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CORRECTION OF FLEXIBLE PLATE SUPERSONIC NOZZLE CONTOURS BY INFLUENCE METHODS

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SUMMARY

Methods of obtaining the contour corrections necessary for the production of a satisfactory flow uniformity at the exit of a flexible plate supersonic nozzle are reviewed. Basic requirements of the methods are considered, and in particular it is found that contour corrections, small in comparison with the boundary-layer thickness, produce flow changes which cannot be described by the simple theory of characteristics. A procedure involving use of experimentally determined jack influences was successfully applied to correction of nozzle profiles in a 12-in. supersonic wind tunnel. Considerations of further application of this method are discussed.

INTRODUCTION

A nozzle designed to produce a uniform supersonic flow field never produces a flow of perfect uniformity because of combined effects of viscosity, defects in fabrication, and possible distortions of the nozzle caused by air loads. The question of whether or not the contour of such a nozzle must be altered to improve the flow will depend upon the magnitude of the initial non-uniformity, the use to which the nozzle is to be put, and the amount of effort required to modify the contour. For nozzles used in wind-tunnel testing the flow uniformity is usually very important, particularly if stability and control testing is involved.

Many wind-tunnel nozzles utilize the solid-block type of construction. Modification of the contours of such nozzles is accomplished by the addition and removal of material. Various methods of calculating the exact distribution of the required corrections along the nozzle surface have been given (Ref. 1, 2, 3, 4). In each the same basic procedure is followed, i. e., determining the required flow corrections by measurement of the flow in the uncorrected nozzle and relating these to the necessary changes in the wall shape by assuming disturbances to be propagated along characteristic lines of the flow. The ideal correction, in general arbitrary, which would produce perfectly uniform flow could theoretically be made by a single application of this process, since the addition and removal of nozzle material could also be made arbitrary. It has been found, however, that corrections so determined are often less than ideal in practice and that the process must be applied repeatedly to obtain a particular degree of flow uniformity (Ref. 2, 4). For very large solid-block nozzles the process becomes costly from a machining standpoint, and the tendency has been to accept without correction the initial contour if the flow variations are within reason.

A considerable number of supersonic wind tunnels have been constructed in which the nozzle contours are formed by bending thin flexible plates between parallel sidewalls by means of jack screws hinged to the plates at intervals. An example of such construction is shown in Fig. 1, which is a view of the nozzle of Tunnel E-1 of the Gas Dynamics Facility at the Arnold Engineering Development Center. Corrections to the contours of such a flexible wall nozzle are easily effected by small movements of the various jack screws. Since the plate behavior is governed by various laws of bending, arbitrary corrections to the nozzle

profiles cannot be made even though there are an infinite number of possible positions of each jack. Arbitrary plate corrections would require an infinite number of jacks. The usual procedure is to correct flexible plate nozzles until a point of diminishing return is reached. This point is considered to be determined by limitations on the contours imposed by the laws of bending and by deflection of the plate between jacks caused by airloads.

A procedure for determining the best possible correction of a flexible plate nozzle based on the influences of individual jacks on the airflow has been given by Puckett (Ref. 5). Because of certain simplifying assumptions made in the development of this method, it has not met with complete success; and most flexible plate nozzles have been corrected by various methods of successive approximation which are partly empirical and partly trial-and-error. Although satisfactory profiles have been obtained by such methods, the procedure is lengthy and time-consuming, especially when applied to nozzles utilizing a relatively large number of jacks. An investigation of the method of Ref. 5 has indicated the major shortcoming to be the neglect of the effect of the boundary layer on the process of transmission of disturbances from the wall into the main flow. A modification of the method of Ref. 5, based on the use of experimentally determined jack influences, has been found capable of producing optimum corrections of a given uncorrected contour with a minimum expenditure of time and effort.

Tunnel E-1, on which the experimental work connected with this investigation was carried out, is a 12-in. intermittent blowdown wind tunnel with a Mach number range of 1.3 to 5.0. It incorporates a flexible plate (0.20 in. thick) nozzle which is shaped by a series of 12 jacks spaced at 6-in. intervals. The tunnel is supplied with air from a 5200-cu ft high-pressure tank and exhausts into a 200,000-cu ft vacuum sphere. Stagnation pressures of one to four atmospheres are available, but operation so far has been limited to atmospheric inlet pressure. The nozzle itself is a modified version of the flexible plate nozzle used in the 12-in. supersonic wind tunnel at the Jet Propulsion Laboratory of the California Institute of Technology.

SYMBOLS

x = longitudinal coordinate along nozzle centerline, measured upstream from end of nozzle.

x_w = coordinate along surface of nozzle, measured upstream from end of nozzle.

x_j = any finite set of points on the nozzle centerline at which an ideal correction is desired.

$C(x)$ = the ideal correction to an airflow which would yield perfectly uniform flow.

$I_i(x)$ = effect of unit deflection of jack i on Mach number or angle of the airflow of point x.

Δ_i = deflection of jack i, positive for deflection of top plate away from axis of nozzle.

n = number of jacks used in the correction.

P_o = stagnation pressure of airstream.

P_o' = stagnation pressure behind a normal shock wave as measured by a total head tube.

ANALYSIS OF INFLUENCE METHODS OF FLEXIBLE PLATE NOZZLE CORRECTION

METHOD PROPOSED BY PUCKETT

The method proposed by Puckett (Ref. 5) is based upon the establishment of curves of influence of each jack on the airflow in the test region of a nozzle and the performance of certain mathematical operations on these curves which theoretically will yield an optimum correction to that nozzle. The change in slope of the surface of the flexible plates caused by unit deflections of each jack can be calculated by the theory of bending of a simply supported continuous beam, assuming that slope changes for small deflections are independent of the initial curvature of the plates. Curves of jack influence on wall slope calculated for the plates of Tunnel E-1 are shown in Fig. 2. These wall-slope changes can be converted to airflow changes in the main stream by assuming propagation along characteristics (Fig. 3). Since both the bending theory and characteristics theory for small changes in the flow predict that the influences will be linear and obey the superposition principle, the net effect on centerline airflow of any combination of jack deflections can be expressed as a linear combination of all the individual jack influences:

$$\sum_{i=1}^{n} \Delta_i I_i(x) \tag{1}$$

For simplicity, attention is focused on corrections of the flow on the nozzle centerline, under the assumption that a betterment of flow there will result in corresponding improvement elsewhere. To further simplify the problem, the deflections Δ_i are restricted to symmetric and antisymmetric deflections of each pair of jacks (top and bottom plate) at the same longitudinal position on the nozzle. A symmetric deflection of a given jack (pair) will introduce identical families of characteristics from each plate. At the intersection of these characteristics on the nozzle centerline Mach number changes exactly twice those of either family will be produced, while the airflow deflections will exactly cancel. Similarly, antisymmetric deflections will produce zero change in Mach number and flow angle changes exactly twice those of either family of characteristics. In the above summation (1), if Δ_i is a symmetric deflection, the influence $I_i(x)$ is the effect on Mach number only; if Δ_i is antisymmetric, $I_i(x)$ is the effect on flow

angle. Thus corrections of Mach number distribution and flow angularity can be made independently.

