APPLICATION OF MAGNETIC AMPLIFIERS
IN THE AUDIO-FREQUENCY RANGE

by
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POLYTECHNIC INSTITUTE OF BROOKLYN
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ABSTRACT

The present report presents the results of an investigation into the steady-state performance of audio-frequency magnetic amplifiers operated from supplies of relatively large internal impedance. Following a review of the fundamental concepts of ferromagnetism, two basic amplifier circuits are analyzed (as illustrative problems) under the assumption of ideal or nearly ideal components. The characteristics of practically available components are often very different from the ideal characteristics, and the effects upon performance of deviations from the ideal are qualitatively discussed.

When magnetic amplifier application is extended from the amplification of very slowly varying signals into the audio-frequency band, it becomes necessary to use carriers in the low radio-frequency range. The radical changes in core materials and dimensions, rectifiers, supply sources and circuitry necessitated by this frequency increased are discussed.

It is shown that conventional magnetic amplifier circuits will not operate satisfactorily from high-impedance supplies. Several new circuits adapted for use with constant-current (infinite-impedance) supplies are introduced, and, in part, analyzed. Experimental results representing a qualitative check of the theory are given.

Finally, a complete audio-frequency amplifier is considered. Its operation is explained by theoretical reasoning and experimental data are presented in verification of the theory.
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I. Introduction

Historical Outline

Within the past half-dozen years there has been a reawakening of interest amongst American engineers and scientists in a class of modulating devices using saturable reactors as the basic nonlinear elements and, therefore, usually called magnetic amplifiers. Less frequently, the term "transductor" has been used, particularly by European authors. Numerous projects have been directed towards research in the theory and applications of these devices, and magnetic amplifiers meeting a wide range of requirements are now commercially available. It may be said in all fairness, however, that industry, and even the engineering profession, are only now recognizing the wide possibilities offered by this type of amplifier. Nevertheless (as has been mentioned in many publications on the subject), the magnetic amplifier is by no means a new development. At least its basic component, the saturable reactor, was known and used only a few years after the turn of the century and has been more or less popular as a control device throughout the intervening fifty years. The designation "magnetic amplifier" appears to have been used first about 1916 by E. F. W. Alexanderson, who is famous for having realized transatlantic radio-telephony through the use of high-frequency alternators. Alexanderson also attempted to modulate the high-frequency output of his alternators by means of a magnetic device and thus did the first work towards the development of audio-frequency magnetic amplifiers.

The advent of the vacuum-tube doomed this approach to the generation of voice-modulated radio waves to oblivion for many years. Although relatively inefficient, fragile, and inherently short-lived, the vacuum tube possesses advantages which otherwise make it nearly ideal as an amplifier component, particularly when service conditions are not overly severe or critical. The magnetic amplifier could not compete at that time. Even the name virtually disappeared from the literature.


2 For example, see entries 421, 422, 423 of that bibliography.


4 Miles, op. cit., offers convincing evidence.
The development of dry metallic rectifiers and greatly improved core materials made eventually possible the construction of amplifiers vastly superior to those known in the early 1900's. German engineers apparently were the first to apply these in military, naval, and aircraft installations. American interest was aroused when records showed that some of these had satisfactory service-free operating records of several years.

The previously unquestioned supremacy of the electron tube is now being challenged by both the magnetic amplifier and the transistor. Clearly, research in the latter two devices is not directed towards developing an overall substitute for the tube, but rather towards creating amplifiers more suitable for specific uses than equivalent electron-tube circuits would be. The phase of research of which the present project is a part is aimed towards the development of magnetic amplifiers, oscillators, and more specialized circuits suitable for work at audio and radio frequencies.

**Definition of the Problem**

Considerable progress has been made in finding satisfactory solutions to the basic problems of low-frequency magnetic amplifiers. While no single method of analysis or design can be said to be universally accepted, a number of approaches which yield theoretical results well in agreement with experimental data do exist. The transient and steady-state performance with resistive and resistive-inductive load, methods of design, different circuit arrangements, and many applications have been discussed in large numbers of articles and papers. (A comprehensive AIEE "Bibliography of Magnetic Amplifier Devices and the Saturable Reactor Art" lists 901 entries.) Discussions of applications in the audio-frequency range are scarce, however, and at still higher frequencies, use of magnetic amplifiers is an isolated phenomenon. The purpose of the present investigation is to extend to some degree the knowledge of magnetic amplifier behavior in the audio-frequency range.

Time limitations required the study to be restricted to steady-state performance only.

Originally it was intended to design and build amplifiers on the model of those which had proven valuable for low-frequency work, to develop a theoretical analysis of their steady-state performance along the lines of the popular ideal analyses, and then to compare theoretical and experimental results. It became apparent, however, during the course of the investigation...
that most of the standard approximate methods break down when the special problems of high-frequency application are faced; and also that the usual circuits are inadequate unless rather extensively modified. In some respects the difficulties are analogous to those that arise in high-frequency vacuum-tube amplifier design and construction. But while with vacuum tubes the problem of amplifying 20 KC is only slightly different from that of working with, say 10 c.p.s.; with the magnetic amplifier, the entire approach must be altered as the signal frequency is increased from very low values to cover the entire audio range.

In particular, the power supplies that must be used commonly have fairly high internal impedance. Methods exist for lowering supply impedance to arbitrarily small values, but these involve the sacrifice of available output voltage, or current, or of power supply simplicity. It was decided not to adopt these methods, but to examine the problem in the presence of high supply impedance. The problem to which this thesis is devoted is thus an investigation of the steady-state performance of audio-frequency magnetic amplifiers operated from high-impedance supplies.

Method of Investigation

The present investigation has an experimental and a theoretical phase, of which the former is by far the most important. The analysis is predicated upon the fundamental laws governing lumped-constant electromagnetic systems. These laws are applied to the case of the saturable reactor and yield nonlinear differential equations. In the case of the low-frequency amplifier, these equations can often be solved to give neat and concise results by the use of only few, entirely realistic, assumptions. Unfortunately, as frequency is increased, certain effects which are initially quite negligible come into the foreground to complicate the problem, so that the mathematical analysis becomes practically unmanageable. Rather than to attempt to find laborious solutions involving many gross approximations, it was consequently decided to focus attention primarily upon the experimentally observable behavior of the high-frequency amplifier.

Experimental Apparatus

The following is a list of the major equipment used in the experimental phase of the investigation:

(1) High-Frequency Power or Carrier Supply.

The unit employed for this purpose was a Williamson audio-amplifier built from a United Transformer Company Model W-10 packaged kit. However, a

7 Completely described in Instruction Manual for Williamson Amplifier Kit, Model W-10, published by United Transformer Company, 150 Varick Street, N.Y.
Freed Model 1958 output transformer was substituted for the UTC Model LS-61, which had proved unsatisfactory. The unit still had good output at 108 KC, at which frequency it was usually used. In all cases, connections were made to the 500-ohm tap of the output transformer.

(2) Audio-Frequency Power or Signal Supply

An amplifier with a 616 working through an audio-transformer in the output stage was used to supply signal power at frequencies up to 20 KC. Connections were again made to the 500-ohm tap.

(3) Audio-Oscillator

A Hewlett-Packard Model 200 C audio-oscillator usually furnished direct drive for the signal supply. The carrier supply was driven from a separate Wien Bridge oscillator2 synchronized to the output of the Hewlett-Packard.

(4) Carrier-to-Signal Synchronizer

This unit, consisting essentially of a driven multivibrator, was interposed between the Hewlett-Packard oscillator and the Wien Bridge oscillator. Its function was to fix the carrier frequency at integral multiple values of the signal frequency. It permitted clear oscillographic presentation of the modulated carrier and of complex hysteresis loops. The unit is more fully described in Appendix I.

(5) Control and Bias Supply

D-c power for these purposes was taken from a motor-generator set, and was varied by use of potentiometers.

(6) Meters

Ordinary panel meters were used for most measurements. Carrier and signal currents were measured with r-f milliammeters. Whenever the carrier current contained a d-c component, it was necessary to block it from the r-f meter by means of a large d-c milliammeter arrangement.

(7) Oscilloscope

A Tektronix Model 511A was employed for all critical observations. Oscillograms were recorded by means of a Dumont Type 255 Oscillograph-Recorder Camera.

---

2 Waveforms, Vol. 17, MIT Radiation Series, p. 120.
(8) Magnetic Cores

The experimental work was done with toroidal ferrite cores which were kindly prepared by members of the staff of RCA Laboratories at Princeton, N. J. Their characteristics are more fully described in later paragraphs.

(9) Rectifiers

Type IN34 Sylvania or Kemtron germanium rectifiers were used for all work.

(10) Resistors, Condensers, and Other Small Components

No special care was taken in selecting these circuit elements, except that carbon composition resistors were preferred to wire-wound units.

A block diagram of the experimental setup is given in Fig. MRI-13349.

Summary of Conclusions

Most of the high-frequency supplies now commonly available have fairly high internal impedance and the rectifiers usable at high frequencies have high forward resistance. Both properties are detrimental to good magnetic amplifier performance. The most useful type of cores have also relatively unsatisfactory magnetic characteristics which adversely affect the performance and make mathematical analysis unmanageable. Further work towards the development of useful high-frequency amplifiers should first of all be directed towards the development of more satisfactory power sources, core materials, and rectifiers.

II. The Low-Frequency Magnetic Amplifier

Fundamental Considerations

A subcommittee of the American Institute of Electrical Engineers has adopted the following definition:

"A magnetic amplifier is a device using saturable reactors either alone or in combination with other circuit elements to secure amplification or control." 9

The integrating circuit used for displaying hysteresis loops consisted of a 100 K resistor in series with a 0.002 \( \mu \)F condenser.

Although it will be shown later that, for an almost trivial reason, the above definition is not logically complete, it nevertheless describes accurately the group of devices now popularly known as magnetic amplifiers. The basic amplifier component is thus the saturable reactor. The rapid review of a few well-known experimental facts will lead to an understanding of its operation.

When a virgin (originally unmagnetized) ferromagnetic body is subjected to a monotonically increasing magnetizing force, the flux density within the body at first increases almost in direct proportion to the magnetizing force, but eventually the increase in flux density proceeds more and more slowly until finally the relation again becomes linear, but with a slope that is very much smaller than the observed maximum slope. The material is said to have become saturated. The relation between the magnetizing force or field intensity \( H \) and the flux density \( B \) (or total flux \( \Phi \)) is given by the mean or normal magnetization curve of the material. This curve describes the first basic non-linearity of a ferromagnetic material, and is, in general, different for different materials. The ratio \( B/H \) at any point on the curve is known as the permeability or ordinary permeability \( \mu \), and the slope of the tangent to the curve \( dB/dH \) is called the differential permeability \( \mu_d \). Other types of permeability are sometimes defined.\(^{10}\) Evidently, permeability is also a nonlinear function of either the field intensity or the flux density. Sample normal magnetization curves for three different materials are given in Fig. MRI-13350-a.

If the field intensity is reduced monotonically to zero, a somewhat different curve is followed, and usually at zero field intensity some residual flux density remains. If cyclic variations of magnetizing force are impressed, the flux density is also found to vary cyclically, but according to a double-valued relationship. The curve observed for very slow cyclic variations is called the hysteresis loop of the material; its area represents an energy loss called the hysteresis loss.\(^{11}\) The loop represents a second basic nonlinearity of the body. Three sample loops are shown in Fig. MRI-13350-b.

When the excursions of the field intensity are made very large, the maximum available flux density is called the saturation flux density \( B_s \); that obtained at zero field intensity the retentivity \( B_r \). The value of field intensity required to reduce the flux density to zero is known as the coercive force \( H_c \).

If the frequency in the cyclic variation of \( H \) is stepped up from a theoretical minimum of zero (d-c loop), the saturation flux density remains quite constant, but the coercive force appears to increase with frequency. This widening of the loop is indicative of additional energy loss in the body; the increase in loss above that due to hysteresis was formally attributed to eddy currents which are set up in the material as a result of electromagnetic induction. Observed results of the eddy-current effect do not, however, completely agree with predicted ones, and it is now generally conceded that other

\(^{10}\) Ferromagnetism, by Richard M. Bozorth, pp. 6-7.

\(^{11}\) Principles of Electrical Engineering, Timbie and Bush.
factors are also responsible. At any rate, the total energy loss is not linear with frequency. This effect characterizes the third type of important nonlinear behavior encountered with ferromagnetic materials.

A fourth type of nonlinearity becomes apparent when the body is heated. All ferromagnetic properties of the material, i.e., saturation, permeability, and hysteresis, exhibit variation with temperature. The permeability, for example, may remain quite constant over a wide temperature range and then drop sharply to a very low value. For another material, there may be a peaking of the permeability vs. temperature curve at some point. All ferromagnetic materials, however, lapse into the paramagnetic state at sufficiently high temperatures. The point at which this change occurs is known as the magnetic transition temperature or Curie point of the material. Sample curves are given in Fig. MRI-13350-c.

Although hysteresis, eddy current loss, and magnetic transition were introduced in the preceding paragraphs (because their understanding will later be required, and because they are, after all, basic ferromagnetic phenomena), only the property of saturation need be considered in the case of the low-frequency (60 or 440 c.p.s.) saturable reactor. High quality magnetic alloys have been developed which exhibit very small hysteresis loss and have Curie points very far above the normal operating range; and eddy current losses are kept low by the use of thin laminations. These alloys often have the additional advantage of exhibiting very sharp saturation: the permeability changes in the ratio of several thousand to one within a very small range of magnetizing force. The magnetization curve has a practically square corner; the material is known as "sensitive".

The Saturable Reactor

The most elementary magnetic control circuit, the single-core saturable reactor, consists of a ferromagnetic core wound with \( N \) power or load turns, and with \( N_c \) control turns. The load turns are connected in series with a load resistor \( R_L \) to a source of constant voltage a-c. The control turns are connected to a source of d-c voltage. A schematic diagram is shown in Fig. MRI-13351-a.

Viewed in somewhat over-simplified terms, the reactance of the load winding is a function of its inductance, which, in turn, is a function of the permeability of the core as given by the magnetization curve. This permeability may be varied by changing the d-c control current and thus the premagnetization or value of field intensity with no a-c applied. Control of the load current may thus be realized.

12 Bozorth, op. cit., p. 783, Fig. 17-8.
13 Bozorth, loc. cit., p. 713. Also see AIEE Standard Definitions of Electrical Terms (book) AIEE, 1942.
The single-core reactor is not useful practically. First, the varying flux in the core induces large voltages in the control winding which cause currents to circulate through control winding and d-c source unless a large impedance is present in the control circuit to prevent the flow of such currents. When such an impedance is present and sufficiently large to make the a-c component of control circuit current negligible, the control current is said to be forced, and the type of circuit operation is designated as constrained magnetization. When the induced currents are permitted to circulate freely, however, the case is one of free magnetization. The free magnetization (or free case) operation of the single-core reactor is trivial (since the reactor would become merely a short-circuited transformer), while constrained operation requires the use of very large chokes in the control circuit.

Secondly, the simple saturable reactor follows the law of equality of ampere-turns on an average basis. The gain on an ampere-turns basis is, therefore, unity. Much higher gains can be realized with the more practical circuits.

### Practical Magnetic Amplifier Circuits

The effect of the troublesome voltages induced in the control winding may be readily eliminated by using either a two-core reactor circuit, as shown in Fig. MRI-13351-b, or by using a three-legged construction for the reactor core (Fig. MRI-13351-c). In the first case, the induced voltages exist, but oppose each other; in the second, no net a-c flux links the d-c winding at all, so that the control winding has no a-c voltage induced in it.

Both constructions have been used for industrial applications.

The ampere-turns gain of a saturable reactor circuit may be raised from unity to very large values (ideally to infinity) by placing feedback windings on each of the cores. The external feedback or "Buchhold" type of magnetic amplifier is thus obtained. The circuit diagram is given in Fig. MRI-13351-d. Feedback may be accomplished also without the use of additional windings by placing rectifiers in series with the reactors. The resulting circuits are known as self-saturating, and are exemplified by the types known as half-wave, full-wave, and doubler circuits. (See Figs. MRI-13351-e, f, and g.) The half-wave circuit uses only a single core, and thus is also not useful from a practical viewpoint.

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The above listing includes the most useful general-purpose magnetic amplifier circuits. Many other circuits, and refinements of the above circuits, have been reported in the literature, but are usually intended for special uses.

**Methods of Analyzing Magnetic Amplifier Circuits**

No single, universally applicable procedure of analyzing magnetic amplifier operation exists, but many individual circuit arrangements have been solved by one means or another, and the theoretical results are often in close agreement with the experimental observations. One method sometimes used with success is analogous to vacuum-tube technique: it combines the use of a reactor no-load characteristic and of load lines to predict performance at a given condition of load. The load-line analysis is based on linear circuit theory; and, therefore, the method appears to give the best results with less sensitive materials, since with these the high harmonics of the load current contribute relatively little to its rms value.

The more frequently used method of analysis is the direct solution, in a more or less approximate manner, of the nonlinear equations which describe the system. Some of the approximations that are commonly made will be given below when this method is illustrated by examples. One of the major difficulties that is encountered is to find a satisfactory representation of the core magnetization curve. An accurate, but very laborious, process is a point-by-point solution from experimentally observed magnetization data. It is somewhat doubtful whether this type of solution is worth the effort that must necessarily be expended, since usually not the hysteresis loop, but the probably less accurate mean magnetization curve is chosen as the working curve. In addition, the salient features of analysis and performance tend to become obscured in a mass of computation.

