THE PHOTOELASTIC DETERMINATION OF STRESS ON TRANSVERSE PLANES OF SYMMETRY FOR THE GENERAL AXI-SYMMETRIC CASE

by

E.A. FOX

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Abstract

The method of O'Rourke and Saenz of treating the gross retardation patterns of symmetrically strained cylinders and spheres as an Abel integral equation is combined with one scattered light measurement to provide a complete solution on transverse planes of symmetry for the general axi-symmetric problem. A simple expression is derived for three-dimensional "notch stresses."

I. Introduction

The standard three-dimensional photoelastic techniques: freezing-slicing, and scattered light probing, have intrinsic limitations. Slicing is destructive of the model, probing requires a multiplicity of measurements, and neither is easily adapted to dynamic loading.

The idea of determining the interior stresses from the integrated relative retardation pattern is tantalizing and has been pursued by several investigators. Poritsky\(^1\) achieved a solution for cylindrical bars in a state of plane strain; Read\(^2\), a solution for cylindrical glass bulbs under restrictive conditions. Kamerer\(^3\) established the integral equation for the relative retardation in the axi-symmetric case from Neumann's\(^4\) equations.

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1 H. Poritsky, Physics 5, 406-411 (1934).
O'Rourke and Saenz\textsuperscript{5} observed that this equation is Abel's integral equation. They concerned themselves with axially symmetric plane strain in long cylinders and radially symmetric stress in spheres, but required a very restrictive sum rule, $C_2 = C_1 + C_3$, in the case of cylinders to obtain a solution. This restriction was removed by Saenz\textsuperscript{6} by interferometric data and by Drucker and Woodward\textsuperscript{7} by the use of oblique incidence.

The following discussion will apply to the general axi-symmetric case on a plane of transverse symmetry of an elastic body.

\section*{II. Preliminary Equations}

Let the normal to the wave front be parallel to the $\gamma$-axis, then the Maxwell-Neumann stress optic law\textsuperscript{8} relates the relative retardation $\delta$, the principal stresses in the plane of the wave front $\rho$, $q$ ($\rho > q$), the orientation $\phi$ of $q$ with respect to $x$, and $y$, the arc tan of the amplitude ratio of the two transmitted waves, as follows:

$$\frac{\partial \delta(x, y, z)}{\partial y} + C(q - q) = 2 \frac{\partial \phi}{\partial y} \cot 2y \sin \delta$$

\begin{equation}
\frac{\partial y}{\partial y} = - \frac{\partial \phi}{\partial y} \cos \delta
\end{equation}

where $C$ is the stress-optic coefficient.

\textsuperscript{5} R. C. O'Rourke and A. W. Saenz, Quart. Appl. Math. 8, 303-311 (1950).
\textsuperscript{6} R. C. O'Rourke, J. Appl. Phys. 22, 872-878 (1951).
\textsuperscript{7} A. W. Saenz, J. Appl. Phys. 21, 962-965 (1950).
Let the axis of symmetry be \( Z \). Then, in a usual notation, the following relations hold:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{2}
\]

\[
\frac{\partial \sigma_\theta}{\partial z} + \frac{\partial \tau_{\theta\phi}}{\partial r} + \frac{1}{r} \tau_{r\phi} = 0 \tag{3}
\]

\[
\frac{\partial u}{\partial r} = \epsilon_r = \frac{1}{E} \left[ \sigma_r - \nu(\sigma_\theta + \sigma_\phi) \right] \tag{4}
\]

\[
\frac{u}{r} = \epsilon_\theta = \frac{1}{E} \left[ \sigma_\theta - \nu(\sigma_r + \sigma_\phi) \right] \tag{5}
\]

where \( u \) is the displacement in the \( r \) direction. Eliminating \( u \) between (4) and (5) we obtain:

\[
\frac{\sigma_r - \sigma_\theta}{r} = \frac{1}{1 + \nu} \frac{\partial}{\partial r} \left[ \sigma_\theta - \nu(\sigma_r + \sigma_\phi) \right] = \frac{E}{1 + \nu} \frac{\partial}{\partial r} \left( \frac{u}{r} \right) \tag{6}
\]