All the possible corrections which can be imposed on the flow in a flexible nozzle by symmetric or antisymmetric adjustment of the n jacks is represented by the finite series (Eq. 1). If measurement of the air flow in the uncorrected nozzle indicates that an arbitrary distribution of Mach number or flow angle corrections, C(x), would produce a perfectly uniform flow, the best correction which can be made in practice will be some approximation to C(x), since the series (Eq. 1) cannot represent any arbitrary function. Assuming that this optimum correction will be that which minimizes, in the sense of least squares, the differences between the actual and ideal corrections over some interval in the test region of the nozzle, this condition can be represented as

$$\int_a^b \left[C(x) - \sum_{i=1}^{n} \Delta_i l_i(x)\right]^2 dx = \text{a minimum} \tag{2}$$

Since this minimum must be with respect to any of the n possible jack deflections,

$$\frac{d}{d\Delta_k}\left[\int_a^b \left[C(x) - \sum_{i=1}^{n} \Delta_i l_i(x)\right]^2 dx\right] = 0, \quad k = 1, 2, \ldots\ldots n \tag{3}$$

When the indicated operations are performed, Eq. 3 reduce to the following system of n linear simultaneous equations in the jack deflections, Δ_i

$$\sum_{i=1}^{n}\left[\int_a^b l_k(x) l_i(x) dx\right]\Delta_i = \int_a^b C(x) l_k(x) dx, \quad k = 1, 2, \ldots\ldots, n \tag{4}$$

Puckett noted that the integrals forming the coefficients of the Δ_i would be fairly tedious to obtain and proposed a variation in which not the best least squares correction is sought, but one which merely matches the ideal correction at a finite number of points. By adjustment of n jacks, the ideal correction C(x) could be imposed on the flow at any n points x_j, which condition is represented mathematically by another system of simultaneous equations

$$\sum_{i=1}^{n} \Delta_i l_i(x_j) = C(x_j), \quad j = 1, 2, \ldots\ldots, n \tag{5}$$

Here the coefficients of Δ_i are simply the ordinates of the influence curves at the points x_j. (By a slightly different procedure the Mach number or flow angularity could be corrected to equal but unpredictable levels at n+1 points by adjustment of n jacks.) By selection of the x_j near peaks of both the influence curves and the ideal correction curve, theoretically the maximum corrections would occur where most needed, and the corrections at points intermediate to the x_j should be small.

ACCURACY OF THE INFLUENCE CURVES

Initial attempts to correct the Mach number distribution of the nozzle profiles of Tunnel E-1 by the finite point method (Eq. 5) were unsuccessful, and an extended investigation indicated the failure was mainly the result of inaccuracies in the calculated influence curves. A comparison of these influence curves and similar curves obtained by total head measurements in the tunnel at a nominal M = 3.5 is given in Fig. 4. It is convenient to represent the change in Mach number by the change in the easily measured pressure ratio, P_0'/P_0, which is a function of local Mach number alone. When it is considered that the use of the calculated influence curves would introduce errors in the nozzle correction at any point equal to the sum of the errors of the individual influence curves at that point, discrepancies shown in Fig. 4 constitute an ample reason for failure of the influence method. This is particularly so at points where the discrepancies are so severe that the calculated and experimental influences are opposite in sign.

The discrepancies were initially thought to result from the simplifying assumptions made in applying the beam theory, such as simple supports, zero edge fixity, and small deflections from a straight initial condition. However, measurements of the actual plate bending characteristics demonstrated that the beam theory was quite accurate. This can be seen in Fig. 5 which presents a comparison of a calculated slope change curve and a corresponding curve given by numerical differentiation of measured plate deflection data. On the influence curves for jack 10 (Fig. 4) the discrepancies caused by errors in the beam theory can be observed. These are small compared to the total discrepancies between calculated and experimental influence curves, indicating that the discrepancies are largely caused by deficiencies of the aerodynamic theory. The source of these deficiencies is obviously the neglect of boundary-layer effects. The upstream displacement of the characteristics network, which is caused by the larger Mach angles associated with the incomplete expansion due to the reduction of effective flow area, would be expected to produce a similar upstream shift of the influence curves. This effect would be increased by upstream shifts of the apparent points of reflection or origin of the characteristics caused by their curvature within the boundary layer. The experimental influence curves for jacks 10, 11, and 12 (Fig. 4) clearly exhibit these upstream shifts. The influence curves for jacks farther upstream, however, are apparently further complicated by the effect of waves emanating so far upstream in the nozzle that they negotiate the boundary layer more than once in their transit to the test section.

A further discrepancy was noted in the reduction of the peaks of the influence curves. The changes in the flow produced at the flexible plate were not transmitted in full strength to the main flow because of some damping effect of the boundary layer entirely distinct from the shifting effect just discussed. In most other practical applications such damping is of relatively minor importance and, consequently, it has been given very little consideration in currently available treatments of the interaction of boundary layers and disturbance waves in supersonic flow. Various investigators have demonstrated theoretically that a weak wave incident on a supersonic shear layer is not reflected as a single wave but as a series of waves which constitute a diffusion of the wave in the downstream

direction. These waves, nevertheless, have a net strength equal to the strength of the single wave that would be reflected in the absence of the shear layer, i.e., the inviscid reflection (Ref. 6, 7, 8). Similar results have been found for theoretical models of a subsonic layer adjacent to a supersonic layer (each being either uniform or shear) except that the diffusion of the disturbance wave extends upstream as well as downstream (Ref. 9, 10).

The experimental studies of boundary-layer shock-wave interactions have indicated that at points removed from the immediate vicinity of the interaction the flow is similar to that which would be expected in the absence of the boundary layer. The extent of the interaction region, in which severe non-linearities such as separation and reattachment occur, decreases as the wave strength decreases. It has been observed, for example, that the non-linear region is completely absent, and the flow is close to the theoretical models for a 7-deg wave incident on a turbulent boundary layer at M = 3.0 (Ref. 11). Thus, theoretical and experimental indications are that very weak disturbance waves are merely diffused and not altered in overall strength by a supersonic shear layer. Reflections which occur entirely within the supersonic portion of the boundary layer will be free of damping, but waves resulting from disturbances at the physical flow boundary will very definitely be subject to damping between the wall and the sonic streamline because of the basic nature of an elliptic (subsonic) flow field. This damping will be significant only when the wall deflections are so small as to be of the same order of magnitude as the subsonic portion of the boundary layer, and this is a possible explanation of the damping of the peaks of the experimental influence curves. Boundary-layer measurements have not yet been made on Tunnel E-1, but at a nominal M = 3.50 and atmospheric supply pressure, the boundary layer is known to be turbulent and about 1 in. thick at the end of the nozzle. The subsonic thickness of a turbulent boundary layer at M = 3.50 is of the order of 1 percent of the total thickness, and this is of the same order of magnitude as the deflections of the flexible plate used in establishing the influence curves of Fig. 4. The damping effect at M = 2.50 was observed to be less than at 3.50.