---


Other representations of the magnetization curve involve the use of power series (usually with a finite number of terms), of transcendental functions, or of series of such functions. The formerly popular Froelich's equation has been all but discarded. The neatest and most elegant solutions, however, are perhaps obtained when the curve is simply approximated by a series of straight lines. Three line segments are often considered sufficient to describe the mean magnetization curve of sensitive materials; four to approximate the hysteresis loop. Figs. MK-13352-a, b, and c show various straight line representations. The curve of Fig. MK-13352-c indicates an abrupt change in differential permeability from infinity to zero at the instant of saturation. No known ferromagnetic material, of course, displays such characteristics; the curve is, therefore, attributed to a hypothetical ideal magnetic amplifier material which is taken to be the limiting case of sensitive materials.

Results of satisfactory accuracy have been obtained by use of all of the above representations, particularly, of course, when good agreement exists between the assumed curve and the actual magnetization data of the material. The straight-line approximations, best suited for sensitive materials, have the additional advantage of yielding solutions of great neatness and simplicity. Although the straight-line mean magnetization curve method is unfortunately not adequate for solution of the rather difficult case of the high-frequency amplifier operated from a high-impedance supply, it will later be applied to some new amplifier circuits, and it is, therefore, deemed advisable to offer some illustrative examples at this point.

Examples of Magnetic Amplifier Analysis

The principles outlined above will now be applied to obtain the performance of the single-core saturable reactor and the half-wave circuit. Although neither circuit is practically useful, their analysis clearly demonstrates the salient features of the theory, and will serve as an adequate model for the analytical study of the novel circuits introduced in Chapter IV.

20 See source cited in footnote 15; Appendices "A" through "C" for Computations with Cubic, fifth order, and Sinh Functions.

21 Timbie and Bush, op. cit.

Case 1. The Single-Core Saturable Reactor

Consider the circuit shown schematically in Fig. MWL-13353-a. The circuit, as shown, represents a rather general case: it will be reduced by successive assumptions to a very much simpler form. Using abbreviations as given in the List of Symbols, and applying Kirchhoff's and Ampere's laws, we may write for the load mesh:

\[ v(t) = L \frac{di}{dt} + Ri + N \frac{d\phi}{dt} \]  

(1)

and for the control mesh:

\[ v_c(t) = L_c \frac{di_c}{dt} + R_c i_c + N_c \frac{d\phi}{dt} \]  

(2)

and finally for the magnetic circuits:

\[ H \frac{dI_m}{dt} = N_i i_c + N_c I_c \]  

(3)

Making the assumptions, (a) that the amplifier is excited by a constant voltage source \( V_m \sin \omega t \) and controlled by means of a variable d-c source \( E_c \), (b) that the linear inductance \( L \) in the load circuit is negligible, and (c) that the control mesh inductance \( L_c \) is large enough to insure proper constraint of the amplifier, we may write:

\[ V_m \sin \omega t = Ri + N \frac{d\phi}{dt} \]  

(1a)

\[ E_c = R_i I_c \]  

(2a)

and

\[ H \frac{dI_m}{dt} = N_i i_c + N_c I_c \]  

(3a)

Assuming finally an ideal magnetization curve for the amplifier core, we may proceed to obtain a solution for the load current in the following manner:

Let attention be restricted to the mode of operation in which the amplitude of the flux excursions produced by the applied a-c alone is always less than the saturation flux \( \Phi_s \). When this condition exists, and when furthermore the control current \( I_c = 0 \), the ideal magnetization curve requires that the load current also be zero at all times. This may be seen from Eq. (3a), which reduces to:

\[ H = 0 = N_i \]  

(3a')
under the given restrictions. In an actual circuit, an exciting or magnetizing current \( I_0 \) will flow. This current, however, is small and practically independent of the control current.

When some nonzero value of \( I_c \) is caused to circulate in the control mesh, the core will be saturated over part of the cycle, the flux will be forced to remain constant at its saturation value, and Eq. (1a) yields:

\[
i = \frac{V_m}{R} \sin \omega t \tag{4}\]

Current continues to flow in accordance with this expression until some time \( t_1 \), when flux begins to decay in the core. \( t_1 \) may be found from Eq. (4) as:

\[
t_1 = \frac{R_1}{\omega} \sin^{-1} \left( \frac{\frac{R_1}{V_m}}{\frac{R_1}{V_m}} \right) \tag{4a}\]

Flux variations can only occur, however, when \( H \neq 0 \), and Eq. (3a) therefore indicates that, whenever the core is unsaturated:

\[
i = i_1 = \frac{N_c}{N} I_c \tag{5}\]

It is possible to obtain an expression for the flux change in the core by integrating Eq. (1a) from the desaturation time \( t_1 \) as the lower limit to any time \( t \) as the upper. There results for the flux at the time \( t \):

\[
\phi = \phi_s - \frac{V_m}{\omega N} (\cos \omega t - \cos \omega t_1) - \frac{R_1}{N} (t-t_1) \tag{6}\]

The saturation time \( t_s \) may now be found by letting \( \phi = \phi_s \) and \( t = t_s \) in (6) and solving the resulting transcendental equation by any suitable method. It is possible to show that the quantity \( V_m/\omega N \) is exactly equal to the amplitude of the symmetrical flux excursions occurring when \( I_c = 0 \); and, henceforth, the substitution

\[
\frac{V_m}{\omega N} = \Phi_m \tag{7}\]

will be used without further explanation.

For steady-state operation, integration of (1a) yields the further result that the load current has a cyclic average value of zero. On the other hand, when two cores are used, and when the load current is expressed in mean values over a half-cycle, there obtains:

\[
I N = I_c N_c \tag{8}\]
This is the law of equality of ampere-turns referred to previously, and indicates that a straight-line relationship of slope \( N_c/N \) exists between load current measured on a half-cycle average basis and control current.

The solution of the saturable reactor circuit is now sufficiently complete for the illustrative purpose for which it was intended. Load current and flux waves have been mathematically described, and can be sketched for any given value of \( I_c \). (See Figs. MRI-13353-b, c.) In addition, a measure of the performance of the circuit, a relation between output and input quantities has been given. Such a curve, whether theoretically or experimentally determined, is known as a transfer characteristic or transfer curve of the amplifier. Its slope is called the gain of the device. When load current is expressed in half-cycle average values, the ampere-turns gain of a saturable reactor is evidently unity. (This holds for the limiting case; for actual amplifiers the gain is always slightly less.) A transfer curve of a saturable reactor is shown in Fig. MRI-13353-d. It will be noted that the indicated linear relation between control and load currents exists only within a limited range. For larger control currents the core remains completely saturated, and the load current is determined solely by the applied voltage and the load resistance.

Since all components of the saturable reactor circuit are entirely symmetrical in their characteristics, the transfer curve displays symmetry about the ordinate.

Case 2. The Half-Wave Circuit

It was mentioned in the section on Practical Circuits, amplifier gain may be increased above the small value realized in a simple saturable reactor by the use of positive feedback. Analysis of feedback circuits based upon the ideal magnetization curve yields the result that a feedback ratio of unity leads to infinite amplifier gain. (In actual circuits somewhat larger ratios are required to accomplish this result.) Since the half-wave circuit is, in fact, a 100% feedback circuit, the \( h_{111} \) analysis must predict infinite gain. Another method, the so-called firing-point analysis, leads to results more nearly in accord with experimental observations, and this approach will, therefore, be used in the present instance.

Consider the circuit of Fig. MRI-13354-a in conjunction with the magnetization curve of Fig. MRI-13354-b, and the assumed rectifier characteristic

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23 Geyger, op. cit.


25 Geyger, op. cit.
of Fig. MRI-1335h-c. Note that the rectifier is taken to be an open circuit when a negative voltage exists across its terminals: it allows no reverse current flow whatever. In the forward direction it exhibits a certain constant resistance $R_f$.

The equation for the load mesh may be written as:

$$V_m \sin \omega t = R_l + N \frac{d\Phi}{dt} + e_x$$  \hspace{1cm} (9)

while for the control circuit we have:

$$I_c = \frac{E_c}{R_c}$$  \hspace{1cm} (10)

and for the magnetic circuit of the core, there applies:

$$H l_m = N_l + N_c I_c$$  \hspace{1cm} (11)

Because of the presence of the rectifier, negative current can never flow in the load circuit; but it is important to determine whether or not the load current can flow continuously under any conditions of circuit operation. If we assume that this is possible, then for all time:

$$e_x = e_f = i R_f$$  \hspace{1cm} (12)

and integration of Eq. (9) over a complete cycle leads to the result:

$$0 = (R + R_f) I$$  \hspace{1cm} (13)

Load current can therefore flow continuously only in a lossless current.

The process of solution can now be initiated by first considering the portion of the cycle, say between time $t_0$ and time $t_1$, during which the load current is continuously zero. In this interval, Eq. (11) yields:

$$H_{o l} l_m = N_c I_c$$  \hspace{1cm} (11a)

Since the load current can only be positive or zero, it follows that $H_0$ is the smallest possible value of field intensity that can occur. The assumed magnetization curve may be used to relate $H_0$ to the corresponding value of flux $\Phi_0$, and the flux must remain at this minimum point until, at $t_1$, the load current again goes positive. Due to the steep slope of the unsaturated portion of the magnetization curve, however, it remains negligibly small until saturation occurs, at time $t_s$. Under this approximation, integration of Eq. (9) results in:
\[(\Phi_s - \Phi_o) = \Phi_m (1 - \cos \omega t)\] (14)

which may be solved for the saturation time \(t_s\).

After saturation, no further flux change occurs, and (9) yields

\[i = \frac{V_m \sin \omega t}{R + R_f}\] (15)

The load current wave thus consists of truncated half-cycles of a sine wave, and resembles closely the plate current of a thyratron with resistance load. The wave shape is sketched in Fig. MRI-1335ii-d.

It is possible, on the basis of the above work, to develop many other expressions descriptive of circuit behavior,\(^26\) but for the present purposes (illustrations of methods of analysis) it will suffice to obtain a single additional result: the gain of the amplifier. It may be assumed for many materials that the unsaturated portion of the magnetization curve is a straight line, i.e., that:

\[\Delta \Phi = m \Delta H\] (16)

It can be shown\(^27\) that the following relation holds between the average load current and the control current:

\[\frac{\Delta \bar{I}}{\Delta I_c} = f \left[ \frac{m N_c N}{(R + R_f)^{1/m}} \right]\] (17)

The amper-turns gain \(G_H\) then is given by:

\[G = \frac{\Delta (N \bar{I})}{\Delta (N_c I_c)} = \frac{fm N^2}{(R + R_f)^{1/m}}\] (18)

Since \(mN^2/\mu_m\) is the unsaturated inductance of the core, we may write

\[G_H = \frac{f L}{R + R_f}\] (18a)

\(^{26}\) E. J. Smith, loc. cit.

\(^{27}\) For a collection of data, see Bozorth, op. cit. Also includes extensive bibliography.
Note that $Q_T$ bears some similarity to the $Q$ of a coil. The equation indicates that $Q_T$, exactly as $Q$, should rise linearly with frequency. It is well known, however, that the $Q$ of the actual coils behaves otherwise; first increasing with frequency, then remaining nearly constant over a more or less extended frequency range, and finally decreasing again. Experimental data will later be used to show that $Q_T$ exhibits a similar behavior.

Sample gain curves of the half-wave circuit are given in Fig. MRI-13354-e. Symmetry about the load-current axis no longer exists, but the curves are generally shifted to the left of that axis and the portion to the right of the minimum point also shows a much larger slope than that to the left. In view of the shift, it is necessary to use an extra bias winding and supply if it is required that minimum load current be delivered at zero control current.

Measures of Amplifier Performance

So far, the ampere-turns gain and the current gain have been introduced as criteria by which amplifier performance can be evaluated. Other criteria exist. The power gain $G_p$ is frequently given. It is the ratio of load power to control power, and is, as the ampere-turns gain, quite constant over the useful range of operation. Another significant quantity is the time constant $T$. If a step change is made in the input, the output will also change, but not instantaneously. Briefly, the time constant is the time required for the output to reach 63% of the final change. The time constant in cycles, $T_n$, is the time constant in seconds multiplied by the source frequency. Sometimes a figure of merit $A$ is used, and is defined as the ratio of power gain to time constant in cycles.

The Influence of Magnetic Amplifier Component Characteristics Upon Performance

A considerable literature exists on the subject of this heading, although discussions are often subordinated to expositions of principles and methods, and are used principally to justify discrepancies between theoretical and observed results. This attitude is understandable in view of the fact that the best available components for low-frequency amplifiers are so very nearly ideal. The small residual deviations from the ideal produce effects which are of second-order importance in all normal cases of operation.

As the frequency of signal (and carrier) is increased, however, these very deviations attain increasing importance until they finally become major factors in determining the performance of the amplifier. They must consequently be examined, at least qualitatively, before the problem of the high-frequency amplifier can even be approached.

The amplifier components to be examined in turn are the reactor, the rectifiers, the source, and capacitors (which are sometimes present).
(1) The Reactor

It is by now amply evident that the saturable reactor is the heart of all magnetic amplifier devices, and therefore deserves first consideration; and the heart of the saturable reactor is its ferromagnetic core. The windings are of course essential parts, but it will be assumed that, in the frequency range under consideration, their properties are true constants, i.e., that skin effect is negligible, and also that distributed capacitance is unimportant.

With regard to the core, it must be further assumed that the only phenomena that need concern us are those listed in the section on Fundamental Considerations, namely, saturation, hysteresis, eddy currents, and magnetic transition. A host of others has been observed and reported in the literature.\textsuperscript{27} Magnetostriction, magnetic skin effect, the Barkhausen effect, and the influence of core geometry are but a few, rather well-known examples. A recently published work of encyclopedic proportions\textsuperscript{28} is available to the reader interested in these more advanced aspects of the science of ferromagnetic materials. But it is the duty of the engineer to winnow the chaff from the wheat, and the above-mentioned subjects simply cannot be further discussed within the limits of this report.

Although it will later appear that the eddy-current problem and the associated question of core material resistivity is of small significance in determining the performance of high-frequency amplifiers, it plays a great part towards indicating the proper choice of core materials and design. It is, therefore, essential to examine this problem rather closely.

Ordinary transformer theory indicates that flux changes within a magnetic core induce emf's which act so as to produce current flow within the conducting portions of the core itself. These eddy currents react back upon the source which produces them mainly in the following ways: (1) the effective permeability of the material is reduced, (2) the energy loss in the material is increased, and (3) there is a time lag between the field strength and the corresponding induction (flux density).\textsuperscript{29} Although undoubtedly the first and third of these effects are important in their influence upon performance; it is the eddy current energy loss which establishes an upper limit to the useful frequency range of any given core.

\textsuperscript{27} For a collection of data, see Bozorth, op. cit. Also includes extensive bibliography.

\textsuperscript{28} Ibid.

\textsuperscript{29} Op. cit., p. 769.
When it is assumed that the given material has permeability and resistivity independent of variation in both space and time coordinates, it becomes possible to solve Maxwell's equations, at least for certain geometries and time variations of applied field. This yields the result that the power dissipated by the flow of eddy currents is inversely proportional to the resistivity \( r \) of the material.\(^{30}\) For engineering purposes, this resistivity will generally be taken as the effective value for the entire core, and this value may differ considerably from that intrinsic to the material itself. The quality of the insulating material between laminations or magnetic particles will be of prime importance in determining this difference.

As was mentioned earlier, eddy currents also cause a widening of the "hysteresis" loop. The observed dynamic loop thus cannot yield the actual relation between the instantaneous core flux and applied field. A method for correcting the observed loop to eliminate this effect has been proposed.\(^{31}\)

Occasionally it is found desirable to calculate simultaneously with the magnetic and loss characteristics of the material. One approach to this problem has been to postulate a complex permeability.\(^{32}\) This method appears to enjoy considerable popularity in Germany, but is used very seldom in this country.

Now, it is obvious that any power dissipated through the agency of eddy currents must be supplied from the power source itself. This will necessitate a flow, in the load winding, of current relatively independent of control current. Generally speaking, the result will be to increase the minimum possible value of load current, to decrease the efficiency, and to alter the wave shape of the amplifier output. The solution of Maxwell's equations also indicates that eddy-current power loss increases as the square of the frequency of the applied field variation. The effect of increasing frequency is thus to aggravate all problems connected with eddy currents. To sum up, the overall effects of eddy currents are (1) to decrease in the available range of amplifier operation caused by an increase in minimum possible load current, (2) a decrease in amplifier efficiency, and (3) distortion. It may be concluded that high eddy current losses are incompatible with good amplifier performance.

With regard to saturation, any deviation in core characteristics from the ideal will adversely affect the performance. No explicit relation exists, but experience has shown that the most nearly ideal core materials offer the highest amplifier gains; and that gain decreases as the magnetization curve exhibits increased rounding at the knee, and smaller slope in the

\(^{30}\) Magnetic Circuits and Transformers, Members of Staff of MIT Department of Electrical Engineering, Chapter V, John Wiley & Sons, N. Y. 1943. Also Bozorth, op. cit., Chapter 17.


unsaturated region. It should be noted that design considerations indicate that low-frequency amplifier cores should exhibit large $B_s$, but this requirement is based upon conditions external to the reactor itself. For the high-frequency amplifier the requirement appears to be the exact opposite.