In the plane of the wave front

\[
P - q = \left[ \left( \frac{\sigma_r}{r^2} + \frac{\sigma_\theta}{r} - \sigma_\phi \right)^2 + q \frac{\tau_{r\phi}}{r^3} \right] \frac{1}{4} \tag{7}
\]

\[
\sin \phi = \frac{2 \tau_{r\phi}}{P - q} \frac{x}{r} \tag{8}
\]

### III. Transverse Plane of Symmetry

Let \( Z = 0 \) be a transverse plane of symmetry. Let \( a, b \) be the inner and outer radii, respectively, of the section of the body cut by \( Z = 0 \).

On \( Z = 0 \), \( \tau_{r\phi} = 0 \). Hence, from (8), \( \phi = 0 \); therefore (1) and (7) become

\[
\sigma_r = \sigma_\theta = 0 \tag{9}
\]

\[
P - q = \left[ \left( \frac{\sigma_r}{r^2} + \frac{\sigma_\theta}{r} \right)^2 + q \right] \frac{1}{4} \tag{10}
\]

\[
\sin \phi = \frac{2 \tau_{r\phi}}{P - q} \frac{x}{r} \tag{11}
\]

---

\[
\frac{\partial \delta(x, y)}{\partial y} - C(p - q)_{\xi} = C \left[ \sigma_x - \sigma_r \frac{x}{r^2} - \sigma_\theta \frac{y}{r^2} \right]_{\xi = 0} \tag{9}
\]

Consider a pencil of circularly polarized light along the path \( x = 0 \).

Then (9) becomes
\[
C \left( \sigma_x - \sigma_r \right)_{\xi = 0} = \frac{\partial \delta(x, y)}{\partial y} = \frac{\partial \delta(r, o)}{\partial r} = S(r) \tag{10}
\]

where \( S(r) \) is Weller's\(^{10}\) scattered light function which is inversely proportional to the spacing of the interference fringes viewed normally to the light path.

Let \( R(x, z) \) be the two-dimensional map of the integrated relative retardation. Put (6) and (10) in (9) and integrate across the chord with respect to \( \gamma \). Let \( t = \frac{b^2 - x^2}{2} \) be the half chord length. Then since all functions are even in \( \gamma \)

\[
R(x, o) = 2 \int_{0}^{t} S(r) \, d\gamma - 2 C \frac{E}{1 + \nu} x^2 \int_{0}^{t} \frac{\, d}{r} \left( \frac{u(r, o)}{r} \right) \, d\gamma
\]

Changing the variable of integration to \( r \) and transposing, there results

\[
\frac{1 + \nu}{2 E} \frac{1}{x^2} \left\{ 2 \int_{0}^{x} \frac{r S(r)}{\sqrt{r^2 - x^2}} \, dr - R(x, o) \right\} = \int_{x}^{b} \frac{\, d}{\sqrt{r^2 - x^2}} \left[ \frac{1}{r} \left( \frac{u(r, o)}{r} \right) \right] \, dr \tag{11}
\]

This is Abel's integral equation,\(^{11}\) which, since the left hand side of (11) vanishes at \( x = b \), has the unique continuous inverse

\[
\frac{d}{dr} \left( \frac{u(r, o)}{r} \right) = \frac{1 + \nu}{\pi E c} \int_{0}^{b} \frac{r S(r)}{\sqrt{r^2 - x^2}} \, dr \quad \text{at} \quad x = \frac{1 + \nu}{\pi E c} \int_{r}^{b} M(r) \, dr \tag{12}
\]

---


where \( M(r) \) is an experimentally determined function. Integrate (12) with respect to \( r \). Then, since \( M(b) = 0 \),

\[
U(r, a) = r \left[ \frac{1}{1 + \nu} \int M(r) + \frac{U(b, a)}{b} \right]
\]

where \( U(b, a) \) is determined by measurement or is computed. (See Section IV.)

