A DESIGN FOR AN INSTRUMENT TO MEASURE
AND COMPUTE THE FREQUENCY DISTRIBUTION
OF FINE-GRAINED SEDIMENTS

by

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and
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INTRODUCTION

Under Contract N6onr-27712 (NR-0844-008), a study was made of the possibility of instrumenting the problem of mechanical analysis using electronic devices. The following account is the result of an effort to explore only the theoretical possibilities and to suggest a practical prototype design.

The determination of the frequency distribution by weight of grain sizes smaller than 0.062 mm. within a sediment is a tedious and time-consuming process. Many attempts have been made to facilitate such measurements (see Krumbein and Pettijohn, 1938). Such a device would soon pay for itself in the man-hours saved by laboratory technicians, and would provide a means of increasing the number of samples analyzed in a given period of time. A better knowledge of sedimentation phenomena would result from such improvements.

By rearranging the equation for obtaining the cumulative frequency of a fine-grained sediment, as this equation is employed in the principle of the Oden Continuous Sedimentation Balance, (Krumbein and Pettijohn, 1938, p. 114 (25)), it was possible to find an expression which could be solved with electronic devices. The instrument designed by the authors employs a settling tube, a sensitive mechanotransducer, a program timing device to establish the required measurement times, two scaling d.c. amplifiers to set limits for the signal voltage levels, a signal voltage retaining or memory stage, two differential amplifiers, a \( \log_{10} \) multiplier or variable gain DC amplifier, and a recording device. The values obtained with this instrument correspond to cumulative weights rather than total accumulated weight as obtained with the Oden Continuous Sedimentation Balance.

THEORETICAL CONSIDERATIONS

The assumption is made that a sediment whose particles vary continuously in size between unknown limits, is uniformly dispersed in water at time \( t_0 \). A pan suspended at level \( h \) in this dispersion will accumulate sediment in such a manner that the total weight of the pan \( W_\text{p} \) will increase continuously with time. Readings of the total weight \( W_\text{p} \) of particles which have fallen on to the pan will form a curve, \( f(t) \), when plotted against time \( t \). According to Stokes equation (Krumbein and Pettijohn, 1938, p. 96), a particle of any given size within this
uniform dispersion will fall with a velocity determined by its size and density, viscosity, density of the water, and the acceleration due to gravity. It is further assumed that particles of given size will settle independently of any other size. At time \( t \) all the particles of a size having a velocity of \( h/t \) will have settled a measured distance \( h \). Those particles of this velocity that were included in the column of water over the pan will have reached the pan along with all the particles having a velocity greater than \( h/t \). Very few of the smaller particles with a velocity less than \( h/t \) will have reached the pan level \( (h) \) but a portion of each of these sizes will be included in the total weight of the sediment collected on the pan.

It can be shown that the partially sedimented portion (velocities less than \( h/t \)) of the total pan weight is equal to

\[
t . f'(t) \quad \text{Krumbein and Pettijohn, 1938}
\]

The expression for the totally sedimented portion (velocities greater than \( h/t \)) is then:

\[
f(t) - t \cdot f'(t) = W_c
\]  

(1)

Since \( f(t) \) cannot be predicted, the equation is in its simplest form. It may be written:

\[
W_T - t \frac{dW_T}{dt} = W_c
\]  

(2)

Where:  
- \( W_T \) = total weight of sediment on the pan at \( t \).  
- \( W_c \) = cumulative weight of the sediment for all sizes with a velocity equal to or greater than \( h/t \), at \( t \).  
- \( t = \) time since \( t_0 \).

In order to find \( W_c \), the desired variable, it is necessary to differentiate \( W_T \) with respect to time. The differential is shown to be:

\[
\lim_{(t_2-t_1) \to 0} \frac{W_T(t_2) - W_T(t_1)}{t_2 - t_1}
\]
However, as \((t_2 - t_1)\) approaches zero, the amount of sediment accumulating becomes very small and, in fact, too small to measure. To insure a measurable quantity of sediment a suitable interval of time should be chosen rather than the instantaneous differential. This interval should be long enough to give a measurable difference without impairing the accuracy of the cumulative weight. Then \(W_T\) is no longer the instantaneous total weight for a given measurement of cumulative frequency, but becomes an approximate value. Nevertheless, it is so close that the error is undetectable in the final computed value for \(W_c\) in the electronic circuitry.

