ATMOSPHERIC REFRACTION ERRORS
FOR OPTICAL INSTRUMENTATION

PRELIMINARY REPORT

TECHNICAL MEMORANDUM NO. 10.4

October 1953

White Sands Proving Ground
Las Cruces, New Mexico
ATMOSPHERIC REFRACTION ERRORS
FOR OPTICAL INSTRUMENTATION

Preliminary Report

TECHNICAL MEMORANDUM NO. 104

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Reviewed by: Dr. J. W. Muehlner

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Reviewed by: Thomas W. Morgan

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FLIGHT DETERMINATION LABORATORY
WHITE SANDS PROVING GROUND
Las Cruces, New Mexico
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ABSTRACT

Differences are brought out between guided-missile refraction geometry and astronomical refraction geometry. A simple relationship between angular refraction of light, refraction error for ground observer, and refraction error for aerial observer is demonstrated. While angular refraction of light depends only on the extent of appreciable density of the atmosphere, the two types of refraction error continue to vary beyond this point, due to geometry.

Basic equations for atmospheric refractive index, angular refraction, and refraction errors for ground and aerial observers are derived from physical principles, without resort to empirical equations or graphical integration. Previously available equations for angular refraction and refraction errors were applicable only to altitudes of less than 10 miles or only to infinite altitudes. Present equations hold for any altitude. Methods of this report are capable of application to any atmospheric temperature profile.
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I. INTRODUCTION

This report makes available part of the basic work which is being done by the Flight Determinator Laboratory on atmospheric refraction errors in the instrumentation of guided missiles.\(^1\) A comprehensive study of atmospheric refraction errors for optical instrumentation, based on a flat earth assumption, will be published subsequently.

II. VALIDITY OF FLAT EARTH ASSUMPTION FOR ATMOSPHERIC CALCULATIONS

The relative mass of the atmosphere at any elevation angle is given approximately by the cosecant of the elevation angle. This relationship is correct for a flat earth and a flat atmosphere.\(^2\)

To obtain an approximate measure of the accuracy of the above flat earth relationship, it was compared with results obtained by a spherical earth equation. Duntley (Ref. 1) gives an equation for relative number of molecules per unit volume at any altitude, which can be rewritten in terms of relative density:\(^3\)

\[
\frac{\rho}{\rho_o} = e^{-\frac{y}{4.11}} \tag{1}
\]

\(\rho\) = density of atmosphere at altitude \(y\)

\(\rho_o\) = density of atmosphere at ground level

\(y\) = altitude above ground level (in miles)

The same equation in terms of slant range (for a spherical earth) would be:\(^4\)

\[
\frac{\rho}{\rho_o} = e^{-\frac{\sqrt{r_o^2 + 2r_o l \sin E + l^2} - r_o}{4.11}} \tag{2}
\]

1) Most of the material in the present report was published for local distribution at WSPG on 2 February 1953.

2) In the present comparison of the relative air masses at various elevation angles for flat and spherical earths, no account is taken of the slightly curved path followed by radiation arriving at a given angle.

3) Actually, an equation of this form is correct only for an isothermal atmosphere.

4) Derivation will be given in a subsequent report.
\( r_o \) = radius of earth to ground level (in miles) 
\( l \) = slant range from ground level (in miles) 
\( E \) = elevation angle 
\[ \rho = \frac{r_o}{l} \epsilon^{-\frac{4}{3}} \]

The total integral of \( \rho \) with respect to \( l \), for a given value of \( E \), is a measure of the total air mass at elevation angle \( E \). Graphical integrations were carried out for several values of \( E \), using equation (2). The resulting totals were divided by the vertical "air mass" obtained as the integral (from 0 to \( \omega \)) of \( \rho \) with respect to \( \gamma \), using equation (1). Figure 1 shows a comparison of the relative air masses at various elevation angles for flat and spherical earths. The following table gives the percentage deviation of the flat earth equation from the spherical earth equation at low elevation angles:

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<th>Elevation angle</th>
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<th>9°</th>
<th>8°</th>
<th>7°</th>
<th>6°</th>
<th>5°</th>
<th>4°</th>
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<td>% deviation of flat earth equation</td>
<td>2.0</td>
<td>2.4</td>
<td>3.0</td>
<td>3.9</td>
<td>6.9</td>
<td>11.8</td>
<td>18.7</td>
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</table>

Where 5% accuracy is sufficient, it appears that a flat earth assumption may be safely used down to about 7° elevation, for those atmospheric parameters which have a linear dependence on intervening air mass.

### III. VARIATION OF REFRACTION PARAMETERS WITH ALTITUDE

#### A. Pressure

A linear decrease in temperature with altitude is characteristic of the troposphere as a whole. For this region the relationship between pressure and altitude may be written (Ref. 2):

\[
\ln \left( \frac{p}{p_o} \right) = -\frac{1}{LR} \ln \left( \frac{T_o + Ly}{T_o} \right) \quad \text{or} \quad \frac{p}{p_o} = \left( \frac{T_o + Ly}{T_o} \right)^{-\frac{l}{LR}} \quad (3)
\]
\[ p = \text{pressure at altitude } y \]
\[ p_0 = \text{pressure at ground level} \]
\[ L = \text{vertical temperature gradient, } \Delta T/\Delta y \]
\[ R = \text{gas constant for air, } p/g\rho T \text{ or } p_0/g\rho_0 T_0 \]
\[ T_0 = \text{absolute temperature at ground level} \]
\[ g = \text{acceleration of gravity} \]
\[ T = \text{absolute temperature at altitude } y \]

**B. Density**

Since
\[
\frac{\rho}{\rho_0} = \frac{p}{p_0} \frac{T_0}{T} 
\]

and
\[
T = T_0 + Ly 
\]

\[
\frac{\rho}{\rho_0} = \left( \frac{T_0 + Ly}{T_0} \right)^{-\frac{1}{LR}} - 1 \tag{4} 
\]

**C. Refractive Index**

If the refractive index of air at ground level is:
\[ n_0 = 1 + a_0 \]

the refractive index at any altitude may be expressed as: 5)
\[ n = 1 + a_0 \frac{\rho}{\rho_0} \tag{5} \]

For the troposphere:
\[ n = 1 + a_0 \left(1 + \frac{L}{T_0 y} \right) - \frac{1}{LR} - 1 \tag{6} \]

5) Proof will be given in a subsequent report.
D. Angular Refraction of Light

For the Earth, it can be shown from Snell's law that: (Ref. 3)

\[
\frac{\cos E}{\cos E_0} = \frac{n_o}{n}
\]  

(7)

\(E\) = elevation angle (direction) of light ray at altitude \(y\)

\(E_o\) = observed elevation angle of same light ray at ground level

Equation (7) may also be written:

\[
\frac{\cos(E_o - A.R.)}{\cos E_o} = \frac{n_o}{n}
\]

(8)

A.R. = angular refraction of light ray between \(y\) and ground level

From equation (8) it is easily shown that: (4)

\[
A.R. = \frac{n_o - n}{n} \cot E_o
\]

where A. R. is in radians

For practical purposes: (6)

\[
A. R. = \left( n_o - n \right) \cot E_o
\]

or

\[
A. R. = \left( 1 + a_o - 1 - a_o \frac{\rho}{\rho_o} \right) \cot E_o = a_o \left( 1 - \frac{\rho}{\rho_o} \right) \cot E_o
\]

For the troposphere:

\[
A. R. = a_o \left[ 1 - \left( 1 + \frac{L}{T_o} \gamma \right)^{-1} \frac{1}{L R} - \frac{1}{\rho} \right] \cot E_o
\]

(9)

It should be noted that this equation gives angular refraction in radians.

