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CONFIDENTIAL
RESEARCH MEMORANDUM

REVIEW OF HIGH-PERFORMANCE AXIAL-FLOW-COMPRESSOR

BLADE-ELEMENT THEORY

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS
WASHINGTON
April 6, 1954

CONFIDENTIAL
The increase in the aerodynamic load of individual blade rows and the increase in required accuracy of design necessary for the realization of axial-flow compressors of high efficiency, high stage pressure ratio, and high flow capacity will require an extensive and accurate knowledge of the flow characteristics of individual blade elements. This report presents a review of current compressor blade-element theory, with particular emphasis on application to the transonic high-performance compressor. A discussion of the significant parameters of total-pressure loss and deviation angle is presented, and an indication of the extent of available knowledge and the problems involved in the determination of blade-element characteristics is given. Some recent results and considerations in this pursuit and suggestions for further avenues of investigation are indicated.

INTRODUCTION

If higher levels of compressor performance (higher weight flow per unit area, stage pressure ratio, and efficiency) for aircraft-engine application are to be achieved, substantial increases in the aerodynamic load of each individual row of blades in the compressor must be obtained. In these high-performance compressors, the individual blade rows will be required to operate within close tolerances at relatively high levels of design blade loading and inlet Mach number. It becomes imperative, therefore, that an extensive and accurate knowledge of the flow characteristics of individual blade rows be obtained. This knowledge should include design-point data, a clear definition of the flow limitations, and an evaluation of the efficiency potentials and sensitivities of compressor blade rows.

In current design practice, the flow distribution at the outlet of compressor blade rows is determined from the flow characteristics of the individual blade sections or elements in conjunction with the requirements
of radial pressure equilibrium. For the most part, the flow characteristics of blade elements have been obtained from investigations of blade sections in two-dimensional cascade. The accumulation of blade-element design data in the transonic region of operation, however, is currently believed to be difficult, because of the general problems involved in cascade operation in this region. Furthermore, the two-dimensional cascade provides no information on three-dimensional and radial effects, which will undoubtedly be more pronounced in the high-performance compressor. Consequently, the compressor designer is faced with the problem of determining accurate blade-section design data for high-performance application.

This report presents a review of current compressor blade-element theory, with particular emphasis on the application to the transonic high-performance compressor. A discussion of significant parameters and concepts is presented, together with an indication of the extent of available knowledge of the flow characteristics of high-performance blade elements. Some recent results obtained in the Lewis laboratory research program are presented, and suggestions are made for further investigation.

SYMBOLS

The following symbols are used in this report:

A  blade-passage area
D  diffusion factor
i  incidence angle, angle between inlet relative air-velocity vector and tangent to blade mean line at leading edge, deg
M  Mach number
P  total pressure
p  static pressure
Re  Reynolds number
r  radius
V  air velocity
β  air-flow angle, measured from axis of rotation, deg
δ  deviation angle, angle between outlet relative air-velocity vector and tangent to blade mean line at trailing edge, deg
\( \eta \)  
adiabatic efficiency

\( \phi \)  
camber angle, difference between directions of blade mean camber line at leading and trailing edges, deg

\( \rho \)  
static density

\( \sigma \)  
solidity, chord-to-spacing ratio

\( \omega \)  
relative total-pressure loss coefficient

Subscripts:

\( \text{av} \)  
average (between max. and 2)

\( \text{id} \)  
ideal state

\( \text{max} \)  
maximum

\( \text{min} \)  
minimum

\( R \)  
rotor

\( S \)  
stator

\( \text{st} \)  
compressor stage

\( z \)  
axial

\( \theta \)  
tangential

\( 1 \)  
rotor inlet

\( 2 \)  
rotor outlet

\( 3 \)  
stator outlet

Superscript:

\( ^\prime \)  
relative to blade element

**ANALYSIS APPROACH**

In current design practice, axial-flow-compressor blades are evolved from a process of radial stacking of individual airfoil shapes called blade profiles or elements. Each element along the height of the blade is designed to direct the flow of the air in a certain direction as
required by the design flow pattern or velocity diagram of the blade row. The basic parameters defining the flow across blade elements are illustrated in figure 1. For application to compressor design, the principal flow characteristics of a blade element are the variations with incidence angle (or angle of attack) of (1) the loss in total pressure across the blade row, which, in conjunction with the work input, determines the efficiency variation of the row, and (2) the air turning angle, which, for a given inlet condition and annulus area ratio across the row, determines the work input and the absolute direction of the outlet air. From the curve of loss against incidence angle can be determined the low-loss or useful range of operation of the element and a desired or design value of incidence angle for design-point blade setting. For a blade of given camber, for any value of incidence angle, the air turning angle can be determined from the value of outlet deviation angle (see fig. 1). For purposes of analysis, the principal blade-element characteristics considered will be total-pressure loss and deviation angle.