Most electronic circuits have a time base of only a few microseconds when performing differentiation. A memory circuit was devised to find

\[
W_T(t_2) - W_T(t_1)
\]

when \((t_2 - t_1)\) is equal to the chosen interval of time which amounts to milliseconds rather than microseconds.

For the equation to be dimensionally correct, \(t \cdot \frac{dW_T}{dt}\) must equal a weight and \((t)\) and \((dt)\) must be in the same units since \((dt)\) is equal to the chosen interval of time in seconds. The total analysis may take 120 hours or 432,000 seconds. \((t)\) is then expressed in units ranging from 0 - 21,600. It is impractical to multiply by 21,600 electronically. To overcome this problem it is possible to employ logarithms to reduce this excessive range.

let \(t = (\log t')\) \hspace{1cm} \(\text{(3)}\)

and \(dt = d(\log t')\) \hspace{1cm} \(\text{(4)}\)

Then substitute equations (3) and (4) into equation (2):

\[
W_m - \log t' \frac{dW_T}{d(\log t')} = W_c
\]

This limits the range of \((t')\) to about 0-5 units instead of 0-21,600, the range of \((t)\).
Since the time increment over which the measurements are made is the same for all observations, \( d(\log t') \) may be considered a constant and the ratio of \( \log (t') \) to \( d(\log t') \) may be predicted and used as a multiplier of

\[
W_T (t_2) - W_T (t_1) \text{ to determine } W_c \text{ from } W_t.
\]

The writers were unsuccessful in finding a means of multiplying

\[
\frac{dW_T}{\log t'} \quad \text{continuously with sufficient accuracy.}
\]

To simplify and render possible this operation, 16 observation times were selected where \( W_c \) was determined because it is possible to multiply by fixed values more accurately.

Since the authors have selected standardized times for sampling, the particle size at those chosen sampling times will depend on the temperature during analysis. The sampling times shown in Table 1 are for 20° centigrade. Variations in temperature could be compensated for either on the data sheet, or by employing temperature control methods.

The interval chosen to give a measurable amount of sediment over \( (t_2 - t_1) \) for obtaining the difference

\[
W_T (t_2) - W_T (t_1) \quad \text{is 20 seconds.}
\]

To clarify this point with an example, at each selected observation time \( (t_n) \) the instrument measures \( W_T \) ten seconds prior and again ten seconds after \( (t_n) \). Where \( (t_n) \) is the second observation time (see Table 1) which is given as 50 seconds after \( (t_0) \) the analyzer measures \( W_T \) at 40 seconds \( (t_1) \) and again at 60 seconds \( (t_2) \) after \( (t_0) \). The measurements give approximate values for \( W_c \) which are believed to be as accurate as those obtained by usual mechanical analysis procedures.
SUGGESTIONS FOR AN ELECTRONIC ANALYZER DESIGN

To instrument the equation for cumulative weight \( W_c \) of a fine-grained sediment sample, equation (2) may be expressed as:

\[
W_T(t_n) = W_T(t_2) - W_T(t_1)
\]

where:

- \( W_T(t_1) \) is the weight of the total accumulation of sedimented sample at the beginning of each observation time \( n \).
- \( W_T(t_2) \) is the weight of the total accumulation of sedimented sample at the end (20 seconds) after \( W_T(t_1) \) of each observation time.
- \( W_c \) is the cumulative weight for each observation time.

\[
W_c = W_T(t_2) - W_T(t_1)
\]

Therefore:

\[
k_n = \frac{\log t'}{\log t'}
\]

and this equation can be expressed electronically. This can be shown by substituting in equation (6) voltage signals \( E_0, E_1, \) and \( E_2 \), which are proportional to weight measured by the device.

\[
E_1n - (E_1n - E_0)k_n = W_2n
\]
where:

\[ E_{0n} = W_T (t_1)_n \]
\[ E_{1n} = W_T (t_2)_n \]
\[ E_{2n} = W_{cn} \]

Components of the Analyzer: The fine-grained sediment analyzer may be represented in a block diagram (Fig. 1) which describes the various units and how they are linked in order of operation. A diagram for a pilot model is shown in (Fig. 2). Further details of design and construction of the pilot model are filed in the general files of the Woods Hole Oceanographic Institution, and are available for further development of the device.