The inaccuracies of the calculated influence curves could be reduced somewhat by modifying the calculations to allow for the boundary-layer effects. The increased Mach angles in the test rhombus could be inferred from the Mach number reduction indicated by computed values of boundary-layer displacement thickness at the end of the nozzle. An allowance for the effect of curvature of the characteristics in the boundary layer could be made by use of the "reflection thickness" concept suggested by Tucker (Ref. 7). These corrections would account for only the upstream shifts of the characteristics network. The theory of interaction between boundary layers and weak waves is not yet sufficiently developed to predict accurately the damping effect which was observed to occur when the boundary layer is much thicker than disturbances causing the waves. In view of the rather large amount of work that would be expended to produce only partial reductions of the inaccuracies of the calculated influence curves, the conclusion is that the easily obtained experimental influence curves are much to be preferred for use in making nozzle corrections. Of course, any reduction in boundary-layer thickness

caused by increased Reynolds number or delayed transition would improve the calculated curves, but the improvement would not likely be sufficient to justify their use unless a completely laminar boundary layer were obtained.

SUPERPOSITION AND LINEARITY OF INFLUENCE CURVES

Other considerations which will affect the success or failure of the influence method of nozzle correction are the related concepts of superposition and linearity of the influences. To enable the expression of the net effect of a number of jack deflections as the simple combination of all the individual influences, both the bending of the plate and the airflow changes must obey the principle of superposition. This requires that the influence of any jack be independent of the position of all the other jacks. Such a requirement incidentally determines that the influence curves have any simple meaning at all. Because there is no general method of solution of non-linear simultaneous equations, it is also required that the jack influences be linear functions of the deflections, Δ_i. Simple beam theory and characteristics theory for small changes in an inviscid supersonic flow both predict that linearity and superposition will occur, but since the boundary layer and any interaction with disturbance waves may behave in non-linear fashion for changes of the order of magnitude for which the main flow will be linear, the extent to which these two requirements are met in any particular case must be determined by experiment.

Linearity was checked on Tunnel E-1 by measuring the influence of a given jack at a given point in the flow for a considerable range of deflection of that jack. It was found that at $M = 3.50$ linearity occurred for a range of about $\pm$ 0.035-in. deflection, depending on the jack in question. Whether the observed departure from linearity was a structural effect or an aerodynamic effect was not determined, but it is suspected to be aerodynamic. The departure from linearity was gradual so that for a small range beyond the linearity limit the correction equations (Eq. 4 or Eq. 5) could be solved as though linear, and the resulting jack deflections modified by small increments to allow for the non-linearity. Such a procedure was attempted in the experimental work described below in the section "Experimental Verification of Nozzle Corrections Using an Influence Method," but the non-linearities were not large enough in that particular case to produce any appreciable change in the final corrected pressure distribution.

A study of a large number of measured P_o'/P_o data for a given plate profile and corresponding data predicted from that of some other profile by means of the experimental influence curves showed that, at least within the linearity limits, the principle of superposition would be obeyed. Any discrepancies observed were found to represent merely the statistical scatter of the probable experimental errors of the individual influence curves. When a number of influence curves are to be combined, the probable errors of the individual curves will propagate as the square root of the sum of the squares; and thus the magnitude of the probable errors of the individual experimental influence curves will become important in cases where a relatively large number of such curves are to be combined.

CONSIDERATIONS OF THE FORM OF INFLUENCE METHOD BEST SUITED FOR TUNNEL E-1 NOZZLE CORRECTIONS

The form of influence method best suited for use on any tunnel apparently would be determined by factors peculiar to that installation, such as relative jack spacing and boundary-layer characteristics. To aid in deciding on the proper method to use for Tunnel E-1, a number of hypothetical corrections were made to an assumed initial uncorrected pressure distribution using the experimental influence curves obtained at M = 3.5 (Fig. 4). Several of the indicated corrected pressure distribution curves (predicted by influence curves and not actual total head measurements) are presented in Fig. 6. The only errors involved are relatively small ones representing the accumulation of random uncertainties of individual experimental influence curves.

Curve (A) represents the pressure distribution of the uncorrected profile. Curve (B) illustrates the fact that the finite point method of determining nozzle corrections (Eq. 5) will produce desired corrections at selected points, x_j, but the overall correction may be very poor because of large over-corrections which are possible at intermediate points. Curves (C) and (D) show that the optimum correction produced by the integrated method (Eq. 4) is a decided improvement for the same number of jacks and that the improvement increases as the number of jacks increases. It was further found that the overcorrections of the finite point method depend upon the location of the x_j, being largely eliminated if the x_j are chosen to coincide with points at which the integrated method (for the same number of jacks) produces the ideal correction, C(x), exactly. Since these latter points were not found to be correlated in any particular fashion, desirable locations of the x_j can only be found by some process involving trial and error. For this reason the integrated method is always to be preferred for nozzle corrections involving small numbers of jacks (3 to 6). For nozzle corrections involving a large number of jacks (10 to 20) this preference may be reversed for two reasons: First, the number of integrations required becomes inconveniently large $\left(\frac{n^2+3n}{2}\right)$ for a large number of jacks. Second, the sensitivity of the overcorrections to the location of the x_j will probably decrease as the number of jacks increases because of the decreased relative spacing of points at which desired corrections can be made exactly.

To avoid permanent deformation the radius of curvature at any point of a flexible plate nozzle must be kept above some safe minimum value. In Tunnel E-1, this structural consideration was found to limit the deflection of a single jack, without moving adjacent jacks, to about a tenth of an inch away from any smooth initial profile. Moreover, the required corrections would naturally be expected to be quite small since the uncorrected profiles are usually the result of detailed calculations or constructions of profiles designed to produce a smooth isentropic expansion to some supersonic Mach number. The nozzle corrections which were found necessary during a trial-and-error correction (not based on influence methods) of Tunnel E-1 nozzle profiles were invariably small (0 in. to 0.040 in.) and decreased in magnitude towards the end of the nozzle. Deflections of the last jack, number 12, were usually about 0.003 in. to 0.005 in., which is consistent with the physical feeling that the nozzle contour should approach the fixed end of the nozzle smoothly with no abrupt changes at the last jack.

The correction equations, however, impose mathematical conditions on the jack deflections with no regard for the physical aspects of the problem, and there is no assurance that the deflections arrived at will be physically reasonable. The solution represented by curve (D) of Fig. 6 included several such unreasonably large deflections. By letting the correction C(x) represent a correction to pressure levels other than the mean of the uncorrected distribution, any number of solutions to the correction equations could be obtained. The jack deflections given by these solutions were linear functions of the selected pressure level, and generally some level could be found for which the Δ_i were more reasonable than for the mean pressure level of the uncorrected profile. But the pressure distribution corresponding to these more reasonable deflections was always quite poor, as can be seen in curve (E) of Fig. 6. Since the deflection of the last jack seemed to be restricted by the fixed end of the plate and since this restriction could not be recognized by the correction equations, the decision was made to omit the last jack from any attempted nozzle corrections.

EXPERIMENTAL VERIFICATION OF NOZZLE CORRECTIONS USING INFLUENCE METHOD

PROCEDURE

Attempts were made to correct the Mach number distributions of the nozzle contours of Tunnel E-1 using the influence method of Ref. 5 modified in accordance with the considerations discussed in the preceding section. Experimental influence curves were used and considerable care was taken to insure their accuracy by taking influence surveys at 2-in. intervals within a total survey length of 18 in. The accuracy was further increased by obtaining the influences for the largest jack deflections possible within the linearity limits. The percentage error of the influence curves can be reduced by this procedure since the probable error in total head measurements is the same regardless of the size of the jack deflection. For reasons concerned with the ease of solution of the correction equations, the number of jacks used was restricted to four, and this restriction required that the integrated method of Eq. 4 be used. Jacks 8, 9, 10, and 11 were those used, because jacks upstream of a nozzle inflection point were found to have less influence on the detailed flow distribution in the test rhombus than jacks downstream and because of the omission of jack 12 for the reason discussed previously.