In establishing a body of descriptive theory for compressor blade elements, use is made primarily of observed flow characteristics measured in the compressor configuration and, where applicable, of two-dimensional potential-flow and cascade sources. From these experimental and theoretical analyses can then be evolved a series of empirical correlations and analytical relations that describe the variation of the principal flow characteristics and flow limitations over a wide range of operating conditions. It is further necessary that the information obtained be incorporated into the compressor design procedure in a simple and accurate manner.

TOTAL-PRESSURE LOSS

Basic Considerations

The efficiency of a compressor stage at its design conditions is determined by the magnitude of the loss in total pressure across the blade elements in conjunction with the work-input level of the rotor. It would appear at first that operation at higher levels of inlet Mach number and stage pressure ratio would necessarily increase the total-pressure losses across the blade rows and, therefore, would result in lower levels of stage efficiency. However, it must be remembered that efficiency is a function of both loss and pressure-ratio level; and the success of the high-performance stage will depend on the relative rates of increase of loss and work input with increasing Mach number. In general, the greater the pressure ratio, the larger the loss may be for the same efficiency.

These considerations can be expressed as equations. From the developments presented in reference 1, the adiabatic efficiency of a...
compressor stage $\eta_{st}$ can be related to the stage absolute total-pressure ratio ($P_3/P_1$) and the relative recovery factors across the rotor and stator, $(P_{2,1d}/P'_{2,1d})_R$ and $(P'_{3,1d}/P'_{3,1d})_S$, respectively, by the equation

$$\eta_{st} = \frac{1}{\left[ \frac{P_3}{P_1} \right]^{\gamma-1} \left[ \frac{P_{2,1d}}{P'_{2,1d}} \right]^{\gamma-1} \left[ \frac{P'_{3,1d}}{P'_{3,1d}} \right]^{\gamma-1}}$$

where the prime refers to conditions relative to a blade element, and subscripts 1, 2, and 3 refer to rotor inlet, rotor outlet, and stator outlet, respectively. For constant streamline radius across the element, the ideal outlet relative total pressures become the respective inlet relative total pressures.

Inasmuch as the magnitude of the loss in total pressure will increase with increasing inlet dynamic head, it has been convenient in compressor analysis to consider the total-pressure loss in terms of a ratio of loss to inlet dynamic head. In this paper, the blade-element loss is expressed in terms of a total-pressure loss coefficient relative to the blade. The relative total-pressure loss coefficient $\bar{\omega}$ across a blade element is defined as

$$\bar{\omega} = \frac{P'_{2,1d} - P'_{2}}{P'_{1} - P_{1}}$$

where all total pressures are circumferentially averaged values. The difference $P'_{1} - P_{1}$ is used rather than the inlet dynamic head, because it is a function only of relative inlet Mach number. For no change in streamline radius across the rotor, the numerator of equation (2) becomes the decrease in relative total pressure across the blade element. A method of computing the relative loss coefficient from stationary measurements of total pressure and total temperature and from the computed relative inlet Mach number is presented in reference 1.

Equation (2) is used as the basic loss parameter for axial-flow-compressor design and analysis, because it is directly applicable to the calculation of adiabatic efficiency and is relatively simple to determine. In this regard, it is felt that the use of the isolated-airfoil concept of the drag force and its associated profile efficiency
(ref. 2) is not an accurate measure of compressor blade performance because of the general difficulty in establishing an accurate mean condition across the element. An example of the use of the loss coefficient in the analysis of the blade-element characteristics of a transonic inlet stage is given in reference 3.

The relative total-pressure loss coefficient \( \bar{\omega} \) of a rotating or stationary blade element is related to the relative recovery factor \( \frac{P_2}{P_1, id} \) and the relative inlet Mach number \( M_1' \) by the equation

\[
\frac{P_2}{P_2, id} = 1 - \bar{\omega} \left( 1 - \frac{1}{\left( \frac{P_2}{P_1, id} \right)^{\frac{y-1}{2}} + \frac{y-1}{2} (M_1')^2} \right)
\]

where, for constant-radius flow, the ideal relative pressure ratio \( \frac{P_2}{P_1, id} \) is unity. The use of these and similar relations in design applications can be facilitated by the construction of suitable charts and plots.

The accurate prediction or control of efficiency in axial-flow compressors has been very difficult to achieve because of a general lack of specific data concerning the various losses occurring in compressor blade rows. Aside from three-dimensional end effects, the principal factors that influence the loss across individual compressor blade elements are recognized as (1) the incidence angle of the approaching air (pressure distribution around inlet), (2) the relative inlet Mach number (shock losses), (3) the blade loading (measure of the strength of the velocity gradients and, therefore, the boundary-layer growth on the blade surfaces), and (4) Reynolds number (form and development of the boundary layer). Some considerations pertaining to these various loss factors are discussed in the following sections.