6) Since \(n\) is generally less than 1.0003 for optical frequencies, the error in this approximation will be less than 0.01%.
Although equation (9) was derived for the troposphere, it may be extrapolated to any altitude. Using the National Advisory Committee for Aeronautics standard atmosphere (Ref. 2) as a working model, checks have been made through the isothermal layer above the troposphere, and through the next layer, which is characterized by a linear increase in temperature with altitude.\(^7\) The maximum deviation of the extrapolated troposphere equation from the complete equation was 1.7%. The relative-density term in the troposphere equation goes to zero at a definite height.\(^8\) Above this height it is necessary to use zero for the density term, and equation (9) then reduces to the well-known astronomical equation for total angular refraction over a flat earth:

\[
\text{A. R.} = \alpha \cot E_o \quad \text{or} \quad \text{A. R.} = \alpha \frac{\cos E_o}{\sin E_o}
\]

At low elevation angles, the total angular refraction will be an approximately linear function of \(1/\sin E_o\), or air mass. A flat earth assumption, then, may be safely used for total angular refraction down to about 7° elevation. It would appear that the same rule may be safely applied to partial angular refraction (equation (9)).

E. Refraction Error for Ground Observer

Figure 2A represents the geometry of the refraction problem for a guided missile, or other object, in the vicinity of the earth. Light from a point on the missile (M) starts toward the ground along the line which forms elevation angle \(E\) with the horizontal. It follows an increasingly curved path and arrives at camera C at observed elevation angle \(E_o\). The difference between the initial and final directions is the angular refraction (A.R.). The difference between the final and true directions is the refraction error (R.E.) for the ground observer at C. The difference between the initial and true directions is the refraction error (R.E.'s) for an observer (or camera) in the missile looking at the ground target (C).

Obviously,

\[
\text{A. R.} = \text{R. E.} + \text{R. E.'s}
\]

\(^7\) The equation used to calculate angular refraction, accurately, through several regions of the atmosphere will be given in a subsequent report.\(^8\) For NACA standard-atmosphere data, the relative-density term \(\left(1 + \frac{L}{\rho_o} \cdot \frac{1}{\rho}\right)\) becomes zero at 145,370 ft. (above sea level).
Figure 2B represents astronomical refraction geometry, where the object \( O \) is effectively at an infinite distance from the earth. Under these conditions, rays from \( O \) arriving at the earth's atmosphere are effectively parallel to the true direction. The (angular) refraction error for an observer at \( O \) would be zero, and A.R. becomes equal to R.E. In observational work on stars or planets, the angular refraction of light in the earth's atmosphere is synonymous with the refraction error (or refraction correction).

To calculate refraction errors for guided missile work, it is necessary to introduce a horizontal coordinate \( x \), which is defined in this report as horizontal range. Corum (Appendix to Ref. 3) gives a flat earth equation from which the (angular) refraction error for a ground observer may be evaluated: \(^9\)

\[
x = \int_0^y \frac{n_o \cos E_o}{\sqrt{n^2 - n_o^2 \cos^2 E_o}} \, dy
\]

For a given value of \( x \), the corresponding corrected value of \( y \) can be obtained by solution of equation (11). Then: \( R.E. = E_o - \arctan \frac{x}{y} \). Terman (Ref. 4) gives essentially equation (11) for radio-frequency propagation in the ionosphere, with derivation references dating back to 1926.