EXAMPLE OF THE FORMATION AND SOLUTION OF THE CORRECTION EQUATIONS

In accordance with Eq. 4, the elements of the following matrix equation were formed by numerical integration of all the possible products of two of the experimental influence curves for M = 3.5 (Fig. 4).

$$\begin{pmatrix} 34.50 & 5.72 & -22.40 & -16.07 \\ 5.72 & 24.52 & -10.47 & -14.04 \\ -22.40 & -10.47 & 33.82 & -6.66 \\ -16.07 & -14.04 & -6.66 & 37.68 \end{pmatrix} = \begin{pmatrix} 78.87 \\ 64.08 \\ -46.70 \\ -82.20 \end{pmatrix} \qquad (6)$$

All the elements have been multiplied by an appropriate constant to give numbers of convenient magnitude. The solving of Eq. 6 by the method of minors is simplified by the fact that the left-hand matrix is always symmetric. The solution yielded the following symmetric jack corrections for the top and bottom plates, rounded off to the nearest 0.001 in. corresponding to the least count of the jack indicators:

$$\Delta_8 = + 0.047 \text{ in.}$$

$$\Delta_9 = + 0.041 \text{ in.}$$

$$\Delta_{10} = + 0.034 \text{ in.}$$

$$\Delta_{11} = + 0.020 \text{ in.}$$

PRESSURE AND MACH NUMBER DISTRIBUTION OF THE CORRECTED CONTOURS

The above jack corrections were applied to the uncorrected contour for M - 3.5, and the total head survey then taken in the tunnel exhibited marked improvement over that of the uncorrected profile (Fig. 7). The overall degree of correction was good, but small discrepancies remained between measured and predicted values of the corrected distribution, indicating that small errors in the experimental influence curves were not completely eliminated. A similar correction procedure was carried out at a nominal M - 2.5, and the resulting pressure distribution curves are given in Fig. 8. The influence curves at this Mach number were not obtained with quite as much precision as at M - 3.5, and this is reflected in a slightly lesser degree of correction of the Mach number distribution. Each of these corrections required the taking of only six total head surveys, i.e., one for the uncorrected contour, four for the influence curves, and one for the corrected contour.

An evaluation of the merit of the influence method should be based on the following considerations:

1. At M - 3.5 the influence method corrected the Mach number distribution from an original variation of ± 0.032 M to a variation of ± 0.013 M (± 0.4 percent) utilizing only six total head surveys. A previous correction of this contour by a trial-and-error process required over 60 surveys to produce a variation of ± 0.014 M. This was an extreme case, but the average number of surveys required for nozzle correction by the trial-and-error process was 20 to 30 for each Mach number.
2. At M - 2.5 the influence method corrected the Mach number distribution from a variation of ± 0.033 M to a variation of ± 0.018 M (± 0.7 percent) using six surveys. No direct comparison with the trial-and-error method is possible because no attempt had been made to correct the uncorrected contour for this Mach number by trial-and-error means. Note that this contour was not a calculated or constructed contour but the result of extrapolating curves of jack position versus Mach number.

CONCLUSIONS

1. An influence method has been verified in which empirical influence curves can be used to determine the corrections to the calculated contours of a flexible wall nozzle which will produce a more uniform flow. The proper form of the method to use will depend on characteristics of the particular installation on which it is to be used. Calculations of influence curves based on beam theory and simple characteristics theory will not be sufficiently accurate if a relatively thick boundary layer is present. Experimental influence curves are so easily obtained that they are to be preferred even in cases where the boundary layer is thin enough to allow an approximate correction for boundary-layer effects.

2. Since the superposition and linearity characteristics will vary from tunnel to tunnel and with Reynolds number and Mach number for a given tunnel, they should be determined experimentally in any given case. The jack corrections given by solutions of the correction equations should fall within the limits of superposition and linearity. The correction equations do not recognize the physical limitation that the deflection of the last jack before the downstream end of the plate be very small, and this jack is logically omitted from the correction equations.

3. Two methods of correcting flexible nozzles by influence methods are available: an integral method which yields an optimum correction, and a finite-point method which yields exact corrections at a finite number of points. The integral method is the preferred method. It is, in fact, required for corrections involving a small number of jacks if reversion to a lengthy trial-and-error process of selecting the locations of the finite points is to be avoided, because the exact location of these points is a critical factor in determining the size of the overcorrections at intermediate points.

4. The finite-point method will give an approximation to the optimum correction of the integral method. The degree of approximation will increase with the number of jacks involved since the sensitivity of the overcorrections to the location of the chosen correction points will decrease as the relative jack spacing decreases. For nozzle corrections involving a large number of jacks, the finite method then may become more practical since the number of integrations required for the integral method increases rapidly with the number of jacks.

5. The above conclusions have been confined to corrections of Mach number distribution by symmetric deflections of the jacks. It is probable that the same conclusions will hold for independent corrections of flow angularity by antisymmetric jack deflections.

REFERENCES

1. Ward, G. N. "A Note on the Production of a Uniform Flow in the Working Section of a Supersonic Wind Tunnel", D. A. E. R. /Airflow/40, Admiralty, 1948.

2. Harrop, R. and Bright, P. I., Salmon, J., Caiger, M. T. "Design and Testing of Supersonic Nozzles", ARC R. & M. 2712, 1953.

3. Meyer, R. E. and Holt, M. "The Correction of Wind Tunnel Nozzles for Two-dimensional Supersonic Flow", The Aeronautical Quarterly, Vol. II, Part III, November 1950, p. 195.

4. LaBerge, J. G. "Design, Calibration and Correction of a 1.6 Mach Number Nozzle", National Aeronautical Establishment of Canada, Laboratory Report LR-41, November 5, 1952. (ASTIA No. AD-2068)

5. Puckett, Allen E. "Design and Operation of a 12" Supersonic Wind Tunnel", Institute of Aeronautical Sciences Preprint No. 160, July 16, 1948.

6. Marble, F. "The Reflection of a Weak Wave from a Supersonic Shear Layer", Part III of "Problems in Shock Reflection", GALCIT Transonic Research Group, Final Report on Contract No. W-33-039-ac-1717 (11592), USAF, July 1948. (ASTIA No. ATI 41302)

7. Tucker, Maurice. "Approximate Turbulent Boundary Layer Development in Plane Compressible Flow along Thermally Insulated Surfaces with Application to Supersonic-Tunnel Contour Correction", NACA TN 2045, March 1950.

8. Chang, C. C. and Chu, B. T. "Propagation of Disturbances Through Shear Layers of Compressible Flow", Part I of Final Report on Supplemental Agreement No. 4(52-141), Contract AF 33(038)-12919, OSR, ARDC, USAF, September 25, 1952, Department of Aeronautics, Johns Hopkins University (ASTIA No. ATI 170700)

9. Tsien H. S. and Finston, M. "Interaction Between Parallel Streams of Subsonic and Supersonic Velocities", Journal of the Aeronautical Sciences, Volume 16, No. 9, September 1949, pp. 515-528.