Incidence Angle

Range of operation. - Typical examples of the variation of relative loss coefficient with air incidence angle in the tip region of a transonic compressor rotor at increasing levels of relative inlet Mach number are shown in figure 2(a). Each curve represents the range of operation of the rotor at constant wheel speed. The Mach numbers shown in the figure are the values at the points of minimum loss. The blade shape of figure 2(a) was a double-circular-arc section of about 6-percent maximum thickness. The loss curves of a double-circular-arc blade obtained in two-dimensional cascade (ref. 4) are shown in figure 2(b) for comparison with the rotor curves.
Figure 2 is typical of general experiences with compressor blade sections, which show that the low-loss range of operation is reduced as the inlet Mach number is increased. In particular, the sharp reduction in low-loss range of operation for values of incidence angle less than the value for minimum loss has been observed in all cases. It can be seen from figure 2 that, in the transonic range of inlet Mach numbers, the design incidence angle must be known within very close tolerances if minimum loss and design weight flow are to be obtained.

The reduction in range of operation at the higher Mach number levels is a result of compressibility effects on the blade section. Because of the high levels of inlet Mach number, small variations in the inlet flow direction above and below the best approach angle (angle of minimum loss) result in very rapid accelerations of the flow around the leading edge of the blade, with the subsequent formation of strong shocks and flow separation. At negative angles of incidence, a choking of the flow occurs as inlet Mach number is increased. Therefore, the loss and range characteristics of the blade element would be expected to depend on the particular thickness distribution in the inlet region of the blade and also on the maximum thickness of the section.

An indication of the effect of blade shape on high Mach number range of operation is given in reference 4, which presents the results of investigations of several blade shapes in two-dimensional cascade up to an inlet Mach number of 0.8. For example, for the circular-arc mean-line blade at fixed values of solidity, camber, and inlet-air angle, the results of reference 4 indicate a superior high-speed-range characteristic for the blade with a thickness distribution formed by circular-arc pressure and suction surfaces (maximum thickness at 50-percent chord) compared with the conventional British C.4 thickness distribution (maximum thickness forward of the midchord position) for very nearly the same values of maximum thickness. Design considerations to be observed in selecting blade shapes for transonic application are discussed in reference 5.

Design value. - The value of incidence angle selected for a given blade shape in a compressor design will generally depend upon the application and off-design characteristics of the unit, and no one fixed concept is best for all cases. However, for purposes of analysis, it is necessary to establish a reference value of incidence angle in the operating range in order to investigate the effects of various flow and blade parameters on the loss-incidence curve and design settings. The reference incidence-angle position selected in this paper is the incidence angle at the point of minimum total-pressure loss coefficient. At very high inlet Mach numbers, the incidence angle at minimum loss will most likely be close to a desired setting.
For conventional subsonic operation, data for design incidence angle have been obtained primarily from two-dimensional cascade tests and also from potential-flow theory. General experience indicates that incidence angle at minimum loss is essentially constant with inlet Mach number up to conventional design values. Results of cascade investigations of NACA 65-series and circular-arc blade sections are given in references 4 and 6 to 9. From the available data, design incidence angle for a given mean-line shape is found to vary primarily with the blade camber and also, to a smaller extent, with solidity, inlet-air angle, and thickness distribution. In general, minimum-loss incidence angle decreases (becomes more negative) with increasing camber and decreasing solidity. Further experimental correlations are desirable to establish the degree of accuracy of the data for the two-dimensionally derived incidence angle when applied to the rotor and stator of the high-speed compressor.

Experimental data obtained from the various transonic rotors investigated at the NACA Lewis and Langley laboratories (ref. 10) indicate a gradual increase in the magnitude of the incidence angle at minimum loss as the Mach number is increased above values of about 0.75 to 0.85 (fig. 2, e.g.). In the absence of practical theory describing the flow distribution in the approach and inlet regions of a blade element in the transonic range, this high-speed characteristic cannot be analyzed accurately. (The calculation methods used in ref. 5 are not sufficiently accurate for this problem.) However, some general observations can be made from a qualitative examination of the problem.

For blade configurations involving high solidities and low blade-chord angles, as is the case for the hub region of a rotor, the necessity of a more positive value of incidence angle becomes apparent from an inspection of the variation of the blade-passage area. In figure 3(a) is shown a typical hub-section blade passage. If the approach direction is at a negative angle of incidence, with the approach area $A_1$ shown by the dashed lines, a minimum area ratio will be formed in the inlet region of the passage. As the inlet Mach number is increased, a choking of the flow will ultimately occur in the minimum section. At some positive angle of incidence $+\alpha$, however, as illustrated by the solid approach lines, the approach area $A_2$ is reduced to a value approximately equal to the passage throat area, and higher values of inlet Mach number can then be tolerated before choking occurs. For the rotor tip region, where the solidity is low and the blade-chord angles are high, however, the previous one-dimensional picture becomes questionable because of the absence of a well-defined blade passage, as indicated in figure 3(b). Furthermore, for this situation, the flow still cannot be represented accurately by considerations of isolated-airfoil flow. At the moment, the determination of minimum-loss incidence angle in the transonic range appears to be primarily an experimental problem.
Inlet Mach Number