Sum (4) and (5), yielding

\[
\sigma_r + \sigma_\theta = \frac{1}{1 + \nu} \left[ E \frac{1}{r} \frac{d}{dr} (ru) + 2 \nu \sigma_r \right]
\]

Finally, solving (6), (10), and (14)

\[
\begin{align*}
\sigma_r(r, a) &= \frac{E}{(1 + \nu)(1 - 2\nu)} \left[ \frac{U(r, a)}{r} + \nu \frac{dU(r, a)}{d r} \right] + \frac{1 - \nu}{C(1 - 2\nu)} S(r) \\
\sigma_\theta(r, a) &= \frac{E}{(1 + \nu)(1 - 2\nu)} \left[ \frac{U(r, a)}{r} + \nu \frac{dU(r, a)}{d r} \right] + \frac{\nu}{C(1 - 2\nu)} S(r) \\
\sigma_r(r, a) &= \frac{E}{(1 + \nu)(1 - 2\nu)} \left[ 2 \nu \frac{U(r, a)}{r} + (1 - \nu) \frac{dU(r, a)}{d r} \right] + \frac{\nu}{C(1 - 2\nu)} S(r)
\end{align*}
\]

where \( U(r) \) is given by (13), and \( S(r) \) by (10).

**IV. Determination of \( U(b, a) \)**

\( U(b, a) \) may either be measured or computed as follows: Put (2) in (9) and integrate with respect to \( \eta \).

\[
\int_0^1 \frac{1}{2C} R(x, a) = \int_0^t \left[ \sigma_r - \sigma_\theta - \frac{3C}{2r} U - \frac{3C}{2} \frac{\partial U}{\partial \eta} \right] d\eta
\]

Integrate the second term by parts, then after some manipulation (See Appendix.)

\[
\frac{1}{2C} R(x, a) = \int_0^b \frac{\sigma_r(r, a) - \sigma_r(b, a) - \frac{\partial U(r, a)}{\partial \eta}}{\sqrt{r^2 - x^2}} \sqrt{r^2 - x^2} d\eta
\]
This is again Abel's integral equation with the unique inverse

\[ G(x, y) = \frac{1}{\pi y} \int_{-\infty}^{\infty} \frac{f(t)}{y - t} dt \]

Put (2) in (6), then

\[ \frac{\partial}{\partial r} \left( \sigma_r + \sigma_\theta \right) = \nu \frac{\partial \sigma_\theta}{\partial r} - (1 + \nu) \frac{\partial \tau}{\partial x} \]

Integrate (20) with respect to \( r \). Then this with (5), (14), and (19) yields

\[ E \frac{d}{dr} \left[ r u(r, \theta) \right] = E (1 - \nu) \frac{u(b, \theta)}{b} + \frac{1 - \nu}{\pi E} \int_{a}^{b} \frac{R(x, \theta)}{\sqrt{x^2 - a^2}} dx + \frac{(1 + \nu)(1 - 2\nu)}{b} G_\theta (r, \theta) \]

or, integrating,

\[ b u(b, \theta) - a u(a, \theta) = \frac{1 - \nu}{2} \frac{u(b, \theta)}{b} (b - a) - \frac{1 - \nu}{\pi E} \int_{a}^{b} \frac{R(x, \theta)}{\sqrt{x^2 - a^2}} dx \]

\[ + \frac{(1 + \nu)(1 - 2\nu)}{bE} \frac{P}{2\pi} \]

where \( P = 2 \pi \int_{a}^{b} \sigma_x (r, \theta) r dr \) = total axial force across \( z = 0 \).

