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PHYSICS AND ENGINEERING OF RAPID DECOMPRESSION
A General Theory of Rapid Decompression

PROJECT NUMBER 21-1201-0008
REPORT NUMBER 3

PROJECT REPORT
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PHYSICS AND ENGINEERING OF RAPID DECOMPRESSION
A General Theory of Rapid Decompression

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Air University
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August 1953
PRECIS

OBJECT:
To discuss the theoretical background of the physical process of rapid
decompression and to present a workable method of determining the im-
portant facts of rapid decompression.

SUMMARY:
Theoretical considerations indicate that the analysis of the entire process
of rapid decompression can be split into two parts. The first one includes
the geometrical and aerodynamic quantities of the system under considera-
tion and can best be represented by a term which is called time-constant,
because it sets the time scale for the entire process. The second part
contains a pressure function, which determines the dependence of the
process upon the pressures involved. The initial rate of pressure change
is discussed and presented in a similar fashion. The experiments are in
good agreement with the theory.
A. GENERAL THEORY OF RAPID DECOMPRESSION

The physical process of rapid or explosive decompression has been the topic of discussion in many papers (1 - 4). The results in those papers were based on a number of assumptions and simplifications which, in many instances, are not fully justified. The object of this report is to arrive at a general theory of rapid decompression and to verify the theoretical results by experiments. Moreover, for practical use, a method will be presented to determine the important phenomena of rapid decompression.

DISCUSSION OF SIGNIFICANT FACTS

An attempt has been made to evolve a general theory of rapid decompression by taking into consideration many of the significant facts. No differentiation will be made between rapid decompression or explosive decompression since no factor has yet been found which really would justify the differentiation. In the following only the term rapid decompression will be used:

1. Temperature

Previous measurements have indicated that the temperature drop associated with rapid decompression can be very great. Temperature changes in the neighborhood of 100°C have been observed (5). Therefore, to treat the process of decompression as an isothermal one is not justified. On the other hand, the process is not an adiabatic one because heat exchange between air and cabin wall is not negligible.

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2. **Humidity**

The influence of humidity on the physical process of decompression was shown in an earlier report (5). Because the temperature drops below the dew-point in the first moments of decompression, the greatest part of the decompression will take place in air of 100 percent humidity regardless of the initial humidity. The heat released from condensing water vapor will also cause a deviation from an adiabatic process of an ideal gas. Therefore, the process should be treated as a polytropic one (i.e., a process that lies between an adiabatic one and an isothermal one).

3. **Back Pressure**

There is a difference between the rapid decompression that takes place in an aircraft and the rapid decompression that occurs in a usual experimental setup. In an aircraft the back pressure is equal to the atmospheric pressure of the ambient air and remains constant during the process of decompression. In experiments, however, two chambers of different size which can be connected are used. The smaller chamber simulates the cockpit of the aircraft cabin and has the same pressure as the cabin. The large chamber has a pressure which is lower than that in the small chamber, thus simulating the pressure differences as they actually exist in the aircraft flying at high altitude. After suddenly opening the connection between the two chambers, the pressure in the larger chamber does not stay constant but rises, because of the airflow from the small chamber. It is this change in back pressure which makes an experimental decompression different from an actual case. Its significance should be considered. In the discussion that follows, \( P_{co} \) always stands for the pressure in the cabin or in the smaller chamber.

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*In the cited reference, an error was unfortunately made concerning the sign in Eq. (2). This error has no bearing on the influence of the humidity as shown in figures 1 and 2 of the reference.*
4. Critical Pressure Ratios

There is a difference of flow pattern through the orifice depending upon the ratio \( \frac{p_c}{p_a} \) of the two pressures on both sides of the orifice. If the pressure ratio increases, the velocity in the orifice also increases. In the true adiabatic case, the speed of sound is attained at a pressure ratio of 1.89 \((1 - \frac{1}{4})\). This speed will not be exceeded even if the pressure ratio is increased more. The pressure ratio of 1.89 is therefore called the critical pressure ratio. Those ratios smaller than the critical are called subcritical and those ratios higher are called the supercritical ratios. In polytropic processes the ratio is smaller than 1.89.