10. Ritter, A. and Kuo, Y. H. "Reflection of a Weak Shock Wave from a Boundary Layer along a Flat Plate. I - Interaction of Weak Shock Waves with Laminar and Turbulent Boundary Layer Analyzed by Momentum - Integral Method", NACA TN 2868, January 27, 1953.

11. Bogdonoff, S. M., Kepler, C. E., and Sanlorenzo, E. "A Study of Shock Wave Turbulent Boundary Layer Interaction at M = 3.0", Princeton University Report No. 222, July, 1953.

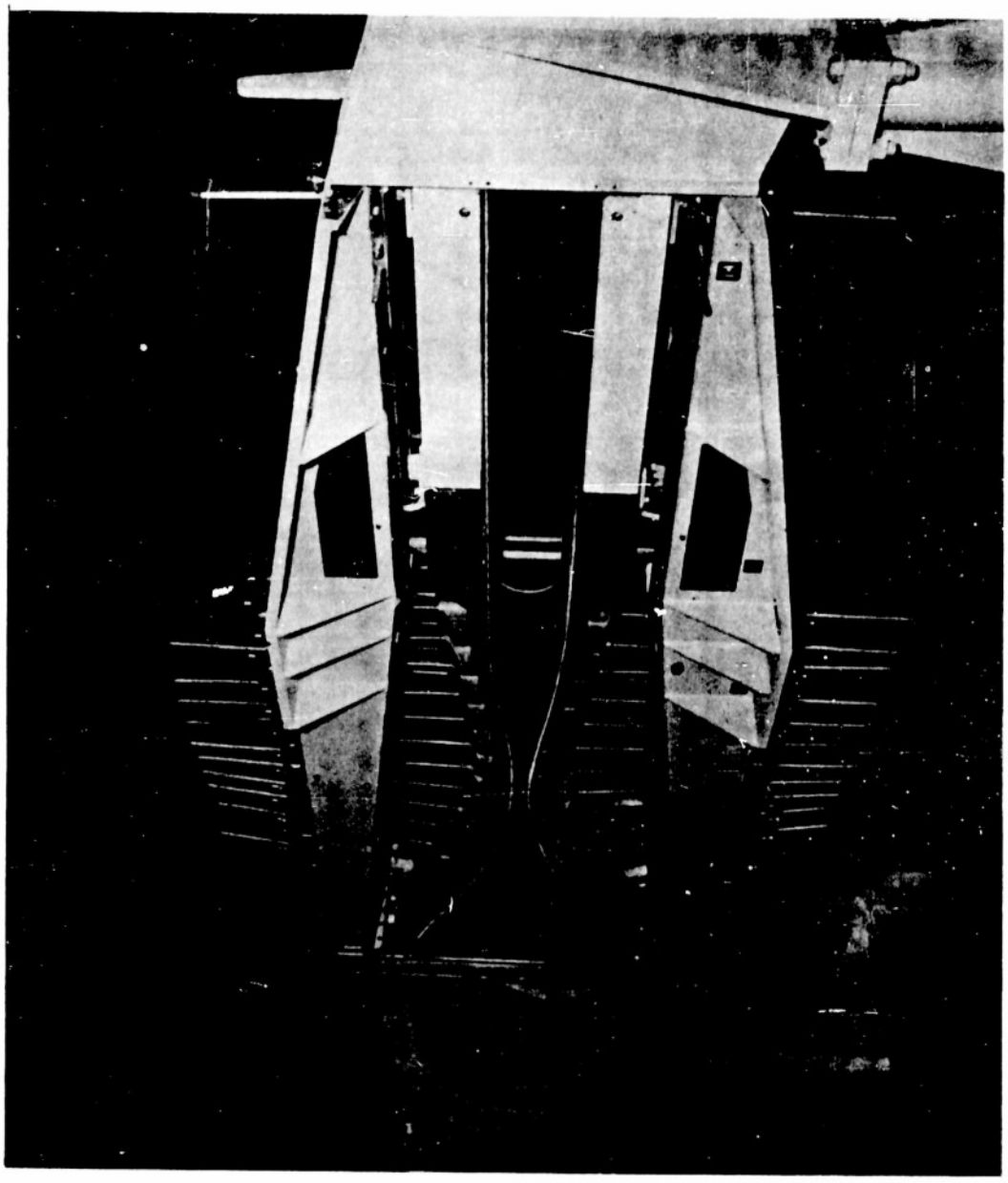