Hysteresis loss produces about the same effects as eddy-current loss in influencing performance. It depends, however, more nearly upon the first, rather than the second, power of the frequency, and is apparently little influenced by the core geometry. Whereas eddy-current power loss may be greatly reduced by properly laminating the core structure, hysteresis loss cannot be materially changed. Being largely beyond the control of the engineer, the desirable reduction of hysteresis losses is more properly an objective of the metallurgical field.

Magnetic transition, finally, is potentially the source of the most radical changes in performance. Fortunately, the commonly used core materials have Curie points far above the normal ambient temperatures. It is possible to conceive, however, of an amplifier in which hysteresis loss is quite large and the Curie point quite low. In such a case, a fortuitous combination of circumstances might easily cause the core to pass into the paramagnetic state. It will be seen that high-frequency amplifiers can undergo such changes. The interaction between hysteresis loss and temperature change in magnetic characteristics is also capable of initiating transients in the output, in other words, noise. This question will be discussed more fully in a later section.

(2) The Rectifiers

As in the case of the reactor core, attention must here also be restricted to a few of the most outstanding rectifier properties, those of forward and reverse resistance, and of self-capacitance. Numerous other phenomena have been encountered in this case also.

The requirement that the rectifier forward resistance shall be several times smaller than the load resistance to be used is almost too trivial to mention, were it not for the fact that high frequency rectifiers tend to have very high forward resistances. It is clear that circuits employing rectifiers in external or self-feedback loops would function very inefficiently if the rectifier forward resistance were at all comparable in value with the load resistance. In addition, the gain of the half-wave circuit and those based upon it depends inversely upon the value of total a-c mesh resistance. (See

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33 See publications by A. G. Milnes, Entries 748, 825, 896, of the Miles bibliography. Also, Lehmann, op. cit.

p. 16.) High rectifier forward resistance thus reduces both the efficiency of circuit operation and the amplifier gain.

The reverse resistance, on the other hand, should be as high as possible. (In the ideal case, infinite reverse resistance is postulated.) The closest approach to such an ideal rectifier is a vacuum tube and, indeed, vacuum diodes have been used for experimental work where it was essential that rectifier leakage be almost completely absent. The practical amplifier is often built for service too strenuous for the vacuum tube and, consequently, metallic rectifiers are made use of. Such rectifiers always exhibit reverse leakage, although the reverse resistance may be very high.

Flow of leakage current will, in general, reduce amplifier gain. In the half-wave circuit, for example, leakage current flows during the time when the core is premagnetized or "net" for the next cycle, and it flows in such a direction as to demagnetize the core. In addition, it slightly reduces the average value of load current, but this effect is small in comparison to the demagnetization.

Rectifier self-capacitance is perhaps the most important from the viewpoint of the high-frequency amplifier, as it not only affects the performance at all frequencies, but also establishes an upper frequency limit to the usefulness of any given rectifier type. The exact effect of capacitance is not readily described, however, as experiment shows that, in a given circuit, the presence of condensers in parallel with the rectifiers may actually increase the amplifier gain. However, when the shunting capacitance becomes excessive, the gain is always reduced. Eventually the amplifier ceases to be useful. For 60 c.p.s. amplifiers, for example, shunting capacitances larger than 1 or 2 microfarads might make the gain impractically low. Now, measurements on selenium cells "indicate a capacitance of about 0.02 microfarad per square centimeter of rectifying area," and, consequently, it is to be expected that selenium cells of the normally available sizes will lose their usefulness at frequencies rather less than 10 kc. In fact, one author states that the capacitive effect (he apparently takes this to be simply a demagnetizing effect like that of leakage) with sensitive amplifiers operated...

35 Lehmann, op. cit.
38 Conrath, op. cit.
from a 400-cycle power supply is sometimes greater than that of leakage.\textsuperscript{39}

(3) The Power Supply

By far the greater portion, as we have seen, of core and rectifier secondary properties adversely affect magnetic amplifier performance. As has been stressed previously, the effects are largely negligible at normal power frequencies, and become important only as the audio-range is approached. Their elimination, however, is closely dependent upon the development of intrinsically better high-frequency core materials and rectifiers, and in this province the electrical engineer must usually follow the lead of the metallurgist and the solid-state physicist. But there exists yet another, and major, obstacle to the construction of satisfactory high-frequency magnetic amplifiers, and that is the lack of suitable power supplies. This problem is entirely within the field of electrical engineering and, as the development of practical h-f amplifiers apparently hinges on its solution, much more attention will probably have to be devoted to it than it has been given in the past.

The statement that amplifier performance is closely related to the characteristics of the power supply would be a trivial truism, were it not indeed the case that this obvious fact has become obscured in the majority of the papers written on the subject. To the best of the author's knowledge, the assumption has been made in every treatment of the amplifier that the supply was of the constant voltage type characterized by low (or zero) internal impedance. In the case of 60-cycle amplifiers, which commonly operate either directly, or through good, low-regulation, transformers, from the power line, this assumption is, of course, fully justified. Because it is so well justified, and so often used, it has so much become a matter of habit that it is occasionally not even mentioned. An example of such omission is the AIEE magnetic amplifier definition itself. (See p. 5.) That definition can logically lead an intelligent layman well versed in the rules of grammar to the obviously fallacious conclusion that a set only of saturable reactors can constitute a magnetic amplifier.

Because of the almost universal use of the above assumption, amplifiers operated from other types of supplies have never, to the best of the author's information, been studied or reported in the literature. External conditions, however, may sometimes dictate the use of other than constant voltage supplied, and therefore the consideration of source characteristics appears to be much more than merely academic in interest.

Let us see what types of supplies in addition to the constant-voltage ones that are usually postulated might be encountered in practice. Broadly speaking, we shall find some that can be represented as combinations

\textsuperscript{39} Ibid.
of entirely linear elements, and thus subject to Norton's and Thevenin's theorems; and others that saturate in some manner or other, and thus possess nonlinear characteristics. All physically realizable supplies are of course, strictly speaking, in the nonlinear group, but may closely approximate linear behavior over some range of operation. Conversely, supplies that show definite nonlinearities when employed over the full normal operating range may be linear over small portions thereof. Vacuum tube supplies tend to fall into this last category.

The two limiting cases of linear supplies are the constant-voltage zero-impedance, and the constant-current, infinite-impedance, types. It is easy to predict the effect upon the performance of any conventional amplifier circuit when its normal constant-voltage supply is replaced by a constant-current supply: it will cease to amplify. The action of normal amplifier circuits depends upon the ability of the reactor to affect the value of load current, and this is manifestly impossible when the current supplied must remain a constant. Some circuits could not even be safely connected to a constant-current supply: The rectifier in the half-wave circuit, for example, would be destroyed by attempting to interrupt the supply current.

This complete failure of conventional amplifier circuits to operate from a constant-current supply is but an example of the influence of supply characteristics upon amplifier performance. Consider another, more realistic, but also more difficult example, that of a supply which can be represented as a true voltage source in series with a constant resistance \( R_g \), connected to an iron-cored coil. For simplicity, all leakage flux will be neglected, so that the equation of the system may be written as follows:

\[
V_m \sin \omega t = R_g i(t) + N \frac{d\phi}{dt} (i, t) .
\] (19)

It is desired to find the wave shapes of current, flux, and coil voltage.

Since the equation is nonlinear, no straight-forward method of solution is possible. In this case, a method of successive approximations was decided upon. It is first assumed that \( R_g = 0 \). The flux and coil voltage will both be sinusoidal, but the current will already be distorted. A common graphical construction\(^{40}\) may readily be used to obtain the current wave shape. It would now be possible to resolve this current into a Fourier series of its component harmonics, but it was considered simpler and possibly more accurate to carry out the entire process graphically. The next step is to multiply the current \( i(t) \) so obtained by \( R_g \), and to subtract the resulting IR term from the supply voltage. This yields the first approximation to the reactor voltage \( V_L \). Since the hysteresis loop requires a knowledge of the flux before a second

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approximation to the coil current may be constructed from it, it is now necessary to integrate the reactor voltage function. This is done graphically by summing areas. The first graphical construction is now carried out for a second time, and thus second approximations to coil current, and voltage, and a third approximation to the flux wave are obtained. From here on, the process is repeated as many times as is necessary to obtain wave shapes which no longer change appreciably under successive approximations. The main steps and results of the process are indicated in the various parts of Figs. MRI-13355 and MRI-13593. Fortunately, the second approximation to the coil current so nearly coincided with the first attempt that it was deemed unnecessary to continue the construction, which, it must be admitted, is rather laborious.

It should be noted that the voltage appearing across the coil itself is quite heavily distorted. In view of the fact that $R_g$ is part of the supply, only this voltage, and not the source voltage $V_m \sin \omega t$, can be observed. Another point of interest is the double peaking of the coil voltage wave form. This effect can become much more pronounced than appears from the example, and in certain cases, the voltage wave is so heavily charged with harmonics that it has the appearance of a repeated train of pulses, rather than of a sine wave.

All these points will later be implemented by experimental evidence. At this instant it was desired only to offer a single example of the possible effect of supply impedance upon the functioning of a magnetic amplifier, and it must in this respect be concluded that, at the very least, certain of the observable wave shapes found to exist during circuit operation will not possess the sharp and clean-cut qualities to which the conventionally operated amplifiers have accustomed the engineer. There is, however, a far more serious disturbance introduced by high supply impedance. Amplifier gain will suffer increasingly as the internal impedance goes up. The extreme of a constant current source has already been discussed. Another example is a vacuum-tube amplifier, such as was employed in the experimental work. Such an amplifier, for a given amount of signal input, might deliver 28 ma rms into a 500 ohm load, and 120 ma rms when practically short-circuited. (These are representative values obtained by actual test.) Such a supply will inevitably thwart the attempts of any normal amplifier circuit to vary the amount of load current over an adequately large range.

To sum up, then, the conventional magnetic amplifier circuits exhibit their best performance only when operated from constant voltage supplies. The presence of any internal impedance leads to reduction in gain and in range of operation and to wave shape distortion; the limiting case of infinite internal impedance makes operation impossible.
(4) Capacitors

When capacitors are included in magnetic amplifier circuitry, it is possible to obtain many different, and often extremely interesting, results. The gain may be increased or decreased, the transfer curve shifted, instabilities introduced, tuning accomplished, etc. Because the results depend greatly upon the exact nature of the circuitry, however, and because they are, indeed, sometimes quite erratic and unpredictable, no general discussion of their influence on performance is possible. Some experimental results will be offered in a later chapter, but no attempt will be made there to give a theoretical verification, for the subject is still for the most part in the experimental stage.

III. Problems Peculiar to High-Frequency Application

So far only a general view of the results of nonideal behavior of amplifier components has been given. Specifically, no attempt has been made to evaluate the relative importance of each deviation from the ideal. The reason for this omission lies in the fact that the relative significance of each property depends to a considerable extent upon the frequency range in which the amplifier is to be used. It is, therefore, necessary to understand how amplifier application outside the usual power-frequency range comes about.

Dependence of Carrier Frequency Upon Signal Frequency Range

In a recent paper on "The Use of Ferrite-Cored Coils as Converters, Amplifiers and Oscillators," V. D. Landon states with respect to the power-frequency amplifier:

In this case the amplification seems to depend on the difference in frequency between the signal and the 60-cycle exciter voltage.

He continues:

Obviously a higher frequency exciter voltage may be used and then the frequency of the signals may be increased.

The first statement is rather questionable since it appears to indicate that the signal frequency may be either higher or lower than the exciter frequency, but even if it is accepted, it does not follow as an obvious consequence that an increase in signal frequency requires a simultaneous increase of carrier frequency. A statement by J. M. Manley is somewhat more to the point:

\[1\] The Use of Ferrite-Cored Coils as Converters, Amplifiers, and Oscillators, V. D. Landon, RCA Review, Vol. 10, September 1949, pp. 387-95.

...the ratio of power taken from the carrier source to that taken from the signal source is always as great or greater than the ratio of their respective frequencies if no losses are considered.

The statement is given without proof. At any rate, there appears to be general agreement that gain or amplification can only be realized when the carrier frequency is in excess of the signal frequency. It is possible to estimate the actual ratio that is required for a reasonable amount of amplification from considerations of time response.

The definition of time constant was given in the section on "Measures of Amplifier Performance" of the previous chapter. Physical reasoning indicates that such circuits as the half-wave circuit cannot possibly have a time constant of less than one cycle of the carrier frequency, since the change in control may occur at a time when flux build-up has already been completed and thus cannot affect the output until the next cycle. Experimental results, however, show that actual amplifiers require at least 3 or 4 cycles of the carrier frequency, and consequently could not adequately follow a signal whose frequency exceeds about one-third of the carrier frequency. More precise figures are given by E. L. Harder and W. F. Horton who investigated the response time of magnetic amplifiers. They write:

Representation of the magnetic amplifier as a single delay system may be tested by comparing its frequency response with that of a theoretical single delay. The theoretical curve for the single delay drops off in voltage response to 50% (voltage) at a signal frequency of 2.55 cycles per second for the test amplifier which has a time constant of 6.5 cycles, measured with a step function. The test amplifier follows the theoretical curve reasonably well, indicating that representation as a single delay is justified.

Table I shows the theoretical possibilities of passing various frequencies. To obtain 87% voltage response (75% power response) even with a two cycle time constant,...... the signal frequency must not be over 1.6% of the supply frequency, i.e., 3 c.p.s. for a 60 cycle supply. Fifty percent power could be obtained at 6% or approximately 5 c.p.s., 25% power at 13.8% or 8 c.p.s. Below this point power varies inversely as signal frequency squared, or voltage inversely as the signal frequency. That is, they drop off at 6 dB per octave. Thus serious attenuation is encountered unless the supply frequency is at least ten times the signal frequency.

TABLE I

Response of Single Delay System

<table>
<thead>
<tr>
<th>Output*</th>
<th>Input</th>
<th>Voltage Power</th>
<th>$f_0$</th>
<th>$1$ cycle</th>
<th>$2$ cycles</th>
<th>$5$ cycles</th>
<th>$10$ cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>86.6%</td>
<td>75%</td>
<td>0.092</td>
<td>0.092</td>
<td>0.016</td>
<td>0.019</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>70.7%</td>
<td>50%</td>
<td>0.160</td>
<td>0.160</td>
<td>0.060</td>
<td>0.032</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>25%</td>
<td>0.276</td>
<td>0.276</td>
<td>0.138</td>
<td>0.055</td>
<td>0.028</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>1%</td>
<td>1.60</td>
<td>1.60</td>
<td>0.80</td>
<td>0.32</td>
<td>0.16</td>
<td></td>
</tr>
</tbody>
</table>

Practically speaking, then, reasonable gains are not possible until the carrier frequency exceeds the signal frequency by a factor of eight or more. Only in this sense does amplification depend upon the "difference in frequency" between the signal and control voltages.

It follows as a result of these considerations that an amplifier designed to cover the entire audio-frequency range, i.e., up to about 15 - 20 kc, must have a carrier supply capable of furnishing somewhat more than the desired load power at frequencies of 120 - 160 kc. And thus, while for the 60-cycle and 100-cycle amplifiers the need for an a-c power supply constitutes at best a distinct advantage and at worst a minor inconvenience, the same necessity is a serious hindrance to the development of satisfactory and practical magnetic audio-amplifiers. The difficulties are connected not only with the supply, but also with the amplifier components themselves. The general concepts of Chapter II can now be applied with the high-frequency amplifier in mind to evaluate the relative significance of the component properties with a view towards designing as successful an amplifier as seems possible at the present time. This evaluation, unfortunately, must be carried out without the guidance of a satisfactory analysis of magnetic audio amplifier performance, for the very magnitude of the secondary characteristics of the components eliminates all possibility of completing a theoretical analysis based on realistic assumptions. The fully (or nearly) idealized case can be discussed, but has little practical value in view of the actual properties of the component parts. All further attempts at analysis will, therefore, be deferred until a discussion of these properties is completed.

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* Based on 100% for d-c signal.

** $\frac{f}{f_0}$ Signal frequency/supply frequency

$T$ time constant in cycles of supply frequency.
I. Component Characteristics at Higher Frequencies

(1) The Reactor Core

It has so far been noted that the carrier frequency must be kept eight or more times larger than the signal frequency, and also that the eddy current power loss increases as the square of the excitation frequency (p. 18). The hysteresis loss only increases in proportion to the excitation frequency, and thus, at least for metallic materials, constitutes the lesser of two evils. At any rate, beyond the careful selection of materials, not much can be done by the magnetic amplifier engineer to affect hysteresis loss.

The requirement for reasonably efficient operation at high frequencies may then, in practical terms, be translated into that of keeping the eddy-current losses at a minimum. In essence only two approaches have been made; one involves the increase of the resistivity of the core material itself; the other the subdivision of the core into suitable portions, such that the length of the eddy current paths is increased, or their cross-sectional area reduced, or both. The second method leads to laminated cores and to powder cores. As it is difficult to increase substantially the intrinsic resistivity of ferromagnetic metals, the first approach has led principally in the direction of nonmetallic ferromagnetic bodies.

Each of the methods mentioned has merits and defects. Powder cores have the principal advantage that their losses are so effectively reduced that they can be used at frequencies of many megacycles. They are, however, in the author’s opinion, quite useless as magnetic amplifier cores. The resinous plastic which binds the powder particles into a solid structure not only acts as an insulator between particles and so reduces losses, but also introduces airgaps throughout the core, and thus sharply cuts its effective permeability.

