta \delta = \delta_\psi + \delta_c = (-3 \text{ feet per mil}) \times (-20 \text{ mils}) = 60 \text{ feet, } \delta \approx 150 \text{ feet, in exact agreement with the solution (14).}

A study of Figure 8 sheds much light on the relative merits of the sight adjustment and integrator adjustment methods and of various combinations thereof. The general trend of Figure 8 may be predicted from Figures 6 and 7.

* The operating limits assumed here are as follows: Minimum dive angle 20°, minimum slant range 4000 feet; maximum dive angle 60°, maximum second altitude 10,000 feet, and maximum slant range that which, according to Report CD-28-105 (Fig. 2) would correspond to 100-foot horizontal error at 150 knots for Mid -0 bomb director. The point \( \tilde{F} \) \( ( \tilde{F} = 7500 \text{ ft., } \phi = 40^\circ \) is close to the “center of gravity” of this operating region.
When either $\alpha$ or $V$ increases, according to Figures 6 and 7, $\delta_a$ increases, whereas $\Delta \delta$ decreases or increases according as $\phi$ is positive or negative; these two error components therefore partially offset each other, as regards variation with $\alpha$ and $V$, when $\phi$ is positive, but increase the total error when $\phi$ is negative. This observation is borne out quantitatively by Figure 8.

At $\phi = -20$ miles (Figure 8a) the curves are densely packed, indicating wide variation of residual error; within the operating limits and for $\pm 25$-knot variation in $V$, $\delta$ is seen to vary approximately between -200 and 150 feet. As $\phi$ increases the curves become sparser, until at $\phi = \pm 30$ (Figure 8d) $\delta$ becomes practically independent of air speed. At $\phi = \pm 10$ miles (Figure 8d), while being somewhat more dependent on air speed, $\delta$ is limited to 50 feet throughout almost the entire operating region and at a fixed slant range seems to be almost independent of $\phi$. The corresponding variation at $\phi = 0$ (Figure 8e) is also between the limits of 50 feet, and more independent of slant range. In all three curves (a), (d) and (e), the worst error conditions occur in the vicinity of 35° and maximum range, the bomb falling short by an amount which approaches 100 feet. These results would, of course, be altered somewhat if different values of $\delta$, $\alpha$, and $V$ were used in the adjustment, but the general trend would be entirely similar.

**CONCLUSIONS**

Aircraft whose sights are adjusted to be correct for machine gun use have sight misalignments which average about -30 miles for those bombing. In view of Figure 6, correction of errors of such magnitude by means of an NPI ($\Delta \delta_{1c}/\Delta \delta_{2c}$) adjustment alone is highly disadvantageous, as this method makes the impact point very sensitive to changes in range, dive angle, and air speed. Adjustment of the sight itself brings much more accurate performance within wide limits, and seems to give an adjustment roughly independent of slant range.

Equally favorable results, however, may be achieved if a combination of sight and integrator adjustments is used. In this method the sight is over-adjusted until misaligned by about $\pm 10$ to $\pm 20$ miles, and this misalignment is in turn corrected by an NPI adjustment. The optimum sight adjustment depends on the choice of a modified value of $\delta$, $\alpha$, and $V$, but when these are 7500 feet, 45°, and 350 knots, the minimum variation of impact error with dive angle is reached at an overadjustment of approximately 40 miles; with air speed at approximately $\pm 30$ miles, and with slant range at approximately 5 miles (sight adjustment alone).

The effect of this combined adjustment is to offset the effect of changes in angle of attack by means of an integrator adjustment so determined as to counteract such variation. An additional advantage of this method lies in the fact that, in practice, steeper dives are generally accompanied by greater air speeds. Since an increase in either dive angle or air speed increases the error due to changes in angle of attack, it follows that a sight adjustment alone would make the impact point doubly sensitive to changes in $\alpha$ or $V$. The combination method reduces the variation of $\delta$ with both $\alpha$ and $V$. However, *except for small dive angles.*
If the minimization of the variation of $\hat{S}$ with slant range at fixed values of $\hat{\alpha}$ and $\hat{V}$ is considered more important, only the sight adjustment, i.e., $\phi = \sigma$, should be made.

The above choice of modal values $\hat{S}$, $\hat{\alpha}$, $\hat{V}$, about which the adjustments are to be made, appears to be satisfactorily centralized with respect to usual operational limits. Any decrease in $\hat{S}$ below 7500 feet would make positive values of $\Delta \hat{S} / \phi$ (Figure 7) considerably more probable than negative values; and a slight increase, perhaps to 8500 feet, would yield a somewhat better balanced distribution of errors.

In practice, however, greater accuracy in the sight adjustment is achieved at short ranges. Since angle of attack is independent of range, adjustment of the sight at $\hat{\alpha}$, $\hat{V}$, and a range less than $\hat{S}$ would not affect the result. The NPI adjustment ($\Delta T_{c} / T_{c}$) could then be made at $\hat{S}$, $\hat{\alpha}$, $\hat{V}$.

It is also recommended that this combination method of sight correction be tested in the field, to determine the agreement of theory and practice at different values of $\phi$. In any such tests, however, one caution is believed to be in order. The impact errors with which we are here concerned are comparable in magnitude with errors due to such factors as pilot aiming and the use of approximate $\psi$-functions. A difference between two observed impact errors, for the same plane under different conditions, will be approximately free of components due to other factors, and is therefore much more reliable for this purpose than any single absolute observed error.