A. A settling tube and balance pan with transmission beam attached mechanically to a transducer.

B. A mechanoelectrical transducer tube or reactance (RCA 5734 was suggested here) with power supply and associated circuitry.

C. First DC scaling amplifier for calibration and range selection of the input signal voltage.

D. A voltage splitter of the compensated type using vacuum tubes instead of purely resistance loads for better isolation.

E. A program timer and time relay switch which provides the function periods or observation times. As part of this component there is a channeling and thermionic-loop-controlled chopper circuit for distribution of the signal voltage at the two moments either end of the 20-second interval.

F. A memory component which retains a signal voltage for exactly 20 seconds then releases it to one of the inputs of the first differential amplifier (G).

G. The first differential amplifier component, where two signal voltages enter simultaneously and a voltage, their difference, is passed on to the variable gain amplifier (II).
FIG. 1
H. A 16-stage \( \log_{10} \) multiplier, or variable gain amplifier, which provides the proper constant \( (k_n) \) each of the 16 observation times to give the product of the constant \( (k_n) \) and the input voltage from (G).

I. The second differential amplifier component, performing the same type of operation as the first one in the final step in the solution of the equation.

J. The second scaling amplifier is identical to the first one, but is necessary to raise the signal voltage \( (E_0) \) to the proper value to provide a full-scale adjustment in the recorder. In conjunction with the first scaling amplifier, this unit can be adjusted to calibrate the instrument accurately.

K. The recording instrument, which should be a highly sensitive direct-writing galvanometer.

Operation of the Analyzer: As the weight from the accumulation of sediment on the balance pan (A) increases, there is an accompanying increase in strain transmitted to the plate shaft of the transducer (B) through a suitable wire link. Changes in strain on the shaft produce proportional changes in output voltage from the transducer. In the remainder of this discussion the output voltage from the transducer and its change through various sections of the circuitry will always be referred to as the signal voltage.

The signal voltage passes from the transducer component into a scaling amplifier (C) permitting accurate calibration in conjunction with a second scaling amplifier (J) to be considered later. Calibration of the signal voltage means that for a specific weight of sediment a finite voltage amplitude will be established and an accurate scaling in terms of grams/volts maintained through the system.

The signal voltage leaves the first scaling amplifier and enters a voltage splitter (D). In the splitter the signal voltage is divided and isolated into two equal voltages which may then be introduced into two different parts of the circuitry simultaneously without mutual interference. The splitting of the original signal voltage does reduce the sensitivity but not enough to affect the necessary accuracy at the recording end of the analyzer. The two half-value voltages enter a chopper-controlled, channeling component operated by a program timing device (E).
The timing control is preset to provide sixteen observation times, mentioned above in the section on theoretical considerations. At each observation time \( n \) the time switch activating a thermionic chopper permits one of the half values of the signal voltage to enter the computer circuitry for a suitable milliseconds interval by way of channel 1. The signal voltage passes directly to the memory component \( F \) where a flip-flop relay permits the signal to enter and remain stored. At the end of the 20-second interval the chopper allows the channeling relay to move to channels No. 2 and No. 3 permitting the two half signals to pass into them.