Fig. 1. The Flexible Plate Nozzle of Tunnel E-1

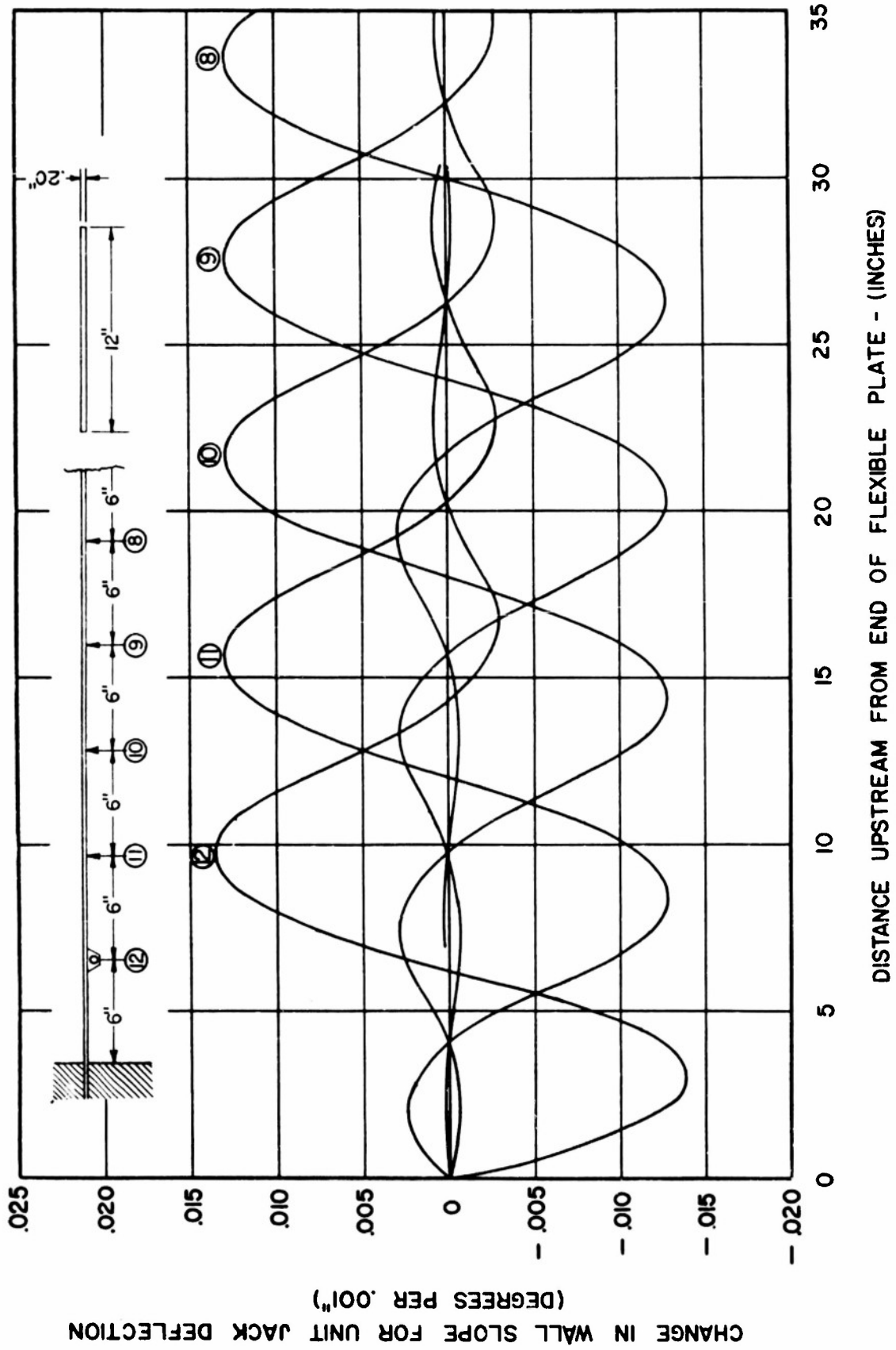
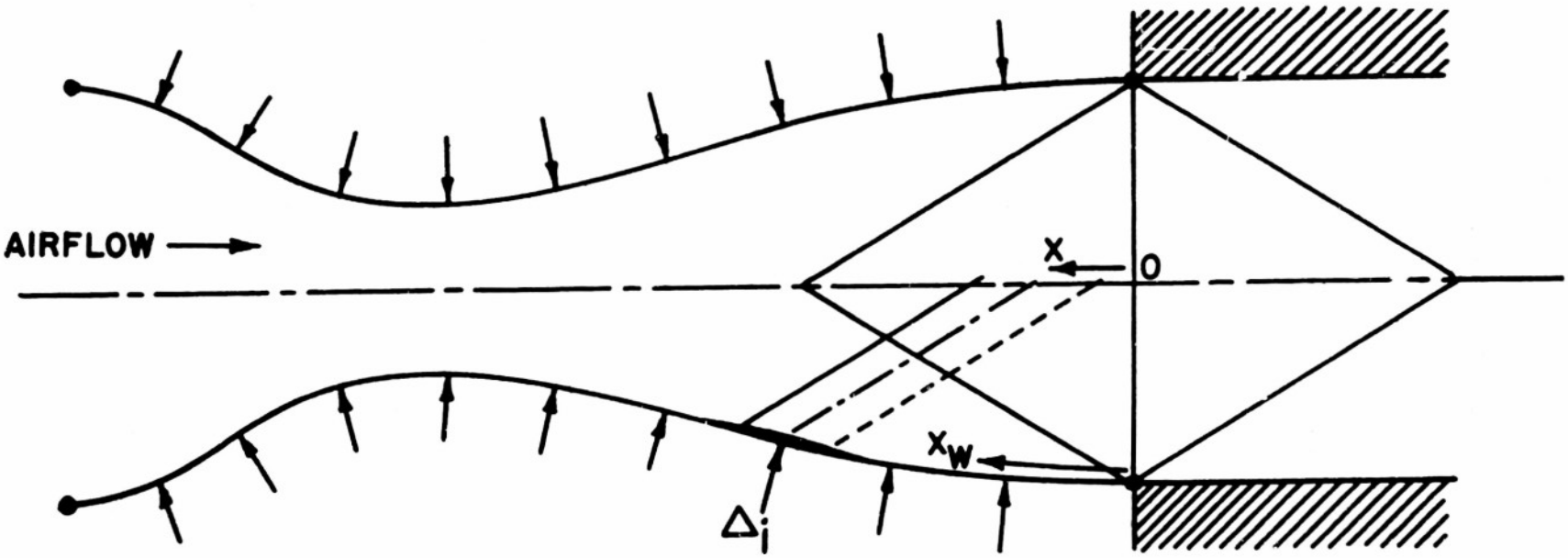


Fig. 2. Theoretical Influence of Jacks on Wall Slope



(a) CORRESPONDENCE OF CENTERLINE POINTS x_1 INFLUENCED BY WAVES FROM WALL POINTS x_{w_1} IS DETERMINED FROM CHARACTERISTICS NETWORK.

(b) $\Delta\theta(x_1) = \Delta\theta(x_{w_1})$

(c) $\Delta M(x_1) = -\dfrac{\left(1 + \frac{\gamma-1}{2} M_1^2\right) M_1}{\sqrt{M_1^2 - 1}} \Delta\theta(x_{w_1})$