The effective permeability $\mu_e$ of a total core made of sufficiently fine iron particles having an intrinsic permeability $\mu_i$ is given by

$$\mu_e = \left( \frac{V_i}{\mu_i} + V_b \right)^{-1}$$

where $V_i$ is the fraction of the total volume occupied by the iron, and $V_b = 1 - V_i$ is the fraction of the volume occupied by the nonmagnetic resin coating between the particle. For a practical core with $\mu_i = 80$ and $V_i = 0.95$ (3-micron particles), $\mu_e$ is only 16. There is little advantage in going to higher intrinsic

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Carbonyl Iron Powders (pamphlet) General Aniline and Film Corporation, 1144 Madison Ave., N. Y. Also, Crolite Magicores (Catalog 46) Henry L. Crowley and Co., Inc., West Orange, N. J.
permeabilities here, because \( V_B \) limits \( \mu_0 \) to a maximum value of 20 even if \( \mu_1 \) were infinitely large. Further reduction in \( V_B \) has been impractical because thinner insulating coating (binder) results in (1) insufficient mechanical strength, and (2) increased electrical contact of the particles. The increased electrical contact increases the effective particle size and eddy-current path-length.\(^{45}\)

The air gaps introduced by the binder, do not, however, much affect the saturation flux density, which, for the powder core, is close to that of the solid ferromagnetic material. The mechanical strength of the powder core is much less than that of the original metal, and this factor makes the production of cores with small cross-sectional areas difficult. Now, as later discussion will make clear, design considerations demand that cores intended for use in high-frequency amplifiers shall have relatively low \( B_s \), as high a permeability as can be achieved, and relatively small cross-section. Although powder cores have proven quite valuable to the radio engineer, they have properties exactly opposite to the desired ones in the above important respects, which apparently more than offset their low-loss quality and make their successful application to magnetic amplifiers unlikely.

The opposite is true of cores made up of metal alloy laminations, usually wound of strip material in a toroidal form. Such cores have been prepared (at least for experimental purposes) from laminations as low as 1/8 mil in thickness, and have reportedly been used at frequencies of several megacycles without excessive losses. These strip-wound cores can be made of the most sensitive materials available to the magnetic amplifier designer, and for this reason alone may eventually be found to be the best solution in the audio-frequency range. They possess essentially the saturation characteristics of the base material, and thus saturate at rather high values of flux density. This apparent design disadvantage is greatly nullified, however, by the fact that such cores can be prepared with exceedingly small cross-sectional areas. (See Appendix II.) Although laminated, strip-wound cores are very delicate, both electrically and mechanically, they are even now preferred by a number of workers in the field.

Incidentally, the tendency has been to use the thinnest available laminations for high-frequency use, and stock as thin as 1 mil for 60-cycle application. No doubt the eddy-current problem has been considered sufficient justification for this practice. Recent research tends to indicate that there may exist, for any given application, an optimum lamination thickness. This hypothesis is more fully discussed in a thesis being written concurrently with the present one by R. Zarcuni, also at the Polytechnic Institute, and therefore cannot be further considered.

Let us now revert to the basically different approach of increasing the intrinsic resistivity of the core material. Historically, metallurgists have attacked the problem of developing better magnetic materials from two points of departure: elemental iron, and a ferromagnetic iron oxide known as magnetite. The latter possesses, even in its natural form, the advantage that its resistivity is about 1000 times that of metallic iron. With this oxide as a basis, ferromagnetic compounds have been developed which are known as ferrospinels or ferrites. It must be emphasized that they are true compounds and entirely different from the mixtures which constitute powder cores. In the final form of manufacture they are homogeneous crystalline structures throughout and contain no binder whatsoever. In physical appearance they usually resemble black or grey masses of ceramic, break with a somewhat shell-like fracture, and have a hardness approximating that of glass.

The physics of ferromagnetic spinels, the methods of their manufacture, and certain of their characteristics, are rather fully described in a paper by R. L. Harvey, I. J. Hegyi, and H. W. Leverenz which is entitled: "Ferromagnetic Spinels for Radio Frequencies". Another publication, "Magnetic Ferrites", by G. L. Snyder, E. Albers-Shoenberg, and H. A. Goldsmith, contains the results of tests on a number of different, commercially available, ferrite materials. The indisputably very important fact of high volume resistivity is stressed there, and a figure of \(10^9\) ohm-cm at room temperature is given for one of the materials. The values given for five others vary between \(2 \times 10^3\) and \(3 \times 10^7\). The method of measuring this resistivity is unfortunately not discussed, and the practical meaning of the values cited is somewhat doubtful in view of the following observations reported by Harvey, Hegyi, and Leverenz:

One might expect that ferrospinels having high electric resistivity would be completely devoid of eddy-current loss. However, this is not borne out by experimental data. For example, two ferrospinels which were measured, were found to have values of volume direct-current resistivity which differed by a factor of \(2 \times 10^4\), and yet they had the same eddy-current loss constant. This leads to the belief that in a polycrystalline structure of this kind, the resistivity may not be uniform throughout the body. Indeed the experimental data strongly indicates that the crystals may have low resistivity and that the crystal boundaries are responsible for the high overall resistivity.

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46 Ibid.
47 Ibid.
Whether the exceedingly high values of resistivity cited above are practically meaningful or not, the experimental fact remains that eddy-current losses in ferrites remain negligible over wide frequency ranges. It has been noted that eddy-current power loss leads to a widening of the apparent hysteresis loop. The cores used by the author for most of the experimental work were tested at frequencies between 20 kc and 100 kc, and visual inspection of oscillographic patterns revealed no detectable widening between these limits. Furthermore, the materials have proved sufficiently good from the standpoint of losses to permit their use in saturable elements at frequencies about 100 mc. The GG3 Increductor is an example of such application.

It is seldom, however, that we get something for nothing. The point to be made now will be most easily presented in conjunction with some triaxial diagrams of the ternary ferrite system NiO - ZnO - Fe₂O₃, which is rather fully described in the Harvey, Hegyi, and Leverenz reference. Consider the diagram of Fig. MRI-13356. (In passing, it is interesting to note that such a triaxial plot is really a planar representation of a five-dimensional curve, with the constituents along three of the axes, the parameter along the fourth, and the independent variable along the fifth. The two-dimensional mode of plotting is perhaps possible only because three of the variables are restricted between the limits of zero and unity, and their sum must always equal unity.) Every point within the equilateral triangle represents a different composition whose ingredients may be read directly from the coordinates of the point.

Now examine the charts for permeability, Q, and Curie point, as given in Figs. MRI-13356, MRI-13357, and MRI-13358, in conjunction with each other. Note that the desirable qualities of high permeability, high Q (or low losses), and high Curie point are not simultaneously realizable in any single mixture of the system. The region of high permeability, for example, coincides with that of rather low Q and very low Curie temperature. The lowest losses occur in a region of undesirably low permeability. The permeability in particular falls off quite sharply with relatively small changes in composition. As a result, the production of reasonably large quantities of cores exhibiting reasonably constant magnetic properties is a rather difficult problem in quality control. The author does not pretend to understand this obstinate attitude on the part of nature which seems determined to thwart the development of satisfactory high-frequency, ferrite-cored, magnetic amplifiers. In any case, whatever core composition is chosen, the selection inevitably represents a compromise between the various desirable properties.

Fortunately, ferrite metallurgy is not limited to the nickel-zinc-iron system. Numerous other metallic oxides can be used in the mixes to alter the resultant characteristics. There is, indeed, no reason to remain within ternary systems. Research is now going on in the laboratories of the larger ferrite-core manufacturers, such as General Ceramics and Steatite Corporation, 49

The C. G. S. Increductor (pamphlet), C. G. S. Laboratories, Inc., (Stamford, Conn.), 1951.
makers of the "Ferramic" materials, and W. V. Philips' Gloeilampen-Fabrieken, originators of the "Ferroxcube" line. Some excellent results have already been reported, among them a "square-loop" material by General Ceramics. It must be recognized that, compared to the work that has been done on ferromagnetic metals, the spinel field is still in its infancy. There is hope that entirely satisfactory materials manufactured under close quality control to insure uniformity will be available in the future. Let us see what single property of the cores most urgently requires improvement.

It has been noted that the eddy-current problem is rather neatly disposed of by the use of ferrite cores; that materials can be obtained which couple reasonably high permeability with relatively low total loss; (see Figs. MRI-13358 and MRI-13359) and it must now be mentioned that the saturation flux densities of ferrite materials are small, being in the range from 1000 to 5000 gauss. In view of all these advantages, what ground still remains for dissatisfaction? The major failing of the available ferrites is that a very low transition temperature usually characterizes the otherwise most suitable mixtures.

In Chapter II, the direct bearing of Curie temperature upon amplifier performance was hinted at, and the possibility of the generation of false signals or noise indicated. As may be seen from Fig. MRI-13359, the Curie point for a portion of the nickel-zinc-iron system can occur below 100°C, and actually falls close to 150°C for what are, in the author's opinion, the most useful mixtures. Even in the interior of the core, the temperature therefore cannot exceed this range of values without greatly modifying the characteristics of the amplifier. More serious still is the fact that over a very considerable temperature range below the Curie point, the magnetic characteristics are often not constant with temperature. The exact nature of the temperature dependence varies with the composition of the material. A few typical curves gleaned from the manufacturers' publications are reproduced in Fig. MRI-13359. Although there only the dependence of the initial permeability upon temperature is plotted, most, if not all, the other ferromagnetic properties also show variation with temperature. The following experimental results were noted with respect to hysteresis and saturation flux density:

It was recognized at an early stage of the experimental work that this temperature dependence would seriously affect the accuracy of all data and conclusions drawn therefrom. Precise temperature control would have involved the


51 Snyder, Albers-Schoenberg and Goldsmith, op. cit.

52 Boucorth, op. cit., Chapter 1h.
expenditure of a great deal of time, however, and was deemed unnecessary, as it would not have led to more practically realistic results. No practical amplifier can be expected to have the advantage of such ambient temperature control, and consequently all measurements were made at the prevailing room temperature. The temperature within the core and in its immediate vicinity was, of course, influenced by the heating of the core, and thus an operating amplifier somewhat tends to establish its own ambient conditions. All results to follow (except where specifically noted) are based on tests in which sufficient time was allowed between readings for practically complete attenuation of all heating transients. When a set of hysteresis loops is taken under such conditions, as shown in the oscillographs of Fig. MRI-13360-a, it is noted that the locus of the endpoints of the loops, which is the a-c mean magnetization curve, displays a slight downward slope for large saturation. This anomaly can be most easily explained by supposing that the saturation flux density for the cores under test decreases with increasing temperature, at least in the range in which the testing was done. This hypothesis is supported by the evidence contained in Fig. MRI-13360-b which shows loops with relatively extensive upper branches taken under two different temperature conditions. The longer loop was photographed as soon as possible after the initial excitation of the core, the shorter after disappearance of the heating transient. The decrease in $B_8$ is clearly visible. Also noticeable is a decrease in the permeability (ordinary as well as differential) over considerable portions of the loop.

The above-mentioned heating transients make themselves felt in the following manner: When a source of power is connected to the winding on the core, the initially cold core material is heated (primarily) by hysteresis losses. For the cores tested, the permeability decreases as a result thereof; roughly speaking, the "reactance" of the coil drops, and consequently the current drawn from the source increases. Actually the current wave is nonsinusoidal, of course, and the increase in, say, its rms value is due mainly to an extension of its peaks. This is particularly true when the flux excursions impressed upon the cold core are almost large enough to bring it into saturation. As the core heats up and its $B_8$ drops accordingly, saturation will actually be accomplished while the supply voltage remains unchanged. This explains the increased peaking of the current wave. The point is illustrated by Figs. MRI-13360-c and d, which show loops and current waves for the cold and hot states, respectively.

The current in rms values may be plotted as a function of time; this was done for a given core for a number of different values of initial current. The graph is shown in Fig. MRI-13361, which also includes the variation of temperature in the immediate vicinity of the core (as measured with a mercury thermometer) in its time-dependence. The curves resemble exponentials; the steady-state value and the initial slope are both functions of the initial current value, but the time required for the transient to damp out seems to be nearly constant for all curves. The largest initial current values lead to the largest differences between initial and final states. This is to be expected since large currents mean large flux excursions and therefore large hysteresis loss.
The curves shown contain breaks which indicate reductions in supply voltage of such magnitude that the current was caused to return to its initial value. It will be noted that portions of the curve following the breaks are nearly horizontal and show that the preliminary heating period has led to a stabilization of the core characteristics, and to a temperature equilibrium.

It is possible for the indicated processes to become cumulative, leading finally to enough temperature rise in the core to cause its lapse into the paramagnetic region. Such action was observed by experimenters at Princeton University who found it necessary to an air-blower to reduce the ambient temperature to safe values.

One of the advantages of magnetic amplifiers sometimes cited in the literature is their readiness to reach full operating conditions very quickly because no warm-up period is needed. Putting aside the question of electronic power supplies for the moment, the same statement can still be made with regard to ferrite-cored amplifiers but must be modified by saying that a temperature stabilization period is needed. Electronic equipment, however, also requires time, sometimes many minutes or even hours, to stabilize and eliminate temperature drifts, so that this factor is not a serious obstacle to the practical use of magnetic amplifiers in place of electronic devices. Under certain critical conditions, it may be found advisable to stabilize the external electrical characteristics of the amplifier by suitable thermostatic temperature control.

The temperature dependence, incidentally, suggests the possible use of ferrite cores as the sensing devices of thermostat elements.

Still another rather undesirable property of the ferrite materials is their high coercive force \( H_c \) which varies from about 0.5 oersted for commercially available bodies to nearly 5 oersted and is thus five to a hundred times larger than the normally observed values for sensitive metals. This leads to large values of magnetizing or exciting current \( I_0 \), and makes the minimum currents possible with ferrite-cored amplifiers correspondingly large. It also drains all power from the previously realistic substitution of the saturation curve for the hysteresis loop which was used in Chapter II to make amplifier performance amenable to analysis. In the author's opinion, all performance calculations upon ferrite-cored amplifiers must be based upon the loop itself, and this is possible, though unwieldy, for the case of d-c signal.

53 Personal conversation with Prof. Johnson, Head of E.E. Department of Princeton University.

54 Snyder, Albers-Schoenberg and Goldsmith, op. cit.


56 A point-by-point solution could be required and numerical or machine calculation would be necessary.
In the case of a-c signal, the difficulties have so far been found unsurmountable. A more complete discussion of the problem follows later in the chapter.

In spite of all their defects, however, ferrites are the only materials now known to exhibit ferromagnetic properties sufficiently good to allow saturable reactor operation at frequencies in the vicinity of 100 mc. And so, although thinly laminated strip-wound cores may eventually dominate the field in the audio band, ferrites will almost certainly hold unquestioned supremacy in any development of radio-frequency magnetic amplifiers.

(2) The Rectifiers

In Chapter II the conclusion was reached that interelectrode capacitance made selenium rectifiers useless at frequencies over a few kilocycles, and further, that vacuum diodes were not desirable for practical applications. What has been said about selenium plates applies of course in a large measure to copper oxide and sulphide rectifiers. (Recently units made up of very thin selenium plates have appeared on the market, and it is claimed that they may be employed up to 200 kc. But the high-frequency response has been obtained at the cost of current capacity -- the ratings of these rectifiers are about 500 microamperes.) At the present time the only remaining alternative is the use of germanium diodes, which, at least in the point contact form, are very similar to the crystal detectors used in the early days of radio. A typical cell consists of a tiny disc of metallic germanium (which is a semiconductor) and of a "cat's whisker" ending in a sharp point which is placed in contact with the germanium. The entire assembly is housed in a small ceramic or plastic cylinder, which may be about 1/2 inch long and 3/16 inch in diameter. The self-capacitance of such an unit is about 1 nmf, which is small enough to permit it to rectify fairly efficiently even in the microwave region. Unfortunately, the forward resistance is about 200 ohms. The following data are given by Kentron Electron Products, Inc., whose diodes were used for most of the experimental work. The figures refer to the popular type IN34 diode, and are given for 25°C.

| Min. Forward Current at +1 V. | 5.0 Ma. |
| Max. Inverse Current at -10 V. | 0.050 Ma. |
| Max. Average Rectified Current | 40 Ma. |
| Max. Peak Rectified Current | 150 Ma. |
| Max. D-C Inverse Voltage | 60 Volts |
| Peak D-C Inverse Voltage | 75 Volts |
| Shunt Capacitance | 1 nmf. |
| Temperature Range | -40°C. to +75°C. |
| Average Life Over 10,000 Hours |

In most important respects, these diodes may be seen to be quite adequate for magnetic amplifier applications. The reverse resistance is high, being about 200,000 ohms; the self-capacitance is small, the reverse voltage that may safely be applied is about twice as large as for selenium rectifiers;
and their life expectancy is far larger than that of hot-cathode vacuum diodes. Only their very high forward resistance makes them less than ideal for magnetic amplifier applications. It is possible of course to use several diodes in parallel, but this tends to be a rather wasteful measure unless the paralleling is also necessary for reasons of current capacity. It is felt that the solution lies rather in designing the amplifier in such a manner that the rectifier resistance becomes negligible in comparison with the useful load resistance.

A junction-type germanium rectifier recently announced by the General Electric Company deserves mention. It has a low forward and high reverse resistance characteristic, but a relatively high shunt capacitance and a strong temperature sensitivity. According to the manufacturer, the maximum operating frequency is 50 kc. It may prove of value in the development of audio-amplifiers in cases where the main requirement is large power output and fidelity is a relatively subordinate issue. In such an application, a carrier frequency of 50 or even 25 kc may be found sufficient, and the G. E. "power rectifier" might be used to advantage.

