On the Two Types of Charge Invariance

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The hypothesis of charge independence requires a very high degree of symmetry in the system of meson and nucleon fields; in particular it requires the invariance of the Hamiltonian of the system relative to the group of rotations in charge space. We know that this hypothesis has still not been definitely confirmed. However, a lower type of symmetry may be considered as established: the invariance of the Hamiltonian relative to rotation around the z-axis in charge space (which corresponds to the law of conservation of the total charge system), and its invariance relative to rotation around the x-axis by 180° (which means invariance relative to the replacement of neutrons by protons and to the corresponding replacement of \( \pi^- \)-mesons by \( \pi^+ \)-mesons and vice versa). In other words, invariance relative to the group \( D_\infty \) in charge space has been established.

The proof of the invariance relative to the group of rotations must essentially include the neutral meson field. The interaction Hamiltonian of the meson and nucleon fields, which satisfies the requirement of invariance relative to the group of rotations, is a scalar in the isotopic space \( g(\tilde{t}, \tilde{q}) \). A quite general type of interaction Hamiltonian satisfying the requirement of invariance relative to the \( D_\infty \)-type group can be written as

\[
g(\tilde{t}_1\tilde{q}_1 + \tilde{t}_2\tilde{q}_2 + \tilde{t}_3\tilde{q}_3 + \tilde{t}_4\tilde{q}_4)
\]

where \( \tilde{q}_3 \) is the third component of a vector in isotopic space and \( \tilde{q}_0 \) is a scalar. If we disregard the possibility of describing the \( \pi^0 \)-mesons simultaneously by two functions \( \tilde{q}_3 \) and \( \tilde{q}_0 \), we must determine whether the neutral meson field is described by the function \( \tilde{q}_0 \) or by \( \tilde{q}_3 \).

The photoproduction of \( \pi^- \)-mesons on deuterons is an effect which is very sensitive to the properties of the symmetry of the wave function of the neutral meson field in isotopic space:

\[
\gamma + d \rightarrow (d + \pi^0, (I), n + p + \pi^-). (II)
\]

In an examination of this effect it was found that when the signs of the interaction constants of the \( \pi^0 \)-mesons with the neutron \( g_n \) and the proton \( g_p \) are opposite, i.e., \( g_n = -g_p \) (which corresponds to the description of the \( \pi^0 \)-mesons by the function \( \tilde{q}_0 \)), then the cross sections of processes (I) and (II) must be of the same order; when, on the other hand, \( g_n = g_p \) (the \( \pi^0 \)-mesons are described by the function \( \tilde{q}_0 \)), the cross section of process (I) is much smaller than that of process (II). However, some assumptions were made in reference 2 which impair the generality of the results: the approximation of weak coupling, the adoption of a concrete meson theory, and the
phenomenological introduction of anomalous magnetic moments, all of which play an important role. We shall show presently that this result can be obtained in general if we base ourselves only on the group properties of the wave functions and of the Hamiltonian in charge space as well as on one very natural assumption, which we shall explain a little later.

We shall write the Schrödinger equation for the system of the meson and nucleon field:

\[ i \frac{\partial \Psi}{\partial t} = H_0 \Psi. \]  \hspace{1cm} (1)

Here \( H_0 \) is invariant, at least relative to the \( D_0 \) group and possibly relative to the group of rotations. In order to introduce the electromagnetic field into \( H_0 \), we must replace all the \( \partial/\partial x_i \) that act on the nucleon functions by

\[ \frac{\partial}{\partial x_i} + ie \frac{(1 + \sigma_3)}{2} A_i, \]  \hspace{1cm} (2)

and all the \( \partial/\partial X_i \) that act on the meson functions by

\[ \frac{\partial}{\partial X_i} + ieL_3 A_i. \]  \hspace{1cm} (3)

Here \( L_3 \) is the charge operator of the meson field which has the following characteristic values: +1 when it acts upon a \( \pi^+ \)-meson field, -1 when it acts upon a \( \pi^- \)-meson field, and 0 when it acts upon a \( \pi^0 \)-meson field. The operator has the transformation properties of a third component of a vector in isotopic space. We shall expand \( H \) in a power series of \( eA_i \) and limit ourselves to the first order (we shall consider only the absorption of the photon). In other words, the interaction with the electromagnetic field is assumed to be weak and viewed as a minor perturbation causing transitions in the system of the meson and nucleon fields. Then the Schrödinger equation takes the form

\[ i \frac{\partial \Psi}{\partial t} = (H_0 + H_\epsilon) \Psi. \]  \hspace{1cm} (4)

The matrix elements of the transition are

\[ (\Psi_f, H \Psi_0), \]  \hspace{1cm} (5)

where \( \Psi_f \) and \( \Psi_0 \) are functions satisfying Eq. (1).

If we use the property of symmetry of \( H_0 \) only with respect to a 180° rotation around the \( x \)-axis in isotopic space, then for both types of symmetry, \( H_\epsilon \) can be broken down into two terms:

\[ H_\epsilon = S + V_\omega, \]  \hspace{1cm} (6)

as can be seen from (2) and (3).