In addition to the effect of relative inlet Mach number on the general range characteristics of a blade element as described in the previous section, significant compressibility effects can also occur at the minimum-loss value of incidence angle. As indicated in reference 5, the principal difficulty with high Mach number flow occurs when the surface shock waves become sufficiently strong to result in a separation of the boundary layer behind the shock. Shock separation in the minimum-loss range of incidence angle is generally indicated by a relatively sharp increase in the value of the minimum-loss coefficient as Mach number is increased. An example of a minimum loss against Mach number curve at design incidence angle for a 10-percent-thick 65-series blade section in a two-dimensional cascade and in the mean-radius region of several compressor stators and rotors is shown in figure 4. Each symbol represents points obtained from a given blade tested at several tip speeds. A noticeable Mach number limitation is indicated for this blade shape. (The data of fig. 4 were obtained from ref. 1 for unstalled blade operation.) As yet, similar loss curves for transonic blade elements are not available. On the basis of limited data (refs. 3 and 11), transonic-rotor tip sections have been operated up to relative Mach numbers of about 1.1, apparently without strong shock losses, at incidence angle for minimum-loss coefficient.

As indicated in reference 5, the primary objective with respect to design in the development of high Mach number blading should be the reduction of maximum blade-surface Mach numbers. In general, this can be facilitated by using thinner blades with thin leading-edge regions and a compatible over-all blade loading as well as a local blade-loading distribution (ref. 12) and by avoiding a choked flow or minimum area within the blade passage.

Blade Loading

Diffusion factor. - In the region of minimum-loss incidence angle, the velocity along the suction surface of compressor blades attains some maximum value in the inlet region of the blade and then decreases to the level of the outlet velocity at the trailing edge. Recent studies of boundary-layer behavior indicate that it is primarily the difference between the maximum surface velocity and the outlet velocity that controls the growth of the boundary layer on the blade suction surface. For high values of pressure rise, this velocity difference may become large and may result in a separation of the boundary layer, a condition referred to as blade stall. As a general design procedure, it is currently impractical to compute suction-surface velocities for various blade shapes over wide ranges of design conditions. In the interests of simplicity, it has been adequate to use an approximate blade-loading parameter based on the suction-surface velocity difference but expressed in terms of the over-all velocities and geometry of the element.
In isolated-airfoil theory, the measure of blade loading and stall is given in terms of the well-known lift coefficient. The lift coefficient, when applied to compressors of high pressure rise, however, has not been universally successful. For the isolated airfoil, since the outlet velocity is always equal to the inlet velocity (free-stream velocity), the suction-surface velocity difference is generally directly proportional to the lift coefficient. In the compressor blade row, because of the over-all pressure rise, the outlet velocity is less than the inlet velocity, and the suction-surface velocity difference is then no longer uniquely proportional to the lift coefficient.

A recently developed blade-loading parameter called the diffusion factor has been successful in correlating cascade and single-stage compressor losses in the minimum-loss range of incidence angle (ref. 1). The basis of the development of the diffusion factor is shown in figure 5. Specifically, the diffusion factor, by means of several simplifying approximations and assumptions, is an approximate relation that describes the maximum suction-surface velocity difference of a typical compressor blade velocity distribution at design incidence angle with the over-all velocity characteristics and geometry of the blade element. The diffusion factor \( D \) is defined as

\[
D = \left( 1 - \frac{V'_2}{V'_1} \right) + \frac{\Delta V'_\theta}{2\sigma V'_1}
\]  

(4)

where \( V'_2 \) and \( V'_1 \) are relative outlet and inlet velocities, respectively, \( \Delta V'_\theta \) is the change in relative tangential velocity across the element, and \( \sigma \) is the solidity or chord-spacing ratio.

Examples of rotor and stator loss correlations obtained from reference 1 for several single-stage experimental compressors are shown in figure 6 for sub-limiting values of relative inlet Mach number. Data for the stator loss correlations (solid symbols) were obtained from hub-, tip-, and mean-radius regions. The loss trends at the rotor hub- and mean-radius regions are similar to that in the stator.