Fig. 3. Propagation of Wall Corrections to Centerline of Nozzle

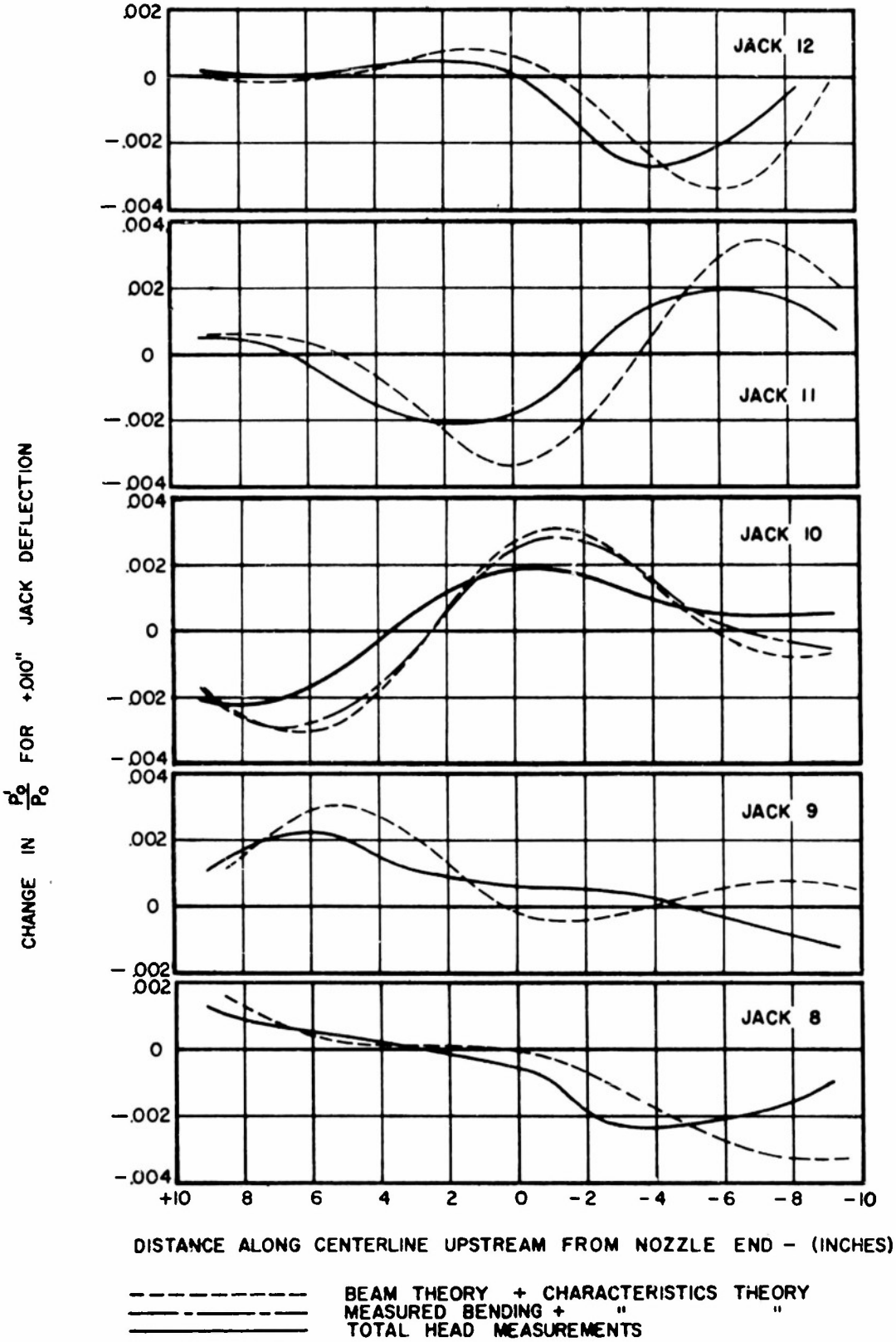


Fig. 4. Comparison of Theoretical and Experimental Jack Influence

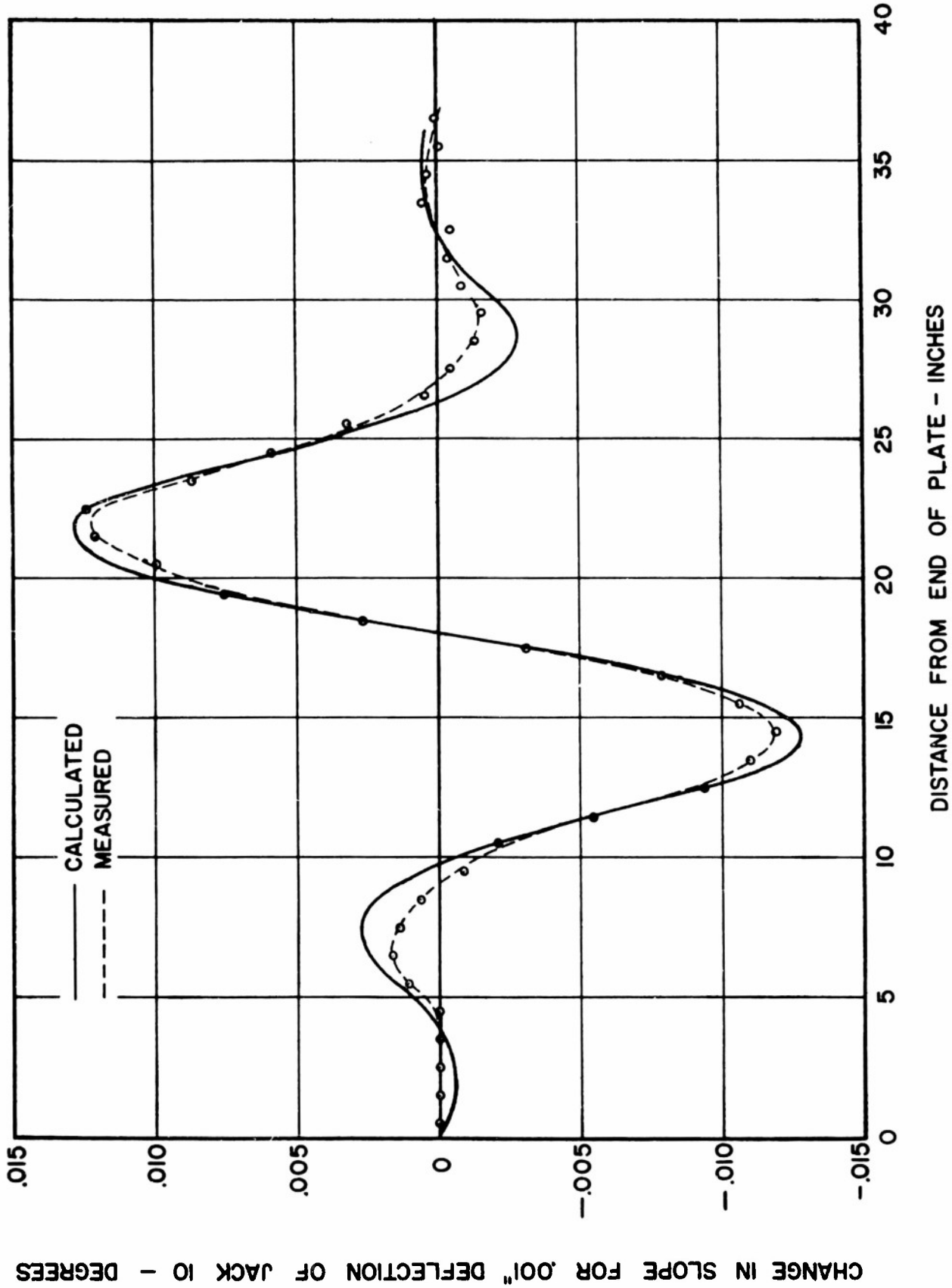


Fig. 5. Comparison of Calculated and Measured Change in Slope of Plate

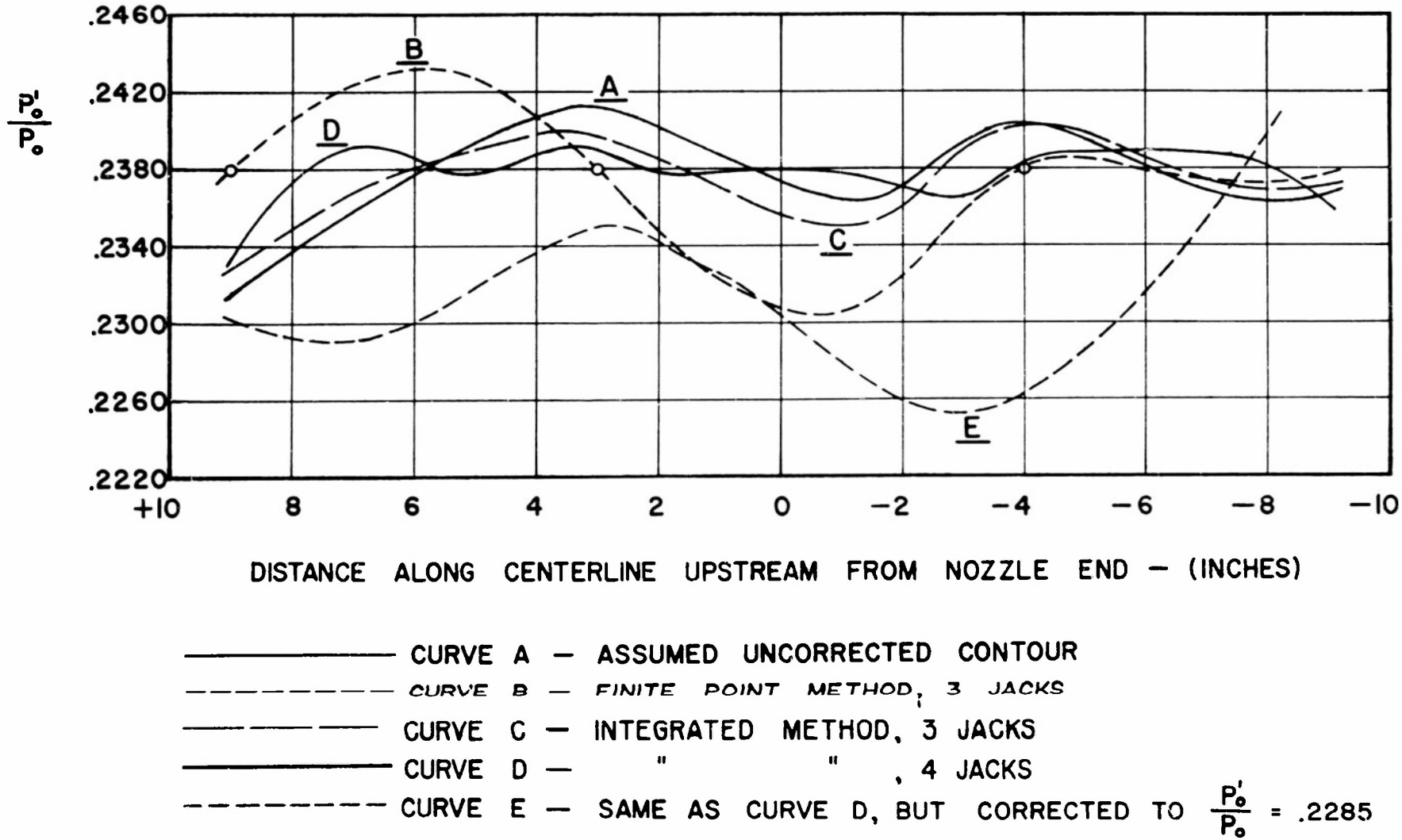