The use of the ferrite materials, hardly to be considered sensitive, seems to make rectifier leakage less injurious to amplifier gain than high forward resistance. It is impossible to turn this state of affairs into an advantage as long as the point-contact rectifier is used, but nothing better is available at the present time.

(3) The Power Supply

The analysis given in Chapter II has shown that the presence of any internal supply impedance whatsoever is detrimental to the good performance of conventional types of magnetic amplifiers. This fact should be kept firmly in mind in the process of developing high-frequency amplifiers, for, when electrical engineers require high-frequency power supplies, nowadays they almost inevitably reach for vacuum-tube amplifiers or oscillators. These, to be sure, have the advantages of convenience, low cost, ready availability, and, usually, simplicity and wide range of control; but they are, broadly speaking, all medium to high output-impedance devices. Their voltage output is, however, in general, quite low. For units employing mainly receiving-type tubes the range may be from fractions of a volt to a few hundred volts, and even with transmitting tubes the output will usually not exceed a few thousand volts. Because of the wealth of different units that may be purchased or specifically built to satisfy certain needs, no more precise figures can be given when the general class of vacuum tube supplies is considered. The significance of low output voltage will be discussed in the section on design considerations.

It is, of course, possible to lower the internal impedance of a vacuum-tube supply to as small a value as is desired by any of several techniques. Most simply, resistors or capacitors may be shunted across the output terminals. The load will then see not the internal supply impedance itself, but the equivalent impedance of the parallel combination, which can, of course, be made arbitrarily low. The obvious disadvantage of this scheme is that available load power is reduced simultaneously with supply impedance. In view of the already low overall economy of electronic systems, so high a price cannot be paid if any practical application is contemplated.

A second method is the use of several cathode-follower stages in parallel as the output device. The internal impedance can be reduced by proper design to a few ohms or even to fractions of an ohm, but the resultant complexity and cost of the supply does not constitute a practical solution. This system has been used for purely experimental work and there proven of some value.

A suitable feedback technique may also be employed, but the output voltage is again affected in the same direction as the impedance.

It seems impossible to escape the conclusion that the intrinsically high output impedance of vacuum-tube devices cannot be sufficiently lowered by any practically satisfactory method. There is yet another aspect to the case against electronic supplies. Not only does their very presence near the magnetic amplifier reintroduce that dangerous vulnerability and fragility which the use of the magnetic amplifier is intended to eliminate, but (this applies especially to amplifiers) also makes the magnetic device totally and ludicrously unnecessary. The vacuum-tube amplifier needed to properly supply a given magnetic amplifier has to have a larger power rating and ten times the frequency range -- it can do a far better job of amplification and requires no subsequent demodulation of the output as does the magnetic amplifier. The practical value of such a combination is less than that of the vacuum-tube amplifier itself; the magnetic amplifier under such conditions is worse than useless.

The author has found only a single passage in the literature which conveys an understanding of this problem. In the concise language of J. M. Manley:

An ideal magnetic amplifier is effective at any frequency. Limitations on values of frequency which can be used in a practical magnetic amplifier may arise from considerations of the carrier source and from the fact of core losses. If the carrier frequency is such as to require vacuum tubes for the generator of sufficient carrier power, then for some applications the advantage of the magnetic amplifier is nullified.

Manley, op. cit.
The following criterion is now proposed in an effort to summarize the above conclusions and to establish a guidepost in the search for more useful types of power supplies. If a given power supply is, or can readily be converted into, an amplifier at the frequency at which operation of the magnetic amplifier is contemplated, then its use as a carrier source can be justified on only one basis, namely, that simultaneous operation of a number of magnetic amplifier channels is intended, and realizable by use of that supply. Only multi-channel operation will enhance the original usefulness of the supply.

Two classes of supplies can be found which satisfy the above criterion, and these are (1) sources which are basically not amplifiers, i.e., principally generators, and (2) "nearly perfect" constant-voltage or constant-current sources. Evidently, the two classes overlap in important instances. Multi-channel operation is possible only with the second class of supply, for, unless the internal supply impedance is either very small or very large, coupling through the supply cannot be eliminated. If coupling occurs, the amplifiers in the various channels can no longer accurately respond to their input signals.

The objection might be raised at this point that it was previously shown that magnetic amplifiers cannot operate at all from constant-current sources. This is true for all conventional circuits, which are specifically suited for use with constant-voltage supplies. The preceding considerations, however, convinced the author that it should be possible to find circuit arrangements well equipped to use with constant-current supplies, and such circuits were indeed found to exist. They are described in the next chapter.

The above criterion now makes possible the selection of particular types of sources conducive to satisfactory amplifier operation. In class 1 (generators) there exists at the present time only the high frequency alternator, sometimes known as the Alexanderson alternator. It is being manufactured for such applications as induction heating, and can develop a substantial amount of power. The frequency range is necessarily limited, but 25 kc appears to be a reasonable value within the useful range. It would be entirely possible to use such a machine in conjunction with the G. E. germanium power rectifier in systems where large power output was required and where frequency response could be sacrificed. Such systems would be most useful where the danger of jarring sensitive ears amongst the listeners is negligible; in the public-address setups of military and naval installations, for example. The low-pass characteristic common to magnetic amplifiers (see p. 25) would lead to a preferential amplification of the low harmonics; thus a pleasingly masculine quality should be imparted to the voices of first sergents and others using the system.

For the sake of completeness, a type of generator should be listed which has been studied experimentally and from time to time discussed in the literature, but which is not too likely to become practically important. This device is the electrostatic generator. Experimental results show that it is almost a true current source over wide ranges of operation. In the past,
experimental models have been built of materials with the low dielectric constants of ordinary insulators, as for example, glass. Recent development of ceramics with dielectric constants thousands of times larger may boost interest in this type of machine, and eventually perhaps make it practical. Most of the literature on this subject is in French, and no active work seems to be going on at the present time.

In class 2 only a single device is at the present time readily available, and that is a vacuum-tube supply utilizing pentodes in the output stage to yield very high output impedance. There is, however, considerable promise of future development. A continuous-control gas tube named "Plasmatron"59 has recently been announced by RCA; this may make possible the construction of low-output-impedance oscillators. Some of the characteristics of this tube are listed in Appendix III. Considerable, though sceptical, interest exists in the magnetic oscillator, which has not, as yet, been developed. It may prove to be essentially a constant-voltage source. Magnetic frequency-multipliers exist, but are not yet available in practical and efficient forms for high multiplication ratios. Finally, it is conceivable that high-power transistor oscillators will eventually be developed. These will probably be constant-current sources.

The above data on supply impedance were not known to the author when the experimental work was initiated, and were discovered only because a standard audio-amplifier with relatively (but not very) high internal impedance was used as a power source. Resultant failure to obtain satisfactory magnetic amplifier performance actually initiated the investigation which led to the above results. It was then recognized that continued use of that supply was incompatible with good performance. Nevertheless, the author completed his experimental work without changing the carrier source on the basis that operation from constant voltage sources was quite well established and adequately described in the literature, while operation from high-impedance sources had apparently never been reported. The impedance characteristics of the audio-amplifier were observed by varying load resistance connected directly across the terminals and recording the load current. When the relation between load resistance and load current is plotted, the curves of Fig. MRI-13362 are obtained. For comparison, a curve corresponding to a constant-voltage source and one for a constant-current source are sketched in. It will be noted that the curves follow neither trend. (As an experiment, a curve for constant power output was computed and plotted alongside the observed ones. It was found to approach them very closely over a large range of loads. The explanation lies in the characteristics of loads and generators matched for maximum power transfer. The characteristic curve is quite flat near its maximum point. For example, if the apparent load resistance $R_L$ differs from the generator resistance $R_0$ by a factor as great as 3 to 1, the power delivered to the load is

By assuming the generator impedance to be purely resistive and the internal voltage source to be a constant, two points on any one of the observed curves may be used to compute a value of generator resistance averaged over the interval which separates those points. It is found that this value varies with the amount of power output, the load resistance, and the frequency of operation. For the normal operating region it is in the order of 300 ohms.

(4) Capacitors

The problems created by the presence of capacitance are even less well defined than for the power-frequency amplifier. (See p. 29.) Here not only lumped capacitance purposely inserted into the circuit may affect the operation, but distributed capacitance may make its presence felt in the upper part of the frequency range studied. If chokes are used to constrain the amplifier, these must be selected to have low values of distributed capacitance in the windings. (A choke rated at 8 henrys was used in some tests to "constrain" the d-c control winding of a half-wave circuit, until it was noticed that unduly high induced currents were flowing. A 60 mh choke was substituted and found to be far more effective.) Capacitive tuning can be used to increase amplifier gain; this is readily accomplished at high frequencies because the necessary condensers are much smaller and cheaper than those needed at power frequencies. Because the use of ferrite materials, high-resistance rectifiers, and high-impedance supplies yields amplifiers with inherently low gain, capacitive tuning may become an important device to improve performance in practical applications. Data given in the chapter on experimental results show that fairly high gains can be achieved, and instability can be induced.

The high dielectric constant of ferrites deserves mention. In conjunction with the high permeability, this property makes for very slow propagation of electromagnetic waves in a ferrite medium. The velocity of propagation is in the order of 30,000 kilometers per second, or about 10 times as slow as in vacuum. Since the losses of some mixtures are sufficiently small to permit use above 100 mc, application as delay lines is possible.

Breakdown of Conventional Analytical Methods

The amplifier analyses presented in Chapter II are predicated upon (1) the substitution of a single-valued magnetization curve for the hysteresis loop, and (2) neglect of all core losses and secondary ferromagnetic characteristics. In the case of the audio-frequency amplifier (especially when

60 Magnetic Circuits, op. cit.

61 Snyder, Albert Schoenberg, and Goldsmith, op. cit.
ferrite-cored) both assumptions cease to be realistic. The large coercive force of ferrites (p. 26) does not, in the author's opinion, permit the contraction of the loop into a single line, or the further assumption that current during flux-buildup is negligibly small (p. 24). While eddy-current losses can still be neglected when ferrites are used in the frequency range under discussion, hysteresis losses should be accounted for by any analysis intended for practical use. Finally, the temperature dependence of core characteristics is not negligible.

If the nature of the carrier supply furthermore necessitates the introduction of an internal supply impedance into the equations, the analysis becomes even more hopelessly unmanageable.

It is possible to analyze the operation of the single-core saturable reactor with d-c signal (as described in Chapter II, Eqs. (1a), (2a), and (3a)), under the assumption of an actual hysteresis loop or a line-segment loop, such as shown in Fig. MRT-13552 a. The procedure is lengthy and laborious. The results do not justify the amount of work involved, for d-c controlled amplifiers can be, and are, operated at power frequencies, where the ideal analysis yields entirely accurate results. The next step is the substitution of a sinusoidal control current for the direct current control. The analysis for this case cannot be carried to completion. A suitable choice of initial conditions, it was found, permits mathematical calculation of operation over only about one cycle of the carrier voltage. The complete description of performance would, of course, necessitate the extension of the analysis to a full cycle of the signal voltage, which has a period ten or more times greater than the carrier voltage.

The mathematics for the a-c controlled saturable reactor with line-segment hysteresis loop, internal supply resistance, and nonnegligible leakage reactance were worked out in some detail. The lower right hand corner of the loop was chosen as the starting point (t = 0), on the supposition that at some time both carrier and signal currents would simultaneously pass through zero in a positively increasing direction. The equations were solved for the right-hand ascending portion of the loop, and for the region of saturation. The beginning of flux decay was studied, and it was found possible to follow the passage of the instantaneous operating point to its lowest position on the hysteresis loop taken on during that cycle of the carrier frequency. It then became necessary to choose a path of further progress of the operating point, for it became clear that the large symmetrical hysteresis loop would no longer be followed. A practical solution to this problem has not yet been found.

In an attempt to answer the question, an oscillographic study was made of the hysteresis loops resulting from application of an a-c signal to (1) the single core saturable reactor circuit, and (2) the half-wave circuit. The clear presentation of these loops was made possible by use of the carrier-to-signal synchronizer mentioned in the equipment list (p. 4). It is believed that complex hysteresis loops of this nature have not before been reproduced.
in the literature, and eight examples are accordingly shown in Fig. KRI-13363. In view of the comments later made in the section on measurement problems, these loops should not be considered as constituting sound quantitative evidence as to core behavior under conditions of a-c control, but they are qualitatively informative and have the force of novelty.

A study of the full collection of loops indicates that the locus of the instantaneous operating point depends upon frequency and amplitude ratios of carrier and signal field intensities, upon the presence and magnitude of feedback and bias, and upon the carrier and signal wave forms. Magnetic data would have to be available for each separate combination of these independent variables for which analysis was desired; this need for individual experimental results eliminates the usefulness of further analysis which, in any case, could then be carried out only for the specific cases for which the test results were already known.

The previously mentioned temperature dependence of the cores introduces still another variation. The complexity of the full practical audio-frequency magnetic amplifier problem places unsurmountable obstacles in the path of any analysis; the problems that can be solved without the use of mechanical or analogue computation methods are based on assumptions too unrealistically severe to have any practical value in this case. Recourse must be had to experiment.

Measurement Problems

Unfortunately, in some respects even the best instruments now on the market do not permit the perfection of an experimental technique sufficiently sound to yield unequivocally meaningful results. Amongst the most significant measurements that can be performed are observations of wave shapes of the various currents, voltages, and fluxes, and of hysteresis loops. When a properly calibrated oscilloscope is used to observe these quantities, direct calculations can be made to obtain measures of the amplifier performance and the significant ferromagnetic properties of the core, and to give an immediate check against the results of theoretical analysis. The oscillographic technique is simple, convenient, and powerful, but it is essential that greatest care be taken to maintain the accuracy of observation.

The hysteresis loop, as observed in the 'scope screen, must of course be inherently in error because of the effects of eddy currents; but if this method of presentation is nevertheless adopted because of its speed and simplicity, it is vital that additional errors due to equipment deficiencies be kept to a minimum. Considerable care must be exercised even when core excitation is supplied at 60 cycles. A recent paper by H. W. Lord sets up minimum standards

for what shall be deemed satisfactory measuring equipment. Lord's criteria are perhaps more conservative than is really necessary; they have also been criticized on the basis that the symmetrical hysteresis loop (towards the measurement of which he has primarily directed his attention) is quite useless as a starting point for performance calculations of the important half-wave circuit and its derivatives. These calculations, it is maintained, are more accurately carried out by use of a "control magnetization" curve which can be accurately measured by use of equipment somewhat less accurate than Lord's criteria demand. Other influences, such as excessive rapidity of core magnetization, may also serve to nullify his contention that a Lissajous figure observed under the precautions stated "truly represents the dynamic hysteresis loop of the specimen under the conditions imposed by the test circuit". Although they certainly do not bring perfect assurance of reliability, Mr. Lord's criteria at least partially fulfill a manifest need for greater standardization and precision in the accumulation of magnetic data, and, in the absence of better standards, will be used in evaluating the equipment used by the author.

The most pertinent of Lord's specifications are that (1) each stage of the "H" axis amplifier (three stages are assumed) shall have less than 45 degrees of phase shift at a high frequency of 250 times test frequency, (2) the integrating circuit have a phase angle of 89.75 to 90 degrees at the operating frequency if only a single R-C integrator is used, (3) each stage of the "B" axis amplifier shall have less than 45 degrees phase shift at 100 times test frequency, and (4) that the "maximum distortion in the voltage wave induced by the flux wave should not exceed 10 per cent. This includes distortion due to the power source and all series resistance in the excitation circuit".

The use of the high-impedance electronic power source in the experimental work of course precluded any possibility of meeting the fourth requirement. As the carrier frequency was in most cases about 100 KC, Lord's criteria (it is implicitly assumed that they are applicable in this range of frequency) demand that the upper half-power point of each stage of the oscilloscope "H" amplifier shall occur at about 25 MC, and of each stage of the "B" amplifier at 10 MC. A scope satisfying such requirements is practically, if not actually, unobtainable today. One of the best wide-band oscilloscopes commercially available at this time, a Tektronix Model 511 A was used. The manufacturer claims the following high-frequency performance:

Vertical Amplifier Band Width: 2 stages, down 3 db (Max.) (from 1 mc response) at 5 cps and 8 mc

Horizontal Amplifier Band Width: DC to 800 KC (3 db down at 800 KC)

63 Lehmann, op. cit.


65 Lord, op. cit.
It must be admitted therefore that the magnetic measurements reported in this thesis are correct only to a first order of approximation.

The magnetic data presented should be taken "with a grain of salt" for still another reason. Even the best scopes on the market sometimes include sources of phase shift not directly related to the amplifiers themselves. It may be impossible to remove such phase shift without extensive and otherwise unwarranted circuit alternations, and external corrective networks may constitute a poor solution unless complete data to aid in their design are available. Even then, they may not be applicable in certain experimental work because they may introduce excessive attenuation of the signal to be displayed. For example, the horizontal attenuation potentiometer of the Tektronix scope used by the author introduced so much phase shift into the "H" signal that a loop of normal appearance could be obtained at only one setting of the "pot". It is not claimed that the loop was correct; only that it had the most "reasonable shape" amongst the many forms of Lissajous patterns that would result simply by turning the horizontal attenuation knob. In essence, it must be granted that with existing high-grade laboratory instruments only very approximate measurements of magnetic data are possible in the frequency range under consideration, and that the data obtained depend in a large measure not upon the technical ability, but upon the discretion of the experimenter.