Put (13) in (21), obtaining

\[ u(b, \theta) = \frac{2b}{(1 + \nu)(1 - \nu)} \left[ a \frac{1 - \nu}{\pi E} \int_{a}^{b} \frac{R(x, \theta)}{\sqrt{x^2 - a^2}} dx + \frac{(1 + \nu)(1 - 2\nu)}{bE} \frac{P}{2\pi} \right] \]

If in particular \( a = 0 \), then

\[ u(b, \theta) = \frac{1}{b\pi E} \left[ (1 - \nu) P - \frac{2(1 - \nu)}{b} \int_{a}^{b} R(x, \theta) dx \right] \]
V. Alternate Expression for $\sigma_z(b, o)$

If it is desired to obtain only the axial stress on the outer boundary of the section $Z = 0$, then a simple expression may be derived directly from the Maxwell-Neumann law. Consider (9). Let $x \to b$. Then $r \to b$, $t \to y \to o$, $\delta(x, y, o) \to \delta(b, o, o) = \frac{1}{2} R(b, o)$, Replot $R(x, o)$ as $R(t, o)$ where $t = (b - x')^t$. Then in the limit

$$\sigma_z(b, o) = \frac{1}{2C} \left[ \frac{dR(t, o)}{dt} \right]_{t=0} + \sigma_r(b, o)$$

where $\sigma_r(b, o)$ is the normal surface traction at $(b, o)$. Thus $\sigma_z(b, o)$ is proportional to the gradient at the boundary of the integrated relative retardation reckoned as a function of the half light path. We observe that (23) is consistent with (19) if $R$ is made a function of $t$. Further, if $u(b, o)$ is known, then (5) and (23) yield

$$\sigma_0(b, o) = E \frac{u(b, o)}{b} + \nu \left\{ 2 \sigma_r(b, o) + \frac{1}{2C} \left[ \frac{dR(t, o)}{dt} \right]_{t=0} \right\}$$

Acknowledgment

The author wishes to thank Professor R. D. Mindlin for suggesting this investigation and for his advice during its course.
Appendix

Derivation of Equation (18)

Integrate (9) with respect to $\eta$.

$$\frac{1}{2C} R(x,0) = \int_0^t \left[ \sigma_t^2 - \sigma_r \sigma_x - \sigma_b \frac{u^1}{r} \right] \, d\eta = \int_0^t \left[ \left( \sigma_t \cdot \sigma_r + (\sigma_r - \sigma_b) \frac{u^1}{r} \right) \right] \, d\eta$$

Then, with (2),

$$\frac{1}{2C} R(x,0) = \int_0^t \left[ \sigma_t^2 - \left( \frac{\partial \sigma_r}{\partial r} + \frac{\partial \sigma_x}{\partial x} \right) \frac{u^1}{r} \right] \, d\eta$$

Integrate the second term by parts, yielding

$$\frac{1}{2C} R(x,0) = \int_0^t \left[ \sigma_t^2 - \frac{\partial \sigma_r}{\partial r} \right] \, d\eta - \left[ \eta \sigma_r \right]_0^t + \int_0^t \left[ \eta \frac{\partial \sigma_r}{\partial r} \right] \, d\eta$$

but $\frac{\partial \sigma_r}{\partial r} = \frac{1}{r}$ and $\left[ \eta \sigma_r \right]_0^t = \int \sigma_r (b,0) \, d\eta$, hence

$$\frac{1}{2C} R(x,0) = \int_0^t \left[ \sigma_t^2 - \sigma_r (b,0) - \frac{\partial \sigma_x}{\partial x} \frac{u^1}{r} \right] \, d\eta$$

Consider

$$\int_0^t \frac{\partial \sigma_x}{\partial x} \frac{u^1}{r} \, d\eta = \int_0^t \left\{ \frac{d}{dr} \left[ \frac{r \sigma_x}{\partial x} \right] + \frac{\partial \sigma_x}{\partial x} \right\} \frac{u^1}{r} \, d\eta = \int_0^t \left\{ \frac{d}{d\eta} \left[ \frac{r \sigma_x}{\partial x} \right] + \frac{\partial \sigma_x}{\partial x} \right\} \frac{u^1}{r} \, d\eta$$

Thus

$$\frac{1}{2C} R(x,0) = \int_0^t \left[ \sigma_t^2 - \sigma_r (b,0) - \int \frac{\partial \sigma_x}{\partial x} \, dr \right] \, d\eta$$

Then, changing the variable of integration to $r$, we obtain (18).