At the same instant the signal voltage from the memory \( E_{0n} \) is released by the flip-flop switch (synchronized by a loop through the chopper unit) and enters one of the input channels of the first differential amplifier \( G \). One of the two half-value signal voltages \( E_{1n} \) enters the other input channel of component \( G \) and the difference of these two voltages appears at the output and passes into component \( H \), the 16-stage logarithmic multiplier. Within component \( H \) the signal voltage \( E_{1n} - E_{0n} \) is multiplied by the constant \( k_n \) for the particular observation time. The product \( (E_{1n} - E_{0n}) k_n \) passes into one of the inputs of the second differential amplifier \( I \) where the other half value voltage \( E_{1n} \) appears at the opposite input, and the difference of these two voltages \( E_{1n} - (E_{1n} - E_{0n}) k_n = E_{2n} \). The signal voltage \( E_{2n} \) passes from \( I \) into the second scaling amplifier \( J \) and in turn to the recording instrument. Only \( E_{2n} \) is actually recorded on the strip graph for each observation time. It should be mentioned here that the duration of the voltage signal passing through the circuitry and to the recording instrument should be sustained for a suitable time in milliseconds to provide a full-scale travel of the recording stylus.

Points on the strip chart recorded by the 16 sweeps of the stylus can be read directly from the calibrated chart coordinates as \( W_c \), the cumulative weight. During each observation time provision can be made to operate a zero level marker to assure accurate calibration of the strip chart for each of the measurements. Values for \( W_c \) are then entered on the data sheet (see Table 1) The percentage of the total fine fractions and then percentage of all fractions is computed to obtain the cumulative percentage of each size. From the latter a cumulative frequency curve may be plotted.

In the original pilot model design the problem of correcting for drift during long period operation imposed on the instrument had not been solved. Since then, self-correcting and recalibrating
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* at 20° centigrade
devices have been made available on the market and may be added to the analyzer to give unattended operation for 120 hours or longer.

Multiple Operation of the Analyzer: The design as proposed here is suitable for analyzing a single sediment sample. Either completely separate analyzer units may be used or a multichannel instrument may be built. The latter would use one common system utilizing components (C) through (J) and be provided with 6 to 12 settling tubes, transducer components, and either separate or multichannel recorders. Such a multiple channel instrument would require a synchronized cam operated timing unit permitting one signal voltage from one sample at a time to enter the common measuring and computing channel. Separate analyzer units offer the advantage of minimizing maintenance problems which would interfere with a large-scale laboratory analysis.

Further Considerations: Some of the theoretical aspects of the problem only are covered in this report and further refinements are possible. The writers had planned to continue to develop the method by adding a preset coefficient stage to allow percentages instead of weights to be plotted. They had also planned to investigate the possibilities of incorporating the measuring element in a centrifuge to reduce the analysis time. It was determined experimentally that other measurement methods were possible. For example, a change of capacitance or dielectric, which could be measured as changes in voltage developed by offsetting a pair of loosely coupled radio-frequency oscillators, will occur in proportion to the change in suspended sediment in the settling column. There is some promise in the field of ultrasonics in which a recent study showed that there may be a good correlation between frequency of resonance (in the supersonic ranges) and particle size. The measurement of resistance, as another parameter, was found to be considerably complicated by polarization which could be partially nullified by using low-frequency alternating current.

These investigations seem to show that a fully automatic, short period, accurate analysis of fine-grained sediments is possible.

Cost Estimates: The original estimate for the cost of parts, materials, and the labor of wiring and assembly, and testing the analyzer without considering further developmental or engineering expense, was in the neighborhood of $2,000. Two fifths of this cost was given to labor. Since this study
was made, in June 1953, a number of packaged components have appeared on the commercial market which would take the place of most of the circuitry necessary for the analyzer. The availability of new transducers with far better discrimination and more ruggedly built, well-regulated d.c. scaling amplifiers, variable gain amplifiers, and subtraction components, as well as fine recording instruments, puts an entirely new light on the success of the analyzer design. The cost of such packaged units would be around $5,000. An additional $1,500 should make it possible to engineer, construct and test the added components which would be needed along with the packaged commercial units. The cost of designing, building and testing the analyzer without recourse to commercial units available, would probably exceed this cost estimate, and require considerably more time to instrument before the device could be used in a sedimentation laboratory. A much closer study of the cost estimates would be made, if any interest were shown to proceed with the development of the analyzer, but it is believed doubtful if the over-all cost would exceed $6,500 for a multichannel system. Furthermore, the packaged commercial equipment purchased for the analyzer could be easily utilized for other types of instrumentation without disturbing the device for which they were purchased.
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