Needless to say, great care must be otherwise exercised in assembling the measurement circuitry. Wire-wound pots should be entirely avoided, and the system must be thoroughly grounded and the effects of stray capacitance held to a minimum. It is of course essential to make correctly polarized connections.

The effects of several minor sources of error upon loop shape are illustrated in the oscillograms of Fig. MRI-1336. Part (e) and (f) show loops obtained during application of a-c signal. Fig. MRI-1336e-f was obtained under magnetic conditions identical with those of Fig. MRI-1336c. Because of the complexity of these loops, it is not immediately apparent that they are also erroneous. The error was introduced by using for the "H" deflection a signal proportional only to the carrier current. When a-c signal is used, the total field intensity in the core is, of course, due to the superposition of the emf's of carrier and signal windings. The voltage applied to the "H" input terminals should, therefore, be proportional to $N_c I_c + N_s I_s$. (It is assumed that no feedback or bias windings exist; if any are present, they may have to be similarly accounted for.) This is not, however, readily accomplished with simple circuitry; if a resistor is used to mix the carrier and signal currents and provide a voltage proportional to their sum, then the results will be accurate only for the special case of unity turns-ratio between carrier and signal windings. The loops shown in Fig. MRI-1336 were taken for a core wound with 75 carrier turns and 120 signal turns. The residual error so introduced was considered acceptable, because, at any rate, the measurements were expected to yield only qualitative results.
Design Considerations

(1) Influence of Frequency Upon Reactor Dimensions.

(It is assumed in the following discussion that the reader is acquainted, at least in a general way, with the range of sizes in which power-frequency reactors are built for some of the common applications.)

Let us assume that a reactor core with a cross-sectional area of 1 sq. in. and a mean length of 7.5 inches is to be incorporated into a half-wave circuit. Take the saturation flux density as 114 kilogausses, which is close to the value given for DeltaMax. The reactor is to be able to withstand 50 rms volts without saturating. Use of the transformer equation

\[ E_{\text{ind}} = 4.44 f N A B 10^{-8} \text{ volts} \]

yields the result that \( N \) should be about 125 turns at an assumed frequency of 100 c.p.s. Now allow the supposition that the core and rectifier characteristics are so ideal that use at 100 KC is practical, and let the frequency be increased to that value while the core dimensions, the winding, and all other circuit parameters are held constant. If the rms voltage needed to saturate the core is now calculated, a value of 50,000 volts is obtained. Evidently a reduction must be made in one or more of the quantities \( N, A, B \), to bring the necessary supply voltage down to some reasonable value, say, 50 volts. If only the core area be reduced, the resulting core will have a cross-sectional area of 0.001 sq. in. Before adopting this alternative let us, however, study the effect of such a step upon the inductance of the core.

A rule-of-thumb useful in designing magnetic amplifiers is that, for minimum acceptable performance, the unsaturated reactance of the reactor shall be at least 6 times the value of the load resistance, and its saturated reactance shall be less than 1/6 of the load resistance. For our purposes, the approximate formula for the d-c inductance of a toroid of rectangular cross-section

\[ L = \frac{\mu N^2 A}{1_m} \]

will be a sufficiently accurate guide. The formula informs us that a reduction in area and a simultaneous increase in frequency, both by a factor of 1000, will not affect the ratio of unsaturated (or saturated) reactance to load resistance. The ideal core that was postulated can be adapted to high frequency operation merely by stripping away 999/1000 of the core.

In the practical case, increase of frequency will adversely affect the permeability and increase the core losses. As long as the core material is not changed, the use of sufficiently thin laminations and reduction of the mean length, and therefore the mean diameter, of the core will partially offset the effects of increased frequency. A recent development along this line is a strip-wound core of a sensitive metal alloy, consisting of a few turns of narrow 1/8 mil strip wound on a ceramic coil-form. The mean diameter is in the order of 1/8 inch.

If, however, as was done by the author, ferrite bodies are chosen because of their high resistivity, a further reduction in permeability must be accepted, and the values obtained are only one-tenth to one-fiftieth of those possible with sensitive metals. Moreover, the processing necessary in their manufacture establishes a lower limit to the cross-sectional area. It is, therefore, indeed fortunate that ferrite cores saturate at flux densities two to ten times smaller than the sensitive metal alloys. However, a well-designed ferrite core intended for use at 100 KC will still be only a few one-hundredths of a square inch in cross-sectional area, and will have a mean diameter of 1/2 inch or less. Few, if any, cores of that size are now commercially available. Appendix II includes an example of the calculation of proper core dimensions.

(2) Constraining the High-Frequency Amplifier.

This subject has been previously mentioned only in the introductory remarks of Chapter I and briefly in the present Chapter when it was mentioned that the chokes used to constrain any d-c windings must be carefully chosen for low distributed capacitance. Since the audio-frequency magnetic amplifier has two or more sources of applied a-c with, possibly, widely different frequencies, and mutual coupling exists between all windings and their circuits, the subject is of considerable importance. The discussion appears in this relatively subordinate position because, at the time of this writing, the subject has not been adequately studied, and no solid evidence has been collected. Only certain suggestions, which appear plausible to the author, will be offered.

It is clear that circulation of induced currents in any one of the windings will bring about the case known as free magnetization. If constrained magnetization is to be maintained, the flow of induced currents at the carrier frequency and at any signal frequency must be blocked in all windings. Perhaps it is necessary at this point to enlarge somewhat the concept of constrained magnetization.

67 Compare data given by Snyder, Albers-Schoenberg and Goldsmith, with those given in the preceding reference.

68 Refer to the same two publications.
The proper job of an amplifier is to produce at its output terminals an enlarged and faithful reproduction of the quantity applied to its input, and to draw the energy required to accomplish this from a reservoir or supply while reacting back upon the input to the least possible degree. In the magnetic amplifier, therefore, the signal frequency component of load current should be the result of nonlinear reactance modulation, and not of transformer action. From that viewpoint, only an amplifier circuit in which the sum of the powers transformed from the signal winding into all other windings is negligibly small should be considered as adequately constrained. Furthermore, to complete the desired isolation of the signal circuit, the circulation of induced currents at the carrier frequency must, as is true for the constrained case with d-c signal, be prevented there (in the signal circuit). When these restrictions are complied with, it is not clear what significance the free magnetization case retains.

In practice it is very difficult to realize complete amplifier constraint. Any d-c bias circuits do not present much of a problem -- a sufficiently large inductance will adequately block the passage of currents at both carrier and signal frequencies, but it is an entirely different matter to accomplish this end in the a-c circuits. A first step should be the choice of circuits in which, by virtue of double-core arrangements, at least prevalent harmonics cancel each other. Then tuned reactive networks can be inserted to block the remaining frequency components. Proper design of these networks involves a knowledge of network synthesis, and sometimes unrealizable large or small values are obtained for the parameters needed to achieve a certain frequency characteristic. The use of lossy components will also greatly reduce the effectiveness of these networks.

As an illustration, let us sketch the design of a network to be inserted in the signal circuit of a magnetic audio-amplifier, to be used at 15 KC. The carrier frequency will accordingly be about 120 KC. Since the flow of signal current should find minimum opposition, a zero of the impedance is chosen at 15 KC. The behavior at zero frequency is immaterial, so that a pole may be allowed there. The simplest network will be obtained if another pole is chosen at infinity; only two elements will be required then. Although the necessary values of inductance and capacitance are readily calculated here, their computation becomes more difficult when a more involved impedance characteristic requires the use of two or more network sections.

IV. Shunt-Fed Reactor Circuits

The following facts were established in the last chapter: (1) The conventional magnetic amplifier circuit arrangements cannot be used with constant-current sources. (2) There now exists at least one type of high frequency power-supply which closely approximates a constant-current source; namely, vacuum-tube amplifiers or oscillators with pentodes in the output stage.
Other types of constant-current sources may be developed in the future. (3) It is relatively difficult to make satisfactory constant-voltage supplies for high-frequency use.

Taken together, these facts constitute the main argument that the development of magnetic amplifier circuits adapted to use with constant-current sources is of practical, and not only of academic, importance to further progress and success in the magnetic amplification of high-frequency signals. On this premise several circuit arrangements so adapted were designed, and they are briefly described in the following paragraphs.

The Shunt-Fed Saturable Reactor

The requirement governing the design of these circuits was that they should work, not some a priori restriction that they should be duals or equivalents of the conventional circuits. There were consequently no rigorous scientific principles to guide the choice of circuitry, and a process of intuitive trial and error had to be used. It seemed plausible to parallel load resistor and saturable reactor with the constant-current supply.* In this manner the single-core shunt-fed saturable reactor circuit of Fig. HKI-13365-a was arrived at. Since an introductory, rather than an intensive study was contemplated, it was then decided to put aside further consideration of the saturable-reactor and external-feedback circuits and to focus attention on self-saturating arrangements.

When source, reactor, rectifier, and load are connected into the conventional half-wave circuit, the sequence of elements is evidently inconsequential, since they are all in series. When the load and reactor are placed in shunt, however, the rectifier may be inserted in at least four different places, as shown in Figs. HKI-13365-b, c, d, and e. Only one of these circuit arrangements, that of Fig. HKI-13365-b, may be written off as useless without further study. In this circuit, as in the half-wave circuit, the rectifier would be required to block every other half-cycle of the current supplied by a constant-current source, and would consequently be destroyed. At least the circuit of Fig. HKI-13365-c may be enlarged into a more useful form employing two reactors, as illustrated in part (f) of the figure.

So far as the author knows, all the six circuit arrangements illustrated are new and have not been previously described in the literature. Any study to be made of their operation must begin with the application of the fundamental principles already employed successfully in the examples of Chapter II to analyze the series-fed saturable reactor and the half-wave circuit.

* On the premise that if, for a constant-voltage supply, control is achieved by varying the division of supply voltage between load and reactor; then, for a constant-current supply, the same and should be attainable by varying the division of supply current between these elements.
The circuit of Fig. MRI-13365-a has been redrawn as Fig. MRI-13366-a and appropriate symbols for the significant quantities have been introduced. The following equations may be written when it is assumed that both load and control meshes are fed by constant-current sources:

\[ I_m \sin \omega t = \frac{V}{R} + i_s \]  
\[ v = N \frac{d\phi}{dt} \]  
\[ i_c = I_c \]  
\[ \Delta L_m = N_i s + N_c I_c \]

In addition, an ideal core material is assumed. Let us first analyze the condition of operation under the restrictions that \( I_c = 0 \) and that the core never goes into saturation. It then follows from the assumed magnetization curve that \( H = 0 \), and from Eq. (4) that \( i_s = 0 \) at all times. Eq. (1) then reduces to:

\[ v = N \frac{d\phi}{dt} = R I_m \sin \omega t \]  

Integration of this equation from time \( t_c \) to time \( t \) results in:

\[ \phi = \phi_o = \frac{-RI_m}{\omega N} (\cos \omega t - \cos \omega t_o) \]  

Substitution of the maximum and minimum values of the flux wave in the last equation leads to:

\[ \phi_{\text{max}} - \phi_{\text{min}} = \frac{-RI_m}{\omega N} \left[ \cos \omega t_{\text{max}} - \cos (\omega t_{\text{max}} + \pi) \right] \]  

As complete symmetry exists (\( I_c = 0 \)), it follows at once that the quantity \( \phi_m \) which, for the saturable reactor circuit of Chapter II was given by \( V_m/\omega N \), is here equal to \( RI_m/\omega N \).

When some definite value of control current (\( I_c = K \)) is applied, then the core will be saturated over part of the cycle, and during that time the following relations will hold:

\[ \phi = \phi_s; \quad \frac{d\phi}{dt} = 0; \quad v = 0; \quad H \neq 0 \]
That is, while the core is saturated, it acts as a short-circuit and absorbs the entire supply current. The analogous statement for the conventional saturable reactor is that the entire supply voltage is absorbed by the reactor while unsaturated, when it acts as an open circuit.

At some time $t_1$, core flux begins to decay. At $t_1$, the field intensity in the core must accordingly vanish, and Eq. (6a) then yields for the corresponding value of reactor current:

$$i_{g1} = -\frac{N_C}{N} I_c$$  \hspace{1cm} (6a)

Eqs. (6a) and (lb) can be combined to give the following expression for the time $t_1$:

$$t_1 = \frac{1}{\omega} \sin^{-1} \left[ -\frac{N_C}{N} \frac{I_c}{I_m} \right]$$ \hspace{1cm} (6b)

During the time the core remains unsaturated, Eq. (1) may be written as:

$$\frac{V}{R} = I_m \sin \omega t + \frac{N_C}{N} I_c$$ \hspace{1cm} (1c)

Integration from time $t_1$ to time $t$ gives:

$$\left( \Phi - \Phi_g \right) = -\Phi_m \left( \cos \omega t - \cos \omega t_1 \right) + \frac{N_C}{N} \frac{I_c}{I_m} \left( t - t_1 \right)$$ \hspace{1cm} (7)

This expression describes the flux waveshape, but can be used to solve for the saturation time $t_g$ by substituting $\Phi_g$ for $\Phi$. The solutions for the reactor current and the flux waveshapes given above are sketched in Fig. MRI-13366-b and c; and they are of course extremely similar to the load current and flux waves in the conventional circuit. But while in the latter case that current feeds the useful load, $i_g$ is here of no practical significance. Only $i_R$ contributes to the load power. This current may readily be computed from Eq. (1) and a knowledge of the reactor current waveshapes. The wave is sketched in Fig. MRI-13366-d. No corresponding waveshape is observed in the conventional circuit.

It will be noted that the above development is exactly parallel in almost every step to the analysis of the conventional circuit given in Chapter II. Except for the description of the output waveshape, the results are identical in nature. Perhaps the most significant change in the load current wave is that core saturation now causes a sharp drop to zero instead of the sharp rise so familiar in the operation of the conventional circuits and often
referred to as "firing". The fact that the leading, rather than the trailing edge of the load current wave now contributes most heavily to the load power seems, however, to be of minor practical consequence.

Integrating Eq. (1) over a complete cycle incidentally yields the result that the average value of reactor current is, as for the conventional circuit, zero. Since the average value of the sinusoidal supply current is also zero, it follows that the load current must have a vanishing average value, too.

It should be noted that, while the parallel combination of reactor and load has a constant current applied, the reactor itself is excited not by a constant current, but by the voltage which appears across the load while the reactor remains unsaturated.

The maximum value of the load current (or voltage) of course occurs in this case while the core remains unsaturated at all times, and therefore at zero value of control current. The transfer curve of this circuit is therefore "upside down" in comparison with that of the conventional saturable reactor circuit. The fact that maximum output is obtained for zero input may have considerable bearing on any practical application of the circuit.

In order to test the results of the above theoretical development, the following circuit was set up: A constant-current source was approximated by connecting a 2500-ohm resistor in series with the 110-volt, 60-cycle power line and limiting the values of load resistors used in the various shunt-fed circuits that were tested to 250 ohms or less. The current drawn from the line could, therefore, vary only in the ratio of 1.1 to 1, and this was considered sufficiently accurate for a qualitative check. In several tests a load of only 100 ohms was used. With such a value, only about 1/5 of the line voltage was available to saturate the core; and a small Hi Mu 80 Core was used which performed satisfactorily. This is unfortunately not too sensitive a material, the knee of the magnetization curve being quite round. Nevertheless, the results of the experiments were quite closely in accord with those of the above ideal analysis.

Oscillograms of the load voltage and reactor current for several values of control current are reproduced in Fig. MRI-13367. The sharp drop in load voltage (or current) that occurs at core saturation is clearly defined in the photographs. Otherwise, general agreement also exists between derived and observed wave shapes. Fig. MRI-13368 is an experimental transfer curve taken with a two-core shunt-fed circuit. Both cores were in parallel with load and supply. This curve is at least qualitatively in agreement with theoretical predictions.
Half-Wave and Full-Wave Control Circuits

The circuit of Fig. MRI-13365-c can be readily analyzed on the basis of the same assumptions about reactor core and rectifier characteristics as were used in the analysis of the half-wave circuit in Chapter II. The supply and control sources will be assumed to deliver constant currents $I_m \sin \omega t$ and $I_c$ respectively. The circuit, with proper labeling of significant quantities is redrawn in Fig. MRI-13369-a. The following equations may be written:

$$I_m \sin \omega t = \frac{V}{R} + i_s$$  \hspace{1cm} (8)

$$i_c = I_c$$  \hspace{1cm} (9)

$$v = e_x + N \frac{d\phi}{dt}$$  \hspace{1cm} (10)

$$H_{m} = N_i i_s + N_c i_c$$  \hspace{1cm} (11)

Substitution of (11) in (8) yields:

$$I_m \sin \omega t = \frac{e_x}{R} + \frac{N d\phi}{dt} + i_s$$  \hspace{1cm} (12)

We wish to determine, as was done for the half-wave circuit, whether the reactor current can flow continuously, or whether it must go to zero during some part of the cycle. Let it be assumed that $i_s$ can be continuous. Because of the presence of the rectifier, it will then have to be continuously positive, and then we may write for the entire period:

$$e_x = e_f = i_s R_f$$  \hspace{1cm} (13)

When Eq. (13) is inserted in (12), and (12) is then integrated over a full period $T$, the result is:

$$0 = \frac{R_f}{R} \int_{0}^{T} i_s dt + \int_{0}^{T} i_s dt$$  \hspace{1cm} (14)

which may be written as:

$$\left(\frac{R_f}{R} + 1\right) I_{DC} = 0$$  \hspace{1cm} (15)

This equation can only be satisfied if $I_{DC} = 0$. The original assumption was, therefore, incorrect, and over some interval, say between $t_0$ and $t_1$, the reactor current must take on zero values. During that interval, Eq. (11) reduces to:
\[ H_0 I_m = N_c I_c \]  

(11a)

The field intensity \( H_0 \) is the minimum value of \( H \) possible with the given \( I_c \) during steady-state operation, and the corresponding minimum value of flux, \( \Phi_0 \), may be read from the magnetization curve of the given core material. The flux remains at this minimum value until the reactor current again begins to increase above zero.