An interesting result of the analysis was the tip-region loss correlation for the rotors. In the other regions of the rotor and at all radial positions of the stators, the loss correlation was similar to that found for the cascade data of reference 6 (ref. 1). The cascade data showed a rather small increase in loss coefficient with diffusion factor up to a value of about 0.6, after which a sharp increase in loss indicating a separated condition was observed. Unfortunately, experimental data at diffusion factors greater than 0.6 in these regions were not available to indicate whether a rise in loss was also present in the compressor configuration.
The markedly different loss correlation in the tip region of the rotor was interpreted as a reflection of the various three-dimensional or end effects (such as tip-clearance flow, wall boundary-layer scraping action, or centrifuging of low-energy fluid) existing in the rotor tip region. The data for the rotor tip-region correlation were obtained at points approximately 12 percent of the radial passage height from the tip of the rotor. In all cases, the measurements were outside the outer-wall boundary-layer regions. Apparently, these end losses are roughly proportional to the blade-element loading as defined by the diffusion factor. The precise manner in which this relation exists is not presently known, and further research is in order on the nature and origin of the rotor tip loss. It may be of some significance to note that, although the stators from which the stator loss correlations of reference 1 were obtained had both rotating (numbers 3 to 6 in ref. 1) and stationary (numbers 1 and 2) hub casings, no significant difference in loss variation was observed within the limits of the data.

The results of the diffusion-factor correlations presented in reference 1 indicate that, at least as far as the inlet stages are concerned, the rotor tip region is the critical region with regard to blade loading. In fact, experience shows that, for typical subsonic and transonic stages, the value of design diffusion factor in the rotor tip region is the principal determinant of the efficiency of the stage. Blade-loading correlations for the middle and outlet stages of multistage compressors have not as yet been determined. Further work is also necessary for establishing loss and loading correlations at off-design incidence angles.

Performance analysis. - A blade-loading parameter such as the diffusion factor is useful in the analysis of experimental data. For example, compressor test data indicate an increase in blade-element loss coefficient as inlet Mach number is increased into the transonic range (fig. 2(a)). This poorer performance might, at first thought, be attributed to shock losses. However, this may not necessarily be the case, since the blade loading also becomes greater as Mach number is increased. For example, in the first three plots of figure 7 are shown the variations of relative inlet Mach number, relative loss coefficient, and diffusion factor with incidence angle in the tip region of a transonic rotor. A plot of loss coefficient against diffusion factor in the minimum-loss range is then shown in the last plot in comparison with the limits of data obtained for blades operating below their limiting (high shock loss) Mach numbers shown in figure 6. Thus, for this rotor the increased loss at the higher Mach number level can be explained on the basis of the increased diffusion factor. If strong shock losses had been present, the data points at the higher speed levels in the last plot of figure 7 would be expected to be at greater magnitude than observed.

The increase in diffusion factor with tip speed observed in the illustrative example of figure 7 was primarily due to a decrease in the
axial-velocity ratio across the rotor row at the higher tip speed. In general, for a fixed annulus area ratio across the rotor, the ratio of outlet axial velocity to inlet axial velocity for a blade element will decrease as the density ratio (i.e. wheel speed) is increased. This effect is essentially a compressibility characteristic which becomes significant at the pressure-rise levels of the high performance stage. In particular, the variation of axial-velocity ratio with tip speed should be considered if work-coefficient or pressure-coefficient characteristics are used for analysis and stage matching in high-performance multistage design. Inasmuch as the variation of blade-element turning angle with incidence angle appears to remain essentially constant over the tip-speed range, a decrease in axial-velocity ratio with speed will result in a rise in the rate of increase of work input with speed (ref. 3). The stage characteristics of average pressure or work coefficient against flow coefficient will, therefore, no longer fall on the same curve over the entire compressor speed range. The effect of a decrease in axial-velocity ratio on the rotor velocity diagram will be most pronounced in the tip region of the rotor (because of the large values of relative air inlet angle). Analysis of blade-element performance should include consideration of the variation of the axial-velocity ratio and its effects on the stage velocity diagram.

Velocity-diagram analysis. - Previously, analysis of desirable velocity diagrams has been conducted to a large extent on the basis of isolated-airfoil concepts of lift and drag and essentially two-dimensional incompressible flow. For example, the well-known concept of the symmetrical velocity diagram was obtained from considerations of these simplified flow conditions for blade elements. Current knowledge of compressor flow characteristics and radial equilibrium considerations, however, indicates the inadequacy of these early concepts for present high-performance applications. In this respect, the use of the diffusion factor and rotor tip velocity diagram will permit a more significant and general evaluation of design velocity diagrams for the high-performance compressor.

An illustration of the use of the blade-element diffusion factor in velocity-diagram analysis is given in the following calculation of compressor stages based on simple radial-pressure equilibrium

\[ \left( \frac{dp}{dr} = \rho \frac{v_r^2}{r} \right) \]  

For a conventional constant-work-input, subsonic inlet stage with inlet guide vanes operating at a specific weight flow of 27 pounds per second per square foot of frontal area (0.5 inlet hub-tip ratio) and a maximum rotor-inlet Mach number of 0.75, a maximum stage
pressure ratio of 1.20 is indicated for a rotor tip diffusion factor of 0.4. The maximum allowable average loss coefficient for the subsonic rotor for a stage efficiency of 0.90 will then appear as shown in figure 8. This figure represents the calculated variation of rotor relative total-pressure loss coefficient that can be allowed at various values of stage pressure ratio in order to maintain a stage efficiency of 0.90. The curves were computed for constant radius from the loss and efficiency equations of the Basic Considerations section.