Here, the first term \( S \), depending only on the interaction of the electromagnetic field with the charge of the nucleon, does not change with a 180° rotation, while the second term \( V_\omega \) changes sign. For the case of charge independence, \( H_\epsilon \) in the form of (6) was obtained in reference 3 from special considerations (in that case, \( S \) and \( V_\omega \) are a scalar and a third vector component, respectively). As can be seen from our derivation, Eq. (6) follows automatically from the general principles of quantum mechanics.

We shall examine process (1): \( \Psi_0 \) describes a deuteron, and \( \Psi_f \) a deuteron and a \( \pi^0 \)-meson. Since the wave function of the deuteron, antisymmetrical in
the proton and neutron coordinates, enters the matrix element twice (in \( \Psi_0 \) and \( \Psi_f \)), the sign of the product of the \( \Psi_0 \) and \( \Psi_f \) functions, under a 180° rotation around the z-axis, will be determined by the transformation properties of the meson function. If the meson function is a scalar (\( \Psi_0 \)) in isotopic space, the product of the \( \Psi_0 \) and \( \Psi_f \) functions does not change sign under the rotation. If, on the other hand, the \( \pi^0 \)-mesons are described by the function \( \Psi_0 \), the product of the \( \Psi_0 \) and \( \Psi_f \) functions will change sign.

We shall assume that the \( \pi^0 \)-mesons are described by the function \( \Psi_0 \). Then, making a 180° rotation in isotopic space and noting the trivial assertion that the value of the matrix elements does not change when the integration variables are replaced, we can write instead of (5):

\[
(\Psi_f, S + V_3 \Psi_0) = -(\Psi_f, S - V_3 \Psi_0) = (\Psi_f, V_3 \Psi_0).
\] (7)

If the \( \pi^0 \)-mesons are described by the function \( \Psi_0 \), we have

\[
(\Psi_f, S + V_3 \Psi_0) = (\Psi_f, S - V_3 \Psi_0) = (\Psi_f, V_3 \Psi_0).
\] (8)

Hence, if we disregard the nucleon recoil effects, \( S = 0 \), the matrix element of transition for the process \( \gamma + d \rightarrow d + \pi^0 \) is zero when the \( \pi^0 \)-mesons are described by \( \Psi_0 \).

The final wave function of two nucleons for reaction (II) may or may not change sign under the 180° rotation; therefore, for reaction (II) there are no selection rules that follow from the requirement of symmetry in isotopic space, except for the condition that, in the case of \( \Psi_0 \), the final triplet state will predominate, while in the case of \( \Psi_0 \) the singlet isotopic state will predominate.

In the above discussion, we assumed that the anomalous magnetic moment of the nucleon (a. m. m.) is caused by \( \pi \)-mesons. Now, if we assume that the a. m. m. is not caused by \( \pi \)-mesons, it can be introduced phenomenologically. This will not change our reasoning. During the computation in the approximation of weak coupling, the a. m. m.'s introduced were equal to those measured in a static magnetic field. The anomalous parts of the magnetic moments of the neutron and proton are equal in magnitude and opposite in sign (with a sufficient accuracy for our purpose). Therefore, the part of \( H_s \) that was caused by the interaction with the a. m. m. had the form

\[
\mu' (\bar{\tau}_1^{(1)} + \bar{\tau}_2^{(2)}),
\]
i.e., it possessed the transformation properties of \( V_3 \). Therefore, the a. m. m. made a large contribution to the cross section for \( g_n = -g_p \) (or \( \Psi_0 \)) and did not contribute at all when \( g_n = g_p \) (or \( \Psi_0 \)). Thus, the results obtained with perturbation theory follow directly from these general considerations. We note that if neutral mesons are described by \( \Psi_0 \), their elastic photo-production on nuclei that are symmetrical with respect to neutrons and protons must be forbidden. The forbidden processes about which we have been speaking must be understoed in the sense that they are forbidden only with an accuracy up to the terms describing the effect of nucleon recoil. We assume that

\[
S \sim \frac{\mu}{M} V_{3a}
\] (9)

and, consequently, the cross sections of the forbidden processes are \((\mu/M)^2\) ~ 40 times smaller than the cross sections of the allowed processes. The natural assumption (9) is fundamental for our results. In this connection,
we computed the cross sections of processes (I) and (II) according to the pseudo-scalar theory, taking radiation damping into consideration, and using the impulse approximation. The theory of radiation damping apparently gives an approximation closer to the true solution of Eq. (1) than the usual perturbation theory. In particular, this theory makes it possible, as we have shown, to find the magnitude and the angle and energy dependences of the cross sections for photoproduction of $\pi^0$-mesons on nucleons without even the phenomenological introduction of the a.m.m. The computations of photoproduction in deuterium, in full agreement with the general considerations formulated in (7), (8), and (9), have shown that the cross sections of processes (I) and (II) are of the same order of magnitude when the $\pi^0$-meson field is described by the function $\varphi_0$, and that the cross section of process (I) is much smaller than the cross section of process (II) when the $\pi^0$-mesons are described by the function $\varphi_0$. Here, the results of the computations change very little with the phenomenological introduction of the a.m.m.

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