During the time \( t_s \) remains at zero, Eq. (8) yields for the load voltage:

\[ v = R I_m \sin \omega t \]  

(8a)

When \( i_s \) again becomes positive, Eqs. (8) and (13) may be combined to give:

\[ I_m \sin \omega t = i_s (1 + \frac{R_f}{R}) + \frac{N}{R} \frac{d\phi}{dt} \]  

(16)

Because of the steepness of the assumed magnetization curve, only a negligible amount of reactor current flows during flux build-up; and therefore, Eq. (16) may be approximated to:

\[ \frac{d\phi}{dt} = \frac{R}{N} I_m \sin \omega t \]  

(16a)

Integrating this equation, we obtain an expression for the flux wave during build-up:

\[ (\phi - \Phi_0) = \Phi_m (1 - \cos \omega t) \]  

(17)

At some time \( t_s \) the core saturates. The saturation time can be obtained by substituting \( \Phi_0 \) for \( \Phi \) in (17), and solving the resulting form

\[ \omega t_s = \cos^{-1} \left[ 1 - \frac{(\Phi_0 - \Phi_m)}{\Phi_m} \right] \]  

(17a)

for \( t_s \). After saturation, evidently:

\[ \frac{d\phi}{dt} = 0 \]  

(18)

\[ i_s = \frac{I_m \sin \omega t}{(1 + \frac{R_f}{R})} \]  

(19)

\[ v = i_s R_f = \frac{R R_f}{R + R_f} I_m \sin \omega t \]  

(20)

\[ i_R = \frac{i_s R_f}{R} = \frac{R}{R + R_f} I_m \sin \omega t \]  

(21)
For sensitive materials, flux decay in the core begins very nearly at \( \omega t = \pi \) and from \( \pi \) to \( 2\pi \), therefore,

\[
i_s = 0 \\
i_R = I_m \sin \omega t
\]  

(22)

For good rectifiers, \( R_f \) is a small fraction of the load resistance \( R \), and may be neglected in comparison with it. Eqs. (19) through (21) then simplify to:

\[
i_s = I_m \sin \omega t
\]  

(19a)

\[
v = R_f I_m \sin \omega t
\]  

(20a)

\[
i_R = 0
\]  

(21a)

The results of the above derivation are summarized in the sketches of reactor current, and load voltage waves given in Fig. MRI-13369, parts b and c. Great similarity to the conventional half-wave circuit is apparent in all waves, except that of the load current, which instead closely resembles the rectifier voltage wave of the half-wave circuit. It can be seen from Eq. (22) that the negative half-cycles of the load current are not affected by the presence of the rectifier-reactor branch, and that no control of these negative alternations is, therefore, possible. The name "Half-Wave Control Circuit" is accordingly suggested. It follows that neither the half-cycle average value nor the rms value of load current can be reduced to zero, and that consequently the transfer curve, will have an upward shift proportional to the area under the negative half-cycle of the current wave if plotted in average values and to the rms value of the supply current if plotted in rms.

By combining two half-wave control circuits as in Fig. MRI-13365-f, it is possible to control both half-cycles of the current wave, and thus to obtain an analogue to the conventional "doubler" circuit. The name "Full-Wave Control Circuit" is proposed as an adequate description of its properties.

The doubler circuit is generally not analyzed as a separate problem, but considered as the superposition of two half-wave circuits whose load currents add to give the total load current, but which do not otherwise interact. Any interactions which do occur between the two half-wave meshes, either through load, or through control circuit, or even through the input transformer, are considered secondary effects, and calculated as correction terms. The same approach can be made here, and it follows at once that the individual reactor currents and fluxes retain the wave shape they have in the half-wave control circuit (and, indeed, in the conventional half-wave circuit), but that the load current is now, as it were, a mirror image of the wave shape observed in the doubler circuit. The wave is sketched in Fig. MRI-13369-d.
In this case also, the theory was subjected to experimental check. Figs. MRI-13370 and MRI-13371 show selected reactor-current and load-voltage wave shapes for the half-wave and full-wave control circuits respectively, and Fig. MRI-13372 shows a gain curve for the full-wave type of operation. These data represent a good qualitative verification of the above theory.

Other Circuit Arrangements

After the expenditure of considerable effort, it was decided to abandon the so far fruitless attempts to derive mathematical analyses for the remaining two, possibly useful, circuits of Figs. MRI-13365-d and e. Experimental results were, however, obtained. They are summarized in Figs. MRI-13373 and MRI-13374, which are largely self-explanatory.

V. The Complete Audio-Amplifier

The author believes that, on the basis of the data compiled in this report, he could, at the present time of writing, do adequate preliminary "paper designs" for two types of magnetic audio-amplifiers, and further, that, given the necessary equipment, he could carry the design through the laboratory phase to the point where these amplifiers might have some practical value. The following paragraphs are intended to sketch the approach that would be employed.

First of all, amplifiers using the conventional circuitry, and therefore constant-voltage carrier sources, could be built for applications where the requirements for high output powers far overshadow any considerations of fidelity of signal reproduction, or where the signal to be reproduced has all its important components at the lower end of the audio-range. An Alexander high-frequency alternator would be specified. The use of this relatively low-impedance source would permit the operation of several channels of amplification, with the amplifiers of course in parallel. Because of the low carrier frequency, a sensitive alloy, strip-wound core would be selected. The tape thickness would, in view of the data indicated on p. 28, have to be determined through an experimental check for best performance. The new General Electric germanium-junction power rectifier would be specified because of its low forward resistance, good rectification efficiency up to about 50 KC, and relatively high power rating. No attempt would be made to design tuning capacitors, but they would be included if, on the basis of experimental evidence, it could be shown that they improved the performance.

Putting aside all questions of patent rights or arrangements, the full-wave circuit would be most advantageous because of the following considerations:
Magnetic audio-amplification requires two distinct steps of which the magnetic amplifier supplies only one, namely, the modulation of the carrier wave with the signal wave. The recovery of an enlarged replica of the signal can be accomplished only by subsequent demodulation of the magnetic amplifier output. In the full-wave circuit, the rectifiers will then perform two distinct functions simultaneously: (1) They supply self-saturation to the amplifier, and (2) they will carry out the rectification portion of the demodulation process. All that need then be done to obtain recovery of signal wave form is to shunt the load with a capacitor of appropriate size so as to read the peak values of the magnetic amplifier output wave across the load. The advantages gained by eliminating the need for an extra demodulator are obvious.

Incidentally, they system of course will yield a faithful reproduction of the signal only if the peaks of the load voltage wave are proportional to the signal. It can be seen from the analysis of the half-wave circuit (Chapter II) that this condition will be met only if the firing angle is restricted to the range between $90^\circ$ and $180^\circ$ for one of the meshes (and $270^\circ$ to $360^\circ$ for the other). If the firing angle advances to values less than $90^\circ$, the average value of rms value of the load current continues to increase, but the peak value remains fixed. It follows that the range of operation of an audio-amplifier intended to reproduce the input signal under the demodulation conditions described above is limited to the lower half of the transfer curve as computed or measured on the basis of rms or average values. If the upper limit of that range is exceeded, the output wave form will have flattened crests. Furthermore, "the signal input must contain a d-c component in order that satisfactory recovery of the signal from the demodulator may be made." This d-c component can be supplied either from the signal source itself, or by a separate bias winding. The need for this d-c level arises because the transfer curve of normal self-saturating circuits without feedback has its useful control range shifted to the left of the zero-control-current axis. Since only the lower half of the transfer curve is useful for the type of application here considered, it is necessary that the quiescent or zero-signal operating point be located about one-fourth of the way up the transfer curve from the minimum point. This requires the presence of bias.

To the left of the minimum point, the transfer curve indicates that negative control currents larger than that required to produce minimum output result in renewed increases in output. For this reason the range of operation for undistorted output is restricted between the lower limit of the minimum load current point and the upper limit of the midpoint of the curve. If the peaks of the signal current exceed these limits, distortion in the output will result. Distortion can also evidently be introduced by a change in bias from the optimum value. The final experimental amplifier (as described below) was intentionally overdriven to test these conclusions. Oscillograms of the resulting output wave forms are shown in Figs. MKL-13382-1, j, k, and the flattening of the peaks and distortion due to passing the minimum current point are clearly discernible.
It may be that "Plasmotron" continuous-control gas tubes, when they become commercially available, will make possible the construction of low-impedance, high-frequency oscillators that can be effectively substituted for the Alexanderson machine suggested for this design. They have a good high-frequency response (See Appendix III) and may make possible the amplification of signals at frequencies up to several hundreds of kilocycles. At the present time, however, only experimental models are obtainable.

The second kind of amplifier that could probably be successfully designed and constructed is based on the use of the full-wave control circuit of the shunt-fed reactor type in conjunction with a constant-current (most likely a pentode amplifier) source. It would be, in contrast to the design just considered, most useful for the amplification of relatively high-frequency signals with low output-power levels. Ferrite cores and germanium point-contact rectifiers would be specified. Since the source current, not the voltage, is constant; multi-channel operation would be accomplished by putting the separate amplifier circuits in series, and not in parallel, with each other. The circuit suggested for this design acts like the doubler circuit, as we have seen, in that its output wave is symmetrical about the zero-axis. Rectifiers other than those used for self-saturation would therefore have to be used for the demodulator. When it is remembered that the transfer curves of the shunt-fed reactor circuits have maximum points instead of minimum points and are in general "upside down" as compared to those of the conventional circuits, the restrictions on the range of operation given above may readily be translated into a form applicable to this circuit.

The design of the final experimental amplifier used by the author did not follow either of the two approaches outlined above, except that the full-wave circuit was used. Otherwise, the construction runs counter to most of the good design practices outlined in this report. The carrier source was the high-impedance Williamson audio-amplifier whose characteristics were described in Chapter III. The parallel combination of six point-contact germanium diodes used in each of the meshes had a forward resistance of about 30 ohms, while the load was only 100 ohms. (This value of load resistance was found, by trial and error, to give the maximum undistorted load power.) A simple tuned circuit was inserted in the signal mesh and served to reduce their flow of induced currents at the second and higher even harmonics of the carrier frequency, but did not completely eliminate the distortion of the signal current wave shape caused by these currents. A noticeable, though unmeasurably small, current at signal frequency was induced into the load mesh.

The author may be accused of "not having practiced what he preaches". In answer he can only say that the serious deficiencies of the circuit did not become apparent until after the completion of the tests; that some of the principles which were violated were not fully understood until the final draft for this report was written and the inter-relations between the many facets of the problem could be studied at leisure; and finally, that, in any case, the construction of a satisfactory test circuit would involve an expenditure of time, effort, and money, which cannot be justified on any theoretical or practical basis known to the author.
Although the data taken with this amplifier circuit evidently are not even remotely indicative of the best performance which it should be possible to realize with well-constructed magnetic audio-amplifiers, they are still useful in some ways as qualitative verifications of the theory developed in this report. In this light, some of the significant test results are presented in the next chapter.

The complete circuit diagram of the final experimental magnetic amplifier is given in Fig. MRI-13375 which also includes a sketch of the cores used in that circuit.

VI. Additional Experimental Data Results and Conclusions

Additional Data

The following experimental data are presented here as supplementary, rather than as principal, evidence in support of the conclusions reached in this report. They will be given in the order in which the subject matter to which they refer appears within the main body of the report.

(1) Frequency Dependence of the Gain of the Half-Wave Circuit

In Chapter II, p. 16, it was stated that the theoretically derived expression for the ampere-turns gain of the half-wave circuit indicates that the gain should increase linearly with the excitation frequency, but that in the practical case, the rise is somewhat slower over a certain range. The gain then reaches a maximum value, and finally again decreases. Since the d-c gain of a given amplifier is the upper limit of the gain possible at any signal frequency, it was considered of interest to determine at what excitation frequency the maximum gain would be realized with the system under consideration. Fig. MRI-13376 shows transfer curves taken at various frequencies under otherwise identical conditions, and also shows a gain-versus-frequency curve. It must be noted that the curves shown may not be true indications of the performance of the magnetic amplifier alone, but may have reflected in them the effect of the power-supply impedance, which increases somewhat with increasing frequency.

(2) The Effect of Internal Power-Supply Impedance Upon Flux, Voltage, and Current Wave Forms

The graphical construction shown in Fig. MRI-13355, and discussed on p. 72 indicates that, when a power supply with appreciable internal impedance is used to excite a saturable reactor, the flux and current wave shapes are little different from those obtained with zero internal impedance, but that the voltage appearing across the reactor is heavily distorted. Fig. MRI-13377
shows oscillograms of flux, current, and reactor voltage waves which are in general agreement with those derived from the graphical construction, and also includes reactor voltage waves for various values of reactor current. It will be seen that the distortion in the voltage wave can become exceedingly severe. At the same time, the flux wave, being the integral of the voltage wave, is much more nearly sinusoidal, for, in the process of integration, the amplitude of each harmonic is reduced in proportion to its order.

(3) The Use of Tuning Capacitors to Improve Amplifier Gain

In Chapters II and III it was mentioned that proper use of capacitors in magnetic amplifier circuits could result in improvement of performance. Tests were made on a two-core series-connected saturable reactor where a variable condenser was shunted across both the cores simultaneously, and on a half-wave circuit. Transfer curves were measured with d-c signal. The curveweshot of Fig. MRI-13378 shows that the gain of the saturable reactor may be almost doubled without appreciable decrease in the useful range of operation. The minimum current also remains almost constant for the range of capacitance included in the test. The minimum point on each of the curves apparently represents the condition of approximate parallel resonance between reactor and condenser. It will be noted that the presence of sufficiently large values of capacitance introduces additional stationary points into the transfer curve, and that, in general, a bias winding will be needed to shift the zero control current axis into the useful range of operation.

For the half-wave circuit, the relatively small gain realised under high-frequency, high-source-impedance operation can be increased markedly through capacitive tuning, as is illustrated by the curves of Fig. MRI-13379. Instability, or snap-action can also be initiated by the use of sufficiently large values of capacitance, as shown by the curve of Fig. MRI-13380.

(h) Frequency Response of the Test Amplifier

The relation of amplifier gain to signal frequency was studied in order to supplement the discussion given at the beginning of Chapter III. The final test amplifier model was used. Tests showed that the highest signal frequency that would yield 20% or more of the maximum a-c current output was about 15 KC. Gain curves taken at various signal frequencies are shown in Fig. MRI-13361, and oscillograms of the output wave with various signal frequencies and constant signal input are shown in Fig. MRI-13382. An analysis of these data on the basis of the figures given by Harder and Horton shows that the time constant of the amplifier was about six cycles of the carrier frequency, or about 0.06 millisecond.
(5) The Magnetic Characteristics of the Ferrite Reactor Core

The locus of the endpoints of the family of hysteresis loops shown in Fig. MRI-13360-a is plotted in Fig. MRI-13383. This curve is consequently the mean magnetization curve as measured at 106 KC. The largest of the loops is also included.

Results and Conclusions

See Fig. MRI-13383. The important results of the present investigation are here summarised:

(1) The operation of the power-frequency magnetic amplifier is readily analysed by certain approximate methods of treating the nonlinear differential equations which describe the behavior. Secondary properties of core materials, rectifiers and supply sources cause discrepancies between the results of the approximate analyses and those of experiment.

(2) These discrepancies become particularly pronounced in the case of the audio-frequency amplifier and can become principal, rather than secondary factors in determining the performance. In particular, the high frequency supplies now commonly available have fairly high internal impedance and the rectifiers usable at high frequencies have high forward resistance. Both properties are detrimental to good amplifier performance. The most useful type of cores have relatively unsatisfactory magnetic characteristics which adversely affect the performance and make mathematical analysis unmanageable. For example, the permeability of these most useful ferrite materials is low, the saturation characteristics are far from ideal, and the temperature dependence of magnetic properties can give rise to heating transients which cause changes in output independently of the input circuit. The high coercive force of most ferrites makes it unrealistic to base the analysis of performance on the mean magnetization curve and necessitates the use of the complete hysteresis loop. For a-c signal, the instantaneous operating point follows not the symmetrical loop, but a multiplicity of minor loops; and an accurate analysis would require a knowledge of magnetic data descriptive of these minor loops. The accumulation of such data cannot be accomplished with sufficient accuracy with the experimental apparatus now commercially available.

(3) The conventional amplifier circuits cannot operate well from high-impedance supplies and cannot operate at all from constant-current sources. It is, however, possible to design amplifier circuits well adapted to operation from constant-current supplies.