For a constant-work-input transonic stage operating at a rotor tip relative inlet Mach number of 1.10 (no inlet guide vanes) at the same tip speed of 1000 feet per second and at a specific weight flow of 31 pounds per second per square foot of frontal area (0.5 inlet hub-tip ratio), a maximum stage pressure ratio of 1.38 can be obtained for the same rotor tip diffusion factor of 0.4 and the same tip solidity of 1.0. (The same stator conditions of 0.75 inlet Mach number and 0.02 loss coefficient were specified in both cases.) The allowable loss coefficient for the transonic rotor (fig. 8) will then be 0.049, somewhat greater than the subsonic value of 0.039. Since, in general, the blade-loading losses will be about the same for both types of rotors (same diffusion factor), and since recent experimental evidence seems to indicate that, for properly designed blading, shock losses remain small for inlet Mach numbers up to at least about 1.1, loss coefficients for the transonic rotor not measurably greater than the loss coefficients of the subsonic rotor are entirely reasonable. Thus, even if the loss coefficient of the transonic stage is increased slightly, it is possible to achieve the same efficiency as with the subsonic stage.

On the basis of the preceding analysis, a better insight into the comparative workings of the subsonic and transonic inlet stages can be obtained. For the conventional subsonic inlet stage with inlet guide vanes (symmetrical-velocity design), the rotation leaving the rotor is such that a large negative radial gradient of axial velocity (decreasing toward tip) is set up at the rotor outlet (see ref. 13, e.g.). As a result of this gradient, a large decrease in axial velocity occurs across the rotor tip, giving rise to a large value of the term \(1 - \frac{V_2}{V_1}\) (see velocity diagram in fig. 5). Consequently, only relatively small values of \(\Delta V_g\) and, therefore, pressure ratio can be allowed for a given value of the diffusion factor (eq. (4)). It is also interesting to note that the radial axial-velocity gradients introduced in the conventional subsonic stage tend to produce large values of diffusion factor at the rotor tip, where very low values are desired, and low values of diffusion factor (because of the increase in axial velocity) at the rotor hub, where relatively high values can be tolerated.
Another detrimental effect of the radial axial-velocity gradient is the high Mach number occurring at the inlet to the stator hub. As a result of this condition, design weight-flow capacity has generally been restricted to avoid excessive stator-inlet Mach numbers. Furthermore, the conventional subsonic velocity diagram is not a very favorable one for part-speed performance (ref. 13).

In the transonic compressor, with the elimination of the rotation introduced by the inlet guide vanes, the steep radial gradient of axial velocity at the rotor outlet is eliminated (for radially constant work input). The absence of a large diffusion in axial velocity across the rotor tip then permits a higher value of the circulation term \( \Delta V_\theta / 2aV_i \) to be achieved before a limiting diffusion factor is attained (see eq. 4 and velocity diagram of fig. 5). The higher permissible value of the circulation term in conjunction with the higher level of \( V_i \) then allows a larger value of \( \Delta V_\theta \) and therefore pressure ratio. At the same time, the thinner blades keep the shock losses down and maintain a comparable level of total loss. The combination of higher pressure ratio and equivalent loss coefficients then maintains the over-all efficiency level at the higher Mach numbers.

The elimination of the axial-velocity gradient and the inlet rotation permits the maintenance of conventional Mach numbers at the hub of the stator at higher weight flows, despite the increased rotor pressure ratio (larger hub \( \Delta V_\theta \)). Favorable part-speed characteristics have generally been observed in the transonic single-stage (refs. 11 and 14) and multistage (ref. 15) units. Thus, in general, the current transonic configuration presents a compressor stage that is inherently capable of producing at least the same efficiencies as conventional subsonic stages at higher levels of pressure ratio and flow capacity.

As more accurate data on loss and performance characteristics become available, a detailed analysis of desired velocity diagrams and performance over the compressor operating range will be possible. In particular, more data are needed on Mach number, blade loading, and three-dimensional losses, as well as the flow characteristics in the latter stages of multistage units.

Reynolds Number

A deterioration of compressor efficiency observed at high altitudes is attributed to the effects of low Reynolds number resulting from the lowered density levels at altitude (ref. 16). The Reynolds number effect occurs in the boundary-layer flow along both the casing and blade surfaces and presumably must also influence the three-dimensional flows and losses in the blade passages; the over-all effect in the multistage compressor is thus the net effect of these various flow changes. Other
factors affecting the Reynolds number problem appear to be turbulence level and compressor size. Further analysis of axial-flow-compressor similarity and Reynolds number considerations in the multistage configuration is in order.