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<td>Mr. E. A. Gerber</td>
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<td>Mr. Martin Goland</td>
<td>Midwest Research Institute</td>
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<td>Kansas City 2, Missouri</td>
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<td>Professor E. Fried</td>
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Contractors and Other Investigators Actively Engaged in Related Research (cont.)

Professor M. Petényi
Northwestern University
Evanston, Illinois

Professor T. J. Higgins
Dept. of Electrical Engineering
University of Wisconsin
Madison, Wisconsin

Professor N. J. Hoff
Dept. of Aeronautical Engineering
Polytechnic Institute of Brooklyn
Brooklyn 2, New York

Professor M. B. Hogan
University of Utah
Salt Lake City, Utah

Professor D. L. Holl
Iowa State College
Ames, Iowa

Dr. J. H. Holliomon
General Electric Research Labs.
1 River Road
Schenectady, New York

Professor W. H. Hoppmann
Dept. of Applied Mechanics
The Johns Hopkins University
Baltimore, Maryland

Dr. Gabriel Horvay
Knolls Atomic Power Laboratory
General Electric Company
Schenectady, New York

Institut de Mathématiques
Université
port. fah 55
Skopje, Yugoslavia

Professor L. S. Jacobsen
Dept. of Mechanical Engineering
Stanford University
Stanford, California

Professor Bruce G. Johnston
University of Michigan
Ann Arbor, Michigan

Professor Thomas R. Kane
25-2 Valley Road
Drexel Hill, Pennsylvania

Professor K. Klotter
Stanford University
Stanford, California

Professor W. J. Krefeld
Dept. of Civil Engineering
Columbia University
New York 27, New York

Professor E. J. Lazan
Dept. of Materials Engineering
University of Minnesota
Minneapolis 14, Minnesota

Professor E. H. Lee
Division of Applied Mathematics
Brown University
Providence 12, Rhode Island

Professor George Lee
Rensselaer Polytechnic Institute
Troy, New York

Professor J. M. Lessells
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Library, Engineering Foundation
29 West 39th Street
New York, New York

Professor Paul Lieber
Dept. of Engineering
Rensselaer Polytechnic Institute
Troy, New York

Dr. Hau Lo
Purdue University
Lafayette, Indiana

Professor G. T. G. Looney
Dept. of Civil Engineering
Yale University
New Haven, Connecticut
Contractors and Other Investigators Actively Engaged in Related Research (cont.)

Dr. J. L. Lubkin  
Midwest Research Institute  
4049 Pennsylvania  
Kansas City 2, Missouri  
(1)

Professor J. F. Ludloff  
School of Aeronautics  
New York University  
New York 53, New York  
(1)

Professor J. H. Macduff  
Rensselaer Polytechnic Institute  
Troy, New York  
(1)

Professor C. W. MacGregor  
University of Pennsylvania  
Philadelphia, Pennsylvania  
(1)

Professor Lawrence E. Malvern  
Dept. of Mathematics  
Carnegie Institute of Technology  
Pittsburgh 13, Pennsylvania  
(1)

Professor J. H. Marchant  
Brown University  
Providence 12, Rhode Island  
(1)

Professor J. Marin  
Pennsylvania State College  
State College, Pennsylvania  
(1)

Dr. W. P. Mason  
Bell Telephone Laboratories  
Murray Hill, New Jersey  
(1)

Professor R. D. Mindlin  
Dept. of Civil Engineering  
Columbia University  
612 West 125th Street  
New York 27, New York  
(15)

Dr. A. Nadal  
136 Cherry Valley Road  
Pittsburgh 21, Pennsylvania  
(1)

Professor Paul K. Naghdi  
Dept. of Engineering Mechanics  
University of Michigan  
Ann Arbor, Michigan  
(1)