Mallinckrodt (Ref. 5) has published a corresponding solution: \(^9\)

\[
x = y \cot E_o + \frac{\cot E_o}{\sin^2 E_o} \int_0^y \frac{v - v_o}{v_o} \, dy
\]

\( v = \) velocity of propagation of radiation at altitude \( y \)
\( v_o = \) velocity of propagation at ground level

Equation (12) may also be written: \(^5\)

\[
x = y \cot E_o + \frac{\cot E_o}{\sin^2 E_o} \int_0^y \frac{n_o - n}{n} \, dy
\]

\(^9\) Derived for radar systems, but directly applicable to optical systems.
For refractive indices very close to unity, it can be shown that equations (11) and (13) are approximate identities. Mallinckrodt's equation reduces the work of integration and has the advantage of being in the form: \( x = \cot E_0 (y + Ay) \). Equation (13) may be simplified to:

\[
x = y \cot E_0 + \frac{\cot E_0}{\sin^2 E_0} \int_0^y (n_0 - n) \, dy
\]

The function \( n_0 - n \) was evaluated for the troposphere in equation (9). Substituting this term in equation (14):

\[
x = y \cot E_0 + \frac{a_o \cot E_0}{\sin^2 E_0} \left[ 1 - \left( 1 + \frac{L}{T_o} y \right)^{\frac{1}{L/R}} - 1 \right] \, dy
\]

Integration gives:

\[
x = y \cot E_0 + \frac{a_o \cot E_0}{\sin^2 E_0} \left\{ y + R T_o \left[ \left( 1 + \frac{L}{T_o} y \right)^{\frac{1}{L/R}} - 1 \right] \right\}
\]

It can be shown geometrically that:

\[
l \left( \text{R. E.} \right) = Ay \cos E_0 \quad \text{or} \quad \text{R. E.} = \frac{\cos E_0 \Delta y}{l}
\]

\( l = \) slant range
\( \Delta y = \) apparent \( y \) minus true \( y \)

10) The error in the function \( n_0 - n \) is covered by footnote 6. The error in the integral will be discussed later in this section.
Substituting for \( \Delta y \) from equation (16):

\[
R.E. = \frac{\cos E_o}{l} \frac{a_o}{\sin^2 E_o} \left\{ y + \frac{RT_o}{y} \left[ 1 + \frac{L}{T_o} y \left( 1 + \frac{L}{T_o} \frac{1}{LR} - 1 \right) \right] \right\}
\]

Since \( l \sin E_o \) is approximately equal to \( y \):

\[
R.E. = \frac{a_o \cos E_o}{y \sin E_o} \left\{ y + \frac{RT_o}{y} \left[ 1 + \frac{L}{T_o} y \left( 1 + \frac{L}{T_o} \frac{1}{LR} - 1 \right) \right] \right\}
\]

or

\[
R.E. = \frac{a_o \cot E_o}{y} \left\{ 1 + \frac{RT_o}{y} \left[ \left( 1 + \frac{L}{T_o} y \right) - \frac{1}{LR} \right] - 1 \right\}
\]

(17)

If roughly approximate values of \( x \) and \( y \) are obtained in the first step of a reduction of trajectory data, the angular refraction error is given by:

\[
R.E. = \frac{a_o}{y} \left\{ x + \frac{RT_o}{y} \left[ \left( 1 + \frac{L}{T_o} y \right) - \frac{1}{LR} \right] - 1 \right\}
\]

(18)

It should be noted that equations (17) and (18) give refraction error in radians.

Although equations (17) and (18) were derived for the troposphere, either one may be used, practically, for any altitude. Using the NACA standard atmosphere (Ref. 2) as a working model, checks have been made through all (nine) regions of the standard atmosphere. 11)

11) The equation used to calculate refraction error, accurately, through all regions of the NACA standard atmosphere will be given in a subsequent report. NACA data for the troposphere are as follows:

\[
T_o = 518.4 \, ^\circ \text{Rankine} \qquad L = -.003566 \, ^\circ \text{F/ft}
\]

Height of troposphere: (to) 35,332 ft. above sea level

Value of \( R \) taken as 33.3583 ft./F

Value of \( a_o \) taken as 0,000,02728 from value given by Epstein (Ref. 7). A better choice for optical frequencies would have been 0,000,02762 from value given by Sears (Ref. 8).
Table I and Figure 3 show a comparison of the FDL complete equation, an empirical equation published by Mallinckrodt (Ref. 6), and the above single-region equations. Refraction error was calculated by each equation for a series of altitudes (and several angles). Of the troposphere equations, (16) was actually used together with:

\[ \text{R.E.} = E_o - \arctan \frac{v}{x} \]

Most of the deviation of the Mallinckrodt equation, within the troposphere, is due to the difference between Cocoa, Florida mean annual temperature (21.2°C) and NACA standard temperature (15°C). It may be seen that Mallinckrodt's empirical equation is accurate only for the troposphere and becomes unusable above about 70,000 feet.