Although blade-chord Reynolds number may not be a significant factor in the effect observed in the multistage compressor, it is of interest that a noticeable variation of loss with Reynolds number exists for compressor blade elements. The effect of blade-chord Reynolds number on the minimum-loss coefficient in compressor blade elements has been established in cascade tests; figure 9 shows typical curves obtained from a 65-series blade in two-dimensional and annular cascade. The cascade curve represents the loss variation presented in reference 1 for a solidity of 1.5 corrected to a solidity of 1.0. The annular-cascade curve was obtained from the loss data at blade mean radius (ref. 17) expressed in terms of the loss coefficient as given by equation (2). Comparable curves for transonic blade elements as well as possible Reynolds number effects on the diffusion-factor correlation are currently unavailable.

DEVIATION ANGLE

As mentioned previously, close design control is necessary over the flow directions at the outlet of each blade row if design pressure ratio and design inlet conditions are to be obtained in each blade row. For analysis and design usage, it is desirable to consider the outlet angle in terms of the deviation angle. As shown in figure 1, the deviation angle $\theta$ is defined as the difference between the angle of the air leaving the blade and the angle of the tangent to the blade trailing edge. The importance of accurate deviation-angle data in compressor design is illustrated in the following calculations of the effect of an error in outlet angle on the performance characteristics of an inlet stage composed of a transonic rotor row and an axial-discharge stator row (same stage as used in the sample calculation in the TOTAL-PRESSURE LOSS section).

The calculated magnitudes of the variations in rotor total-pressure ratio and diffusion factor resulting from a misdesign or error in rotor deviation angle at the hub and tip are shown in figure 10. The increased deviation-angle sensitivity of the rotor tip region compared with the hub (a reflection of the effects of the higher wheel speed and relative air angles at the tip) is a further reason for considering the rotor tip as the critical region in transonic-compressor design.

The effects of stator deviation-angle error on stator diffusion factor and on Mach number and incidence angle relative to a succeeding rotor row are illustrated in figure 11 for the design condition of no
outlet rotation. Fortunately, the effect of a given stator deviation-angle error on the rotor incidence angle is considerably reduced, particularly in the rotor tip region where close control over design incidence must be maintained (rotor-inlet Mach number will be transonic). For finite values of stator-outlet rotation, as in the case of the conventional stage, the corresponding rotor incidence-angle errors will be greater; but at the same time, the relative inlet Mach numbers will be smaller. The situation for the stator, therefore, does not seem to be as critical as for the rotor tip region.

In compressor design, it is desirable to be able to predict the deviation angle over the range of variation of incidence angle of the element. In this respect, the use of the deviation angle rather than the turning angle is preferred, since the variation of deviation angle with incidence angle is considerably less (depending on solidity) than the variation of turning angle. The problem of deviation-angle prediction, for convenience, has been divided into two phases: (1) the determination of deviation angle with blade and flow geometry at some design or reference incidence angle (in this case, at the incidence angle for minimum loss) and (2) the determination of the change in deviation angle with incidence angle at fixed geometry. This latter effect will be a function primarily of the blade geometry and the changes in loss. In the ensuing discussion, only the first aspect of the problem will be considered.

The most accurate source of deviation-angle design data is direct investigations of compressors operating over wide ranges of flow conditions. However, data from rotating units are limited and relatively difficult to obtain. Fortunately, experiences have indicated that Mach number level has a negligible effect on the magnitude of the deviation angle as long as the losses are kept low (ref. 12). Therefore, if attention is restricted to the incidence angle at minimum-loss coefficient, it may be possible to utilize low-speed considerations in predicting deviation angles for transonic application. In view of the fact that deviation angle at minimum-loss incidence angle varies substantially with blade camber, solidity, and air inlet angle, the determination of deviation-angle data over a wide range of blade configurations is a large undertaking. Accordingly, the possibility was investigated of using potential-flow theory and cascade data in establishing the basic trends of variation of the deviation angle.

The use of a circular-arc mean line as a satisfactory camber shape for transonic application (ref. 5) simplifies the problem to a large extent because of the availability of limited cascade data and potential theory for this blade shape. Low-speed deviation-angle data for circular-arc mean lines can be obtained from the theory of reference 18 and from Carter's rule (ref. 7). Carter's rule is given in terms of blade camber angle $\phi$ and solidity $\sigma$ by
\[ \delta = m \varphi \left( \frac{1}{\sigma} \right) \]  

where \( m \) is a variable depending on the angle between the blade chord and the axis. Values of \( m \) obtained from potential theory are given in reference 7. Very limited compressor data seem to indicate that Carter's rule may be a promising approach for transonic blade design, but the comparisons to date are far too limited to be conclusive.