Professor N. M. Newmark  
207 Talbot Laboratory  
University of Illinois  
Urbana, Illinois  
(1)

Professor Jesse Ormondroyd  
University of Michigan  
Ann Arbor, Michigan  
(1)

Professor W. Osgood  
Illinois Institute of Technology  
Chicago 16, Illinois  
(1)

Dr. George B. Pegram  
313 Low Memorial Library  
Columbia University  
New York 27, New York  
(1)

Dr. H. P. Petersen  
Director, Applied Physics Division  
Sandia Laboratory  
Albuquerque, New Mexico  
(1)

Mr. R. E. Peterson  
Westinghouse Research Laboratories  
East Pittsburgh, Pennsylvania  
(1)

Professor A. Phillips  
School of Engineering  
Stanford University  
Stanford, California  
(1)

Professor Gerald Pickett  
Dept. of Mechanics  
University of Wisconsin  
Madison 6, Wisconsin  
(1)

Dr. H. Poritsky  
General Engineering Laboratory  
General Electric Company  
Schenectady, New York  
(1)

Professor W. Prager  
Graduate Division of Applied Mathematics  
Brown University  
Providence 12, Rhode Island  
(1)

Dr. Frank Press  
Lamont Geological Observatory  
Palisades, New York  
(1)
Contractors and Other Investigators Actively Engaged in Related Research (cont.)

RAND Corporation
1500 4th Street
Santa Monica, California
Attn: Dr. B. L. Add
(1)

Dr. S. Paynor
Armour Research Foundation
Illinois Institute of Technology
Chicago 16, Illinois
(1)

Professor E. Reissner
Dept. of Mathematics
Massachusetts Institute of Technology
Cambridge 39, Massachusetts
(1)

Professor H. Reissner
Polytechnic Institute of Brooklyn
99 Livingston Street
Brooklyn 2, New York
(1)

Dr. Kenneth Robinson
Combustion Engineering, Inc.
200 Madison Avenue
New York 16, New York
(1)

Professor Leif Rongved
Dept. of Engineering Mechanics
Pennsylvania State College
State College, Pennsylvania
(1)

Professor M. A. Sadowsky
Dept. of Mechanics
North Hall
Rensselaer Polytechnic Institute
Troy, New York
(1)

Professor M. G. Salvadori
Dept. of Civil Engineering
Columbia University
New York 27, New York
(1)

Mr. Arnold Schacknor
20-35 Seagirt Boulevard
Far Rockaway, New York
(1)

Dr. F. S. Shaw
Superintendent
Structures & Materials Division
Aeronautical Research Laboratories
Box 431, G.P.O., Melbourne
Victoria, Australia
(1)

Dr. Daniel T. Sigley
American Machine and Foundry Company
511 Fifth Avenue
New York, New York
(1)

Professor C. B. Smith
Department of Mathematics
Walker Hall
University of Florida
Gainesville, Florida
(1)

Professor C. R. Soderberg
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts
(1)

Professor R. V. Southwell
Imperial College of Science and Technology
South Kensington
London S.W. 7, England
(1)

Professor E. Sternberg
Illinois Institute of Technology
Chicago 16, Illinois
(1)

Professor J. J. Stoker
New York University
Washington Square
New York, New York
(1)

Mr. R. A. Sykes
Bell Telephone Laboratories
Murray Hill, New Jersey
(1)

Professor P. S. Symonds
Brown University
Providence 12, Rhode Island
(1)

Professor J. L. Synge
Dublin Institute for Advanced Studies
School of Theoretical Physics
64-65 Merrion Square
Dublin, Ireland
(1)

Professor P. K. Teichmann
Dept. of Aeronautical Engineering
New York University
University Heights, Bronx
New York, New York
(1)

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<table>
<thead>
<tr>
<th>Name</th>
<th>Institution and Address</th>
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<tr>
<td>Professor S. P. Timoshenko</td>
<td>Professor Alexander Weinstein</td>
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