The small deviation of equation (16) in the troposphere is due to simplifying \( \frac{n_o - n}{n} \) to \( n_o - n \) (within the integral term). Equation (16) shows a deviation approaching 1% in the isothermal region above the troposphere, but this deviation decreases at higher altitudes. The pressure term in equations (16), (17) and (18) goes to zero at a definite height. Above this height it is necessary to use zero for the pressure term. In Figure 3, the curve for the single-region equation is drawn to show what happens if the real part of the complex number, which develops above this height, is used instead of zero.

The angular refraction of light varies only within the extent of appreciable density of the atmosphere. The refraction error for a ground observer continues to vary beyond this point, due to the purely geometric factor of vertical distance. In equation (17), R.E. does not become equal to total A.R. until \( y \) is infinite.

If \( y \) is fixed at the value where the pressure term becomes zero, equation (17) becomes:

\[ \text{R.E.} = a_o \cot E_o \left( 1 - \frac{RT_o}{y} \right) \quad \text{or} \quad \text{R.E.} = a_o \frac{\cos E_o}{\sin E_o} \left( 1 - \frac{RT_o}{y} \right) \]

12) A comparison based on Cocoa, Florida data will be given in a subsequent report.

13) For NACA standard-atmosphere data, the pressure term \( \left( 1 + \frac{L}{T_o} y \right) \frac{1}{\rho_o} \frac{1}{\gamma} \) is used. The density at 370 ft.
If range is now increased to maintain this same value of \( y \) as the elevation angle is decreased, the refraction error will be an approximately linear function of \( 1/\sin E \) or air mass.

For very long ranges, then, the flat earth assumption may be safely used for angular refraction error down to about \( 7^\circ \) elevation. It would appear that the same rule may be safely applied for ordinary ranges.

Where the lower regions of the troposphere show an irregular temperature profile, it may be desirable to break up the troposphere into a series of regions with different temperature gradients (either positive or negative). The integral term in equation (15) then becomes a series of similar integrals.\(^\text{14}\)

**F. Refraction Error for Aerial Observer**

From equation (10):

\[
R.E. = A.R. - R.E. \quad (19)
\]

Substituting equations (9) and (17) in equation (19) and collecting terms:

\[
R.E. = \alpha_0 \cot \theta \left( \frac{RT}{y} - \left( 1 + \frac{L}{T} \right) \frac{R}{L} - \frac{1}{1 + \frac{RT}{y} \left( 1 + \frac{L}{T} \right) y} \right) \quad (20)
\]

It should be noted that equation (20) gives refraction error for an aerial observer in radians.

From previous considerations, equation (20) may be used for any altitude and may be safely used down to about \( 7^\circ \) elevation.

\(\text{14})\) The use of the above refraction-error equations to fit any temperature profile of a stationary atmosphere (including possible isothermal regions) will be covered in detail in a subsequent report.
REFERENCES


Figure 2A. Refraction Geometry for Guided Missile

Figure 2B. Astronomical Refraction Geometry
<table>
<thead>
<tr>
<th>Apparent Altitude (ft.)</th>
<th>Apparent Elevation ((^\circ))</th>
<th>% Deviation of Other Equations from FDL Complete Equation (sec. of arc)</th>
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**TABLE I**
Comparison of Refraction Errors for Ground Observer Calculated by Various Equations

NACA Data (except for Maltinocchiyo equation)
Initial Distribution of this Report is as follows:

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