In order to investigate the applicability of equation (5) as a generalized design rule, correlations were attempted with available cascade data. A plot of deviation angle against camber angle as obtained from equation (5) for a solidity of unity is shown for several inlet-air angles in figure 12 for a typical low-speed variation of minimum-loss incidence angle. Experimental values of deviation angle at minimum-loss incidence angle obtained in two-dimensional cascade with porous-wall suction (ref. 19) for a camber angle of 30° are also shown in the figure. The cascade values are somewhat greater than the Carter's rule values at this point.

Inasmuch as little porous-wall cascade data are available for the circular-arc blade, use was made of the extensive low-speed cascade data for the NACA 65-series blade presented in reference 6 in further evaluating the applicability of the design rule. The 65-series mean line is very close to a circular arc in shape and can be expressed in terms of an "equivalent" circular arc having the same maximum camber, as shown in figure 13. Thus, although the magnitudes of the equivalent deviation angle may not be precisely correct for the true circular arc, an indication of the problems and trends involved in the correlation approach might be obtained. (A comparison between the characteristics of the true circular arc and of a 65-series equivalent circular arc of the same camber angle is given in ref. 19.) The variation of equivalent deviation angle with equivalent camber angle computed from the data of reference 6 is shown in figure 14 for a solidity of unity. The data points for the largest values of camber angle at inlet-air angles of 45° and 60° were omitted because of excessive losses at these conditions.

Although a similar trend of variation of deviation angle with camber angle is observed in the experimental data, values of deviation angle at zero camber are not zero, in contrast to the zero values predicted from equation (5) and the theory of reference 16. The finite values of deviation angle at zero camber are attributed to the effects of finite blade thickness (the deviation angle is theoretically zero for zero thickness). Experimental data from references 6 and 20 indicate that the zero-camber deviation angle increases with increasing thickness, solidity, and inlet-air angle, trends which are confirmed by examination of potential-flow theory. An upward displacement of the theoretical
deviation angles obtained from references 7 and 18 depending on solidity and inlet angle should, therefore, be expected for compressor blades of usual thickness.

The preliminary results indicated herein suggest the possibility of the use of potential theory (ref. 18 or 7) in conjunction with (1) experimentally determined deviation-angle characteristics of the uncambered (symmetrical) profile and (2) loss corrections as a simplified approach to the problem of the establishment of a basic framework for deviation-angle design data. Considerable work along these lines as well as experimental investigations in the actual compressor may be required before reliable deviation-angle data over wide ranges of operation can be presented.

CONCLUDING REMARKS

Although knowledge concerning the general flow characteristics and limitations of high-performance compressor blade elements is currently far from complete, the limited data presently available are nevertheless capable of producing compressor stages of increased pressure ratio and specific weight flow without sacrifice in efficiency. More detailed and extensive data will be necessary in further developments of the high-performance stage so that stages can be designed more accurately.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, December 9, 1953

REFERENCES


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Figure 1. - Blade-element properties.
(a) Blade element in rotor tip region.  
(b) Blade element in two-dimensional cascade (ref. 4).

Figure 2. - Typical variations of blade-element loss coefficient with incidence angle.
Figure 3 - Typical compressor blade-passage area.

(a) Hub section.

(b) Tip section.
Figure 4. - Variation of total-pressure loss coefficient at minimum-loss incidence angle with inlet Mach number for NACA 65-series blade element (ref. 1). (Symbols represent data obtained with various blades.)
Figure 5. - Basis of development of diffusion factor (ref. 1).
Figure 7. Illustration of blade-element loss analysis for rotor tip region.
Figure 9 - Effect of blade-chord Reynolds number on blade-element loss for NACA 65-12(10) blade.

Solidity, 1.0; inlet-air angle, 45°.
Figure 10. Effect of error in relative outlet-air angle for typical transonic rotor.
Figure 11. - Effect of error in stator-outlet angle for typical transonic stage. Stator hub-tip ratio, 0.53.
Figure 12. Variation of deviation angle with camber angle for circular-arc mean line at solidity of 1.0 according to Carter's rule (ref. 7).

Data points from two-dimensional cascade (ref. 19).
Figure 13. - Equivalent circular arc for NACA 65-series blades.
Figure 14. Experimental variation of equivalent incidence angle with equivalent camber angle for BCA 65-series blades.
This report presents a review of current compressor blade-element theory with particular emphasis on application to the transonic high-performance compressor. A discussion of the significant parameters of total-pressure loss and deviation angle is presented, and an indication of the extent of available knowledge and the problems involved in the determination of blade-element characteristics is given. Some recent results and considerations in this pursuit and suggestions for further avenues of investigation are indicated.
This report presents a review of current compressor blade-element theory with particular emphasis on application to the transonic high-performance compressor. A discussion of the significant parameters of total-pressure loss and deviation angle is presented, and an indication of the extent of available knowledge and the problems involved in the determination of blade-element characteristics is given. Some recent results and considerations in this pursuit and suggestions for further avenues of investigation are indicated.