The results of this analysis, furthermore, de-emphasize the necessity for exact alignment of the flight and sight lines. An approximate alignment will generally suffice provided the error is on the side of over-adjustment, which is subsequently corrected through an NPI adjustment.
FLIGHT AND TRAJECTORY DIAGRAM WITH MISALIGNED SIGHT

Figure 1
Spatial Contour Map of:
HORIZONTAL IMPACT ERROR ($\delta_h$) RESULTING FROM A GIVEN SIGHT ERROR ($\phi$)

(Range-dive angle contours of constant ratio $\delta_h/\phi$, as indicated)

Independent of air speed ($V$) and of pull-up acceleration ($K$)

Figure 2
Spatial Contour Map of:

MPI ADJUSTMENT ($\epsilon$), i.e., PERCENT CHANGE IN TIME TO TARGET, REQUIRED TO OFFSET GIVEN SIGHT ERROR ($\phi$)

<table>
<thead>
<tr>
<th>V-air speed</th>
<th>$n = (v/350\text{ knots})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 knots</td>
<td>1</td>
</tr>
<tr>
<td>350 knots</td>
<td>1</td>
</tr>
<tr>
<td>250 knots</td>
<td>2</td>
</tr>
</tbody>
</table>

(Range-dive angle contours of constant ratio $\epsilon/\phi$)

No. 3 (Flight dependence on $\gamma$)

Figure 3

180 knots
500 knots
250 knots

$\epsilon/\phi$ in $\%$ per mile: 0.5
Spatial Contours of
HORIZONTAL CAPTURE (r) AT H.O.V. AND DIVE PERCENT CHANGE (δ) IN
TIME TO TARGET

(Range-45° elevation contour of constant rate δ/c_c)

K-8 (Height dependency on K)
Relative Angle of Attack Nomogram

For determining changes in angle of attack

Example: For F6F plane with its nominal weight of 2,400 lbs.

1. At 350 knots (dot-dashed lines),
   \[ \phi = 14.9 \text{ mls when } \alpha = 40^\circ \]
   \[ \phi = 16.9 \text{ mls when } \alpha = 30^\circ \]
   Hence decreasing dive angle from 40° to 30° increases angle of attack by 2.0 mls.

2. At 40° dive (solid lines),
   \[ \phi = 15.9 \text{ mls when } V = 350 \text{ knots, } 20.9 \text{ mls when } V = 300 \text{ knots, } 
   \]
   Hence decreasing air speed from 350 to 300 knots increases angle of attack by 5.5 mls.

When weight of plane differs from nominal weight as given on line III, determine new point on line III by joining appropriate point on line I with point on II representing correct weight.
Spatial Contour Map of:
HORIZONTAL IMPACT ERROR (\(S_a\)) RESULTING FROM CHANGES IN ANGLE OF ATTACK, WHEN SIGHT IS PROPERLY ALIGNED AT DIVE ANGLE 40° AND AIR SPEED 350 KNOTS.

(Range-dive angle contours of constant \(S_a\), as indicated)

**AIR SPEEDS**
- 325 knots
- 350 knots
- 375 knots

**DIVE ANGLE**
- 45°
- 60°
- 75°
- 90°

**P&W Plane**
- Gross Wt.: 18,400 lbs.
- (Independent of \(K\))

**HORIZONTAL RANGE IN KILOFEET**
- 0°
- 15°
- 30°
- 45°
- 60°

Figure 8
Spatial Contour Map of
HORIZONTAL IMPACT ERROR ($\Delta \delta = \delta_x + \delta_c$), NOT INCLUDING THAT DUE TO ANGLE OF ATTACK,
RESULTING FROM FIXED KPI ADJUSTMENT ($\delta_c$) SO DETERMINED AS TO OFFSET A SIGHT
ERROR ($\phi$) AT SLANT RANGE 7.5 KILOMETERS, DIVE ANGLE 45°, AND AIRSPEED 350 KNOTS

(Range-dive angle contours of constant $\Delta \delta/\phi$, as indicated)

Airspeeds
325 knots
350 knots
375 knots

Figure 7
Spatial Contour Map (composite of Figures 6 & 7) of:
HORIZONTAL IMPACT ANGLE ($\theta = 8 \theta + 20$) RESULTING FROM
MIXED INT. (SWEEPS 80 DUE W. AT 100 UPON A
SIGHT AT $\phi$) AT SLANT 20° VELOCITY 75 KILOMTR.
258, 300, 40°, AND 375 KNOTS
(Range-sive angle contours of constant $\theta$, within operational limits,
given in units of 50°)

AIR SPEEDS

FGF Plane
Cross St. 12, 000 lbs.

K=3

\[ \phi = -20 \text{ mils} \]

\[ \phi = -10 \text{ mils} \]

\[ \phi = 0 \]

Figure 8

5008-19
Key

10,000' Altitude

*(\(12,000'\), short range)

60° dive angle

\(90°\)

20°

4000'

Theoretical maximum range

for Mod 0 bomb director

at 350 knots
THE END

Microfilmed
by
The Library of Congress
Photoduplication Service
Because of our limited supply, you are requested to return this copy when it has served your purpose so that it may be made available to other requesters. Your cooperation will be appreciated.

Notice: When Government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded or any other implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.