(4) It is possible, by careful choice of amplifier components, to build practically useful audio-frequency amplifiers from certain types of applications. These amplifiers will accurately reproduce a sinusoidal signal within a limited range of signal input, and will yield a distorted output of
the amplitude if the signal exceeds this range. The frequency response is best at d-c and falls off monotonically for untuned circuits. The maximum signal frequency for useful application is about one-tenth of the carrier frequency.

Recommendations

(1) If it is intended to continue and enlarge the analytical phase of the present investigation, the first step should be the accumulation of magnetic data descriptive of core behavior under application of a-c to both carrier and signal windings. These data should be measured as precisely as possible, and, carefully studied with a view toward developing realistic approximations which would allow solution of the nonlinear system equations.

(2) If practical applications are planned, a concerted effort should be made to assemble the best available supply sources, rectifiers, and core materials for the experimental apparatus. Above all, the source used should be (depending upon the choice of circuitry) either a constant-voltage or a constant-current supply. Operation of magnetic amplifiers from supplies with fairly, but not very, high internal impedance appears to be only of academic interest, but of no practical importance.
APPENDIX I

Signal-To-Carrier Frequency Synchronizer

Early in the course of the experimental work, it was decided that it would be desirable to obtain oscillographic presentations of the complex hysteresis loops that arise under conditions of a-c signal. Clear loops are evidently obtained under such conditions only if the carrier frequency is an integral multiple of the signal frequency; otherwise, the entire inside area of the loop, as seen on the 'scope, will be covered with a smear of nonrepetitive patterns. Synchronization was accomplished by injecting pulses at the desired carrier frequency into the grid of the first stage of the Wien Bridge oscillator at the point where the grid is connected into the bridge circuit. (For a discussion of Wien Bridge oscillators, see Waveforms, Vol. 17, MIT Radiation Series.) The oscillator itself was free-running and preset to run at very nearly carrier frequency without synchronization, so that the synchronizing pulses primarily served to prevent drift between signal (Hewlett-Packard) and carrier (Wien Bridge) oscillators.

The pulses were provided by the synchronizing circuit proper. The following process was used to shape these pulses: The signal-frequency sine wave was half-wave rectified, and the resulting pulses amplified and clipped to yield a fair approximation to a square wave which was then differentiated and again rectified to give trigger pulses at the signal frequency. These were then injected into one of the grids of a free-running multivibrator set to operate at the carrier frequency. The square-wave multivibrator output was then treated in the same manner as the square wave obtained by amplification and clipping from the input signal.

The circuit diagram of the entire synchronizer is shown in the Fig. MRI-13528. Undoubtedly more effective and perhaps simpler ways could have been found to accomplish the same end, but the circuit as shown represents at least a workable solution.
APPENDIX II

Sample Calculation of Saturable Reactor Core and Winding Dimensions

Let it be assumed that a supply delivering 50 V rms at 100 KC is to be used with a load resistance of 100 Ω; that the unsaturated reactance of the reactor shall be 6 times the load resistance; that the supply voltage shall be sufficient to cause flux swings in the core equal in amplitude to 75% of the saturation flux; that the unsaturated permeability of the core is 500; and that its saturation flux density is 5000 gauss. (Representative figures for ferrite cores.)

The useful formulas are:

\[ L \approx \frac{\mu A N^2}{1m} \text{ henrys} \]

and

\[ E_{\text{rms}} = h.h \cdot f \cdot N A B \times 10^{-8} \text{ volts} \]

We have

\[ \omega L \approx \frac{\omega \mu A N^2}{1m} = 600Q \]

and

\[ 50 = h.h \cdot f \cdot N A B \times 10^{-8} \]

The unknowns are \( A \), \( l \text{ mean} \), and \( N \), and only two equations are available. The solution is also influenced, of course, by the resulting window size and necessary wire size, and, therefore, by the signal winding. Only the load winding wire size can be found from the given information.

Let us rewrite the equations in the form:

\[ \frac{N^2 A \text{ (sq. meters)}}{r \text{ mean (meters)}} = \frac{600}{f \mu_\text{rel}} \]

\[ NA = \frac{50}{h.h \cdot f \cdot B \text{ (sq. meter)}} \]

or, in the indicated MKS dimensions:

\[ \frac{N^2 A}{r \text{ mean}} = \frac{600}{10^5 \times \ln x 10^{-7} \times 500} \]

\[ = 0.096 \times 10^2 = 9.6 \]
and,

\[ NA = \frac{50}{\mu H \times 10^5 \times 0.3 \times 0.75} = 50 \times 10^{-5} \]

Let us pick a cross-sectional core area of 0.05 sq. cm = \(5 \times 10^{-6}\) sq. meter
Then

\[ N = \frac{50 \times 10^{-5}}{5 \times 10^{-6}} = 100 \text{ turns} \]

and

\[ r_{\text{mean}} = \frac{100^2 \times 5 \times 10^{-6}}{9.6} = 0.52 \text{ cm} \approx 0.2 \text{ inches} \]

The calculation shows that under representative conditions of high-frequency application, cores of extremely small cross-sections and mean diameters are required for proper design. The dimensions arrived at are near the lower limit for practical ferrite cores, while the given exciter frequency is not even enough to permit amplification of signals throughout the audio-range.
The Plasmatron, a Continuously Controllable Gas Tube

The "Plasmatron", a gas tube allowing continuous control of anode current (as previous gas tubes do not) was recently announced by RCA and a theoretical, as well as experimental study of this device was published just prior to the time of writing this report. (Proc. I.R.E., Vol. 40, No. 6, (June 1952) pp. 655-659.) Some of the statements made by the authors (Johnson and Webster) are here reproduced:

"... it is entirely possible and practicable to have both low-impedance operation and continuous control in one tube. The plasmatron is such a tube. It can deliver, and continuously control, large currents at anode potentials of a few volts. This tube, now in the developmental stage, shows excellent promise of fulfilling the long-standing need for a tube that will continuously control large currents at low voltages. In this category are found such applications as motor control, direct loud speaker drive, inverters, and the like.

The principle of operation of the plasmatron lies in the use of a plasma as a conductor between a hot cathode and an anode. This plasma, however, is generated by an auxiliary discharge (which operates independently of the work circuit) and not by the anode current as is the case with a thyatron.

Particular significance should be attached to the last statement, for it leads to the plausibility argument that it may be possible to build continuous control tubes comparable in size and otherwise similar to present-day ignitrons. A device having a steel case and devoid of a filamentary cathode would easily have sufficient ruggedness to be used along a magnetic amplifier in most applications.

The plasmatron may be used either as a diode or as a triode; but the diode frequency-response is poor and shows a strong drop-off at slightly over 10 KC. In triode operation, the frequency-response is flat to beyond 100 KC, but:

"A tube could operate as a Class C oscillator at frequencies as high as 17 mc."

The control characteristic for diode operation is quite linear, that for triode operation nearly hyperbolic, as shown in Fig. MRI-13529. When used as an amplifier, the output may, therefore, be nonsinusoidal for sine wave input, but this minor disadvantage may readily be accepted in view of the other great advantages the tube offers.

The power gain for both diode and triode operation is about 17 db.
# LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>area</td>
</tr>
<tr>
<td>B</td>
<td>magnetic flux density</td>
</tr>
<tr>
<td>(E_c)</td>
<td>d-c control voltage</td>
</tr>
<tr>
<td>(E_{\text{ind}})</td>
<td>voltage induced by sinusoidally varying flux in a concentrated winding</td>
</tr>
<tr>
<td>(e_f)</td>
<td>rectifier voltage drop in forward direction</td>
</tr>
<tr>
<td>(e_x)</td>
<td>voltage difference across rectifier</td>
</tr>
<tr>
<td>(f)</td>
<td>frequency in cycles per second</td>
</tr>
<tr>
<td>H</td>
<td>magnetic field intensity</td>
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<tr>
<td>I</td>
<td>time average of instantaneous current (i)</td>
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<tr>
<td>(I_m)</td>
<td>maximum value of sinusoidal current wave</td>
</tr>
<tr>
<td>(i)</td>
<td>instantaneous value of current</td>
</tr>
<tr>
<td>(i_s)</td>
<td>instantaneous current in a saturable reactor element</td>
</tr>
<tr>
<td>(i_R)</td>
<td>instantaneous current in a resistance</td>
</tr>
<tr>
<td>L</td>
<td>inductance</td>
</tr>
<tr>
<td>(l_m)</td>
<td>mean length of magnetic path in core</td>
</tr>
<tr>
<td>m</td>
<td>constant of proportionality</td>
</tr>
<tr>
<td>N</td>
<td>number of turns in a winding</td>
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<tr>
<td>Q</td>
<td>quality factor of a storage element</td>
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<tr>
<td>R</td>
<td>resistance</td>
</tr>
<tr>
<td>(R_f)</td>
<td>rectifier forward resistance</td>
</tr>
<tr>
<td>T</td>
<td>time constant in cycles of supply frequency</td>
</tr>
<tr>
<td>(t)</td>
<td>time in seconds</td>
</tr>
<tr>
<td>(V_m)</td>
<td>maximum value of a sinusoidal voltage wave</td>
</tr>
<tr>
<td>(v)</td>
<td>instantaneous voltage</td>
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<tr>
<td>(\mu)</td>
<td>permeability</td>
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<tr>
<td>$\rho$</td>
<td>resistivity</td>
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<tr>
<td>$\phi$</td>
<td>total flux instantaneously linking a winding</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>saturation flux</td>
</tr>
<tr>
<td>$\phi_m$</td>
<td>maximum flux</td>
</tr>
<tr>
<td>$\omega$</td>
<td>radian frequency $= 2\pi f$</td>
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</tbody>
</table>

Note: Subscript "c" denotes control mesh quantities, subscript "l" denotes load mesh quantities, subscript "x" rectifier quantities, and subscript "s" saturable reactor quantities.
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A- NORMAL MAGNETIZATION CURVES
1- DELTAMAX
2- FERRITE (FERRAMIC "A")
3- POWDERED IRON

B- HYSTERESIS LOOPS
1- 68 PERMALLOY
2- USS TRANSFORMER STEEL 52
3- FERRAMIC "A"

C- MAGNETIC TRANSITION
1- IRON
2- FERRITE
3- Fe₃O₄

CURVES ILLUSTRATING CERTAIN PROPERTIES OF FERROMAGNETIC MATERIALS. M.R.I.-13350
TYPICAL MAGNETIC AMPLIFIER CIRCUITS

A - SATURABLE REACTOR

B - TWO-CORE SATURABLE REACTOR

C - THREE-LEGGED REACTOR

D - EXTERNAL FEEDBACK AMPLIFIER

E - SELF-SATURATING AMPLIFIER - HALF-WAVE TYPE

F - SELF-SATURATING AMPLIFIER - FULL-WAVE TYPE

G - SELF-SATURATING AMPLIFIER - DOUBLER TYPE

M.R.I.-13351
SIMPLIFIED REPRESENTATION OF MAGNETIC DATA.

M.R.I.-13352
A-CIRCUIT DIAGRAM

B-LOAD-CURRENT
WAVE-FORM

C-FLUX WAVE-FORM

D-TRANSFER CURVE

THE SIMPLE SATURABLE REACTOR.

M.R.I.-13353
THE HALF-WAVE CIRCUIT
Steps in the graphical construction of flux, current, and reactor voltage waves.
FINAL RESULTS OF THE GRAPHICAL CONSTRUCTION OF FLUX, CURRENT, AND REACTOR VOLTAGE WAVES.
PERMEABILITY $\mu$ AS A FUNCTION OF COMPOSITION OF A TERNARY FERRITE SYSTEM.
TOTAL "Q" AT 500 KC AS A FUNCTION OF COMPOSITION OF A TERNARY FERRITE SYSTEM.
Curie Temperature in Degrees C. as a Function of the Composition of a Ternary Ferrite System.
VARIATION OF INITIAL PERMEABILITY WITH TEMPERATURE FOR THREE PERLITES AND MAGNETIC IRON OXIDE.
A- FAMILY OF HYSTERESIS LOOPS

B- EFFECT OF CORE HEATING ON LOOP SHAPE, PERMEABILITY, AND SATURATION FLUX DENSITY

C- EFFECT OF CORE HEATING ON CORE SATURATION (CONSTANT APPLIED VOLTAGE)

D- EFFECT OF CORE HEATING ON REACTOR CURRENT WAVE

EFFECTS OF TEMPERATURE CHANGE ON REACTOR PERFORMANCE

M.R.I.-13360
EXTERNAL CHARACTERISTICS OF WILLIAMSON
AUDIO-AMPLIFIER CARRIER SUPPLY AND OF IDEAL SOURCES
SATURABLE REACTOR

HALF-WAVE CIRCUIT
Hysteresis Loops with A.C. Signal Input

SCALE: HORIZ. 0.6" = 1 OERSTED; VERT. 1" = 2000 GAUSS

M.R.I.-13363
A - 60-CYCLE PICKUP

D - FAULTY INTEGRATION

B - WIRE-WOUND POTENTIOMETER
USED FOR "H"-DEFLECTION

E - "H"-SIGNAL PROPORTIONAL TO CARRIER ONLY

C - EXCESSIVE STRAY CAPACITANCE FROM CRT PLATES TO GROUND

F - "H"-SIGNAL PROPORTIONAL TO CARRIER ONLY

LISSAJOUS PATTERNS ILLUSTRATING ERRORS IN HYSTERESIS LOOP MEASUREMENT TECHNIQUE

M.R.I-13364
A-SHUNT-FED SATURABLE REACTOR

SOME MAGNETIC AMPLIFIER CIRCUITS DESIGNED TO BE OPERATED FROM CONSTANT CURRENT SOURCES.

M.R.I.-13365
A-CIRCUIT DIAGRAM

B-REACTOR CURRENT

C-FLUX WAVE

D-LOAD CURRENT OR VOLTAGE WAVE

THE SHUNT-FED SATURABLE REACTOR.
REACTOR CURRENT AND LOAD VOLTAGE WAVES FOR SHUNT-FED SATURABLE REACTOR.

M.R.I-13367
THE HALF-WAVE AND FULL-WAVE CONTROL CIRCUITS.
REACTOR CURRENT AND LOAD VOLTAGE WAVES FOR HALF-WAVE CONTROL CIRCUIT.
REACTOR CURRENT AND LOAD VOLTAGE WAVES FOR FULL-WAVE CONTROL CIRCUIT.

M.R.I.-13371
LOAD VOLTAGE AND REACTOR CURRENT WAVES FOR CIRCUIT OF FIGURE 17-D

M.R.I.-13373
LOAD VOLTAGE, REACTOR CURRENT, AND RECTIFIER CURRENT WAVES FOR CIRCUIT OF FIGURE 17-E.
CIRCUIT DIAGRAM OF FINAL EXPERIMENTAL AMPLIFIER.

M.R.I.-13375
CONTROL CURRENT - D.C. ma

D.C. GAIN OF THE HALF-WAVE CIRCUIT AS A FUNCTION OF CARRIER FREQUENCY. (R_L = 20 Ω, N = 60, N_c = 75)

M.R.I.-13376
Saturable reactor voltage, flux, and current waves observed with high-internal-impedance supply.

M.R.I.-13377
EFFECT OF CAPACITIVE TUNING
ON THE TRANSFER CURVE OF THE
SATURABLE REACTOR.

\( R_L = 100 \Omega; \ N = 60; \ N_c = 75; \ \text{CARRIER FREQUENCY 108 KC} \)
TRANSFER CURVES SHOWING THE EFFECT OF TUNING CAPACITANCE ON THE GAIN OF THE HALF-WAVE CIRCUIT.

LEGEND:

--- $C = C$
--- $C = 0.0005 \mu F$
--- $C = 0.0010 \mu F$
--- $C = 0.0020 \mu F$

NOTE: ONE MESH OF FINAL TEST AMPLIFIER USED; CAPACITOR CONNECTED DIRECTLY ACROSS LOAD WINDING.

M.R.I-13379
UNSTABLE TRANSFER CURVE OF HALF-WAVE CIRCUIT OBTAINED BY USE OF EXCESSIVE TUNING CAPACITANCE.

($R_L = 100\Omega$, $N = 120$, $N_C = 75$, $C = 0.008\mu f$, $F = 100$ Kc, 6
SIX IN 34 DIODES USED IN PARALLEL.)

M.R.I. - 13380
RELATION OF OUTPUT TO INPUT OF COMPLETE TEST
AUDIO-AMPLIFIER AS A FUNCTION OF SIGNAL FREQUENCY.

M.R.I.-13381
OUTPUT WAVE DISTORTION CAUSED BY OVERDRIVE

OUTPUT WAVESHAPES OF COMPLETE TEST AMPLIFIER FOR CONSTANT SIGNAL AMPLITUDE AND FOR OVERDRIVEN OPERATION.

NOTE: HORIZONTAL LINE IN EACH OSCILLOGRAM IS THE ZERO D.C. LEVEL.
HYSTERESIS LOOP AND MEAN MAGNETIZATION CURVE FOR THE FERRITE SAMPLES USED IN ALL TESTS. MEASURED BY OSCILLOGRAPHIC TECHNIQUE AT 100 KC.

NOTE: CROSSOVER AT POINTS "A" AT LEAST PARTLY DUE TO HEATING EFFECTS.

M.R.I.-13383
ANODE CURRENT—GRID VOLTAGE CHARACTERISTIC

AUXILIARY CURRENT 10 mA
ANODE POTENTIAL 8 VOLTS

GRID VOLTAGE IN VOLTS

TRIODE CONTROL CHARACTERISTIC
OF PLASMATRON GAS TUBE.

M.R.I.-13529