FIGURE 6 — CENTERLINE PRESSURE DISTRIBUTION FOR SEVERAL HYPOTHETICAL NOZZLE CORRECTIONS, NOMINAL MACH NUMBER = 3.50

Fig. 6. Centerline Pressure Distribution for Several Hypothetical Nozzle Corrections, Nominal M = 3.50

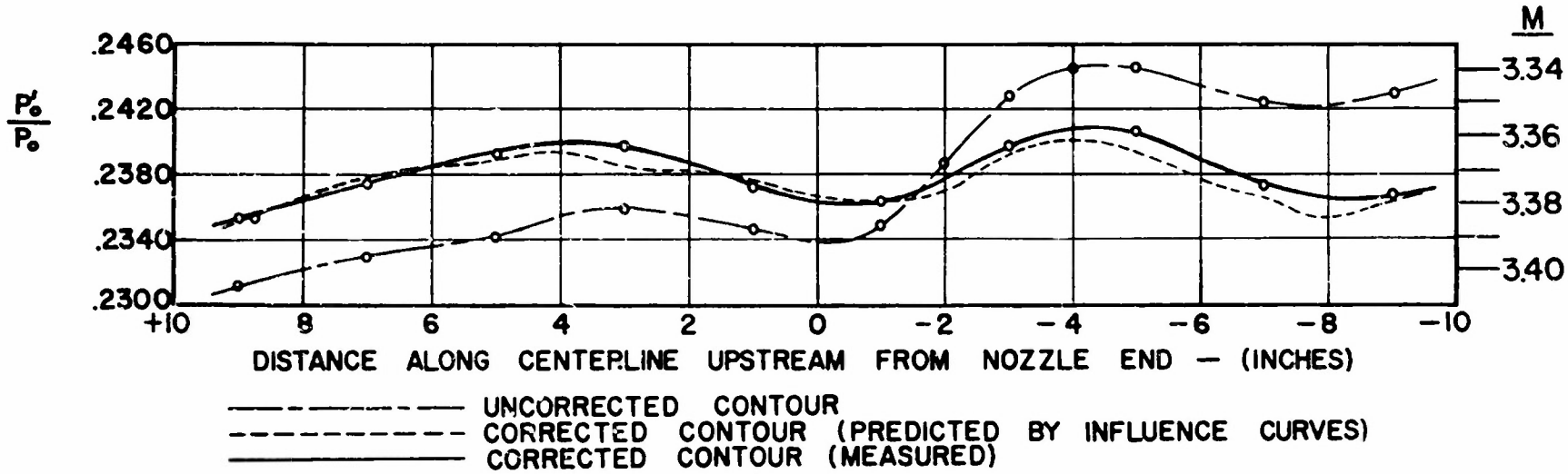


Fig. 7. Correction of Pressure Distribution at M = 3.50

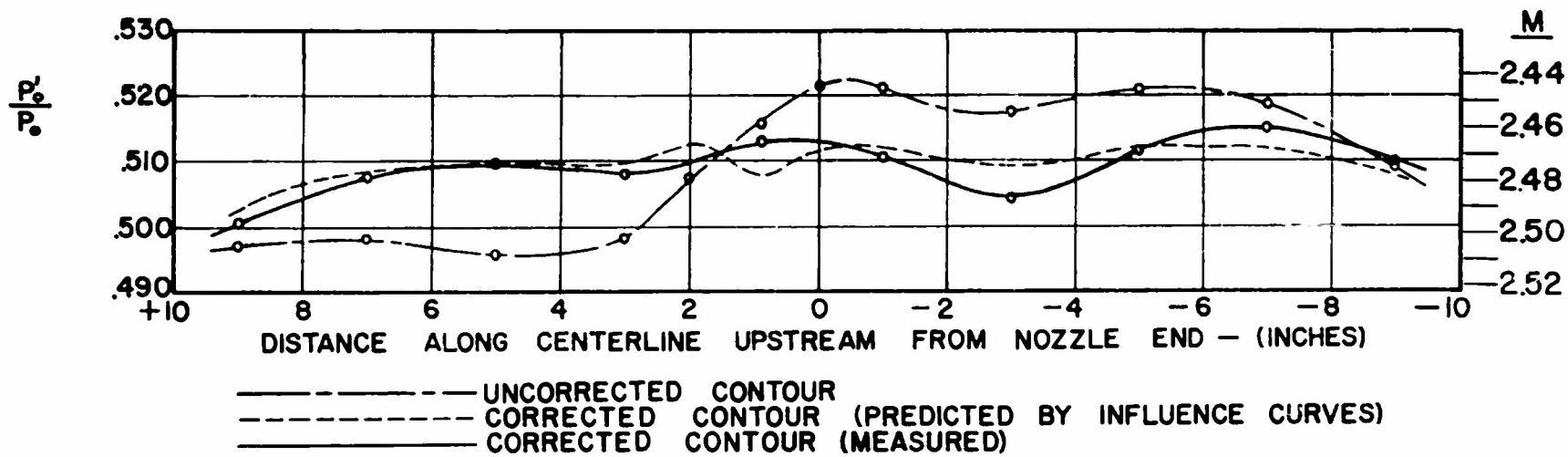


Fig. 8. Correction of Pressure Distribution at M = 2.50