5. Effective Cross Section

The practical flow of air through an orifice deviates from the theoretical flow. The reasons for such deviations are numerous, such as reduced velocity in the orifice due to friction or formation of eddies at sudden changes in cross section. All those modifications are usually accounted for by the introduction of an average coefficient of orifice, which depends upon the shape of the orifice. It is also possible to include pressure losses in ducts and bends in this coefficient. The coefficient of orifice thus reduces the geometrical cross section of the orifice to the so-called effective cross section.

In an experimental setup which includes all the above enumerated factors, it is possible to determine the thermodynamic process by measuring temperature and pressure. Knowing the nature of the thermodynamic process, it is then possible to draw conclusions concerning the aerodynamic facts of rapid decompression such as the pressure losses and the aerodynamic properties of the orifice.
Some of the basic concepts and all of the experimental results are discussed in this section. A full account of the theoretical deductions, however, is given in the appendix.

**BASIC CONCEPTS**

In an approximative fashion, the pressure change \( \Delta p \) in terms of the initial pressure \( p_{co} \) can be written as follows:

\[
\frac{\Delta p}{p_{co}} = \frac{\Delta V}{V_c} \tag{1}
\]

\( \Delta V \) being the volume of air which has passed through the orifice and \( V_c \) designating the volume of the cabin. The loss \( \Delta V \) can be expressed by

\[
\Delta V = A \cdot w \cdot \Delta t \tag{2}
\]

if \( A \) is the area of the orifice, \( w \) the rate of flow in the orifice, and \( \Delta t \) the time element. Inserting Eq. (2) into Eq. (1) yields

\[
\frac{\Delta p}{p_{co}} = \frac{A}{V_c} \cdot w \cdot \Delta t \tag{3}
\]

The velocity \( w \) is a function of pressure and density. The density can be eliminated by introducing the speed of sound as a characteristic of the flow. The velocity \( w \) can thus be expressed by

\[
w = c \cdot f\left(\frac{p_f}{p_{co}}\right) \tag{4}
\]

\( c \) being the speed of sound, with \( f \) a function indicating the dependence of the rate of flow upon the final pressure \( p_f \) after decompression and the initial pressure \( p_{co} \). It may be noted that the speed of sound is not necessarily attained as speed in the orifice. The numerical value of \( f\left(\frac{p_f}{p_{co}}\right) \) is never greater

\( \Delta p, \Delta V, \text{ and } \Delta t \) are used for infinitesimally small changes of pressure, volume, and time.
than 1.0, indicating that the speed \( w \) never exceeds the speed of sound. With Eq. (4) in Eq. (3)

\[
\frac{\Delta p}{p_0} = \frac{A \cdot c}{V_c} \cdot f\left(\frac{p_f}{p_0}\right) \cdot \Delta t
\]

(5)

is obtained. Since information is sought about the time required for a certain drop in pressure, Eq. (5) is solved for

\[
\Delta t = \frac{V_c}{A \cdot c} \cdot \frac{\Delta p}{p_0} \cdot \frac{1}{f\left(\frac{p_f}{p_0}\right)}
\]

(6)

Despite the readers' possible reluctance or antipathy against something expressed in mathematical terms, it is suggested that Eq. (6) be checked for the combination of units. Since both sides of an equation must have the same units, the right-hand side of Eq. (6) therefore should appear in units of time. The last term in Eq. (6), containing the pressure, obviously is without units because only pressure ratios are used. Hence, the term \( \frac{V_c}{A \cdot c} \) must appear in units of time which in fact is the case and can be easily demonstrated by factoring out

\[
\frac{V_c}{A \cdot c} \cdot \frac{ft^3}{ft^2 \cdot ft/\text{sec}} = \text{sec}
\]

Considering different cases of rapid decompression, with identical pressure ratios involved, the term \( \frac{\Delta p}{p_0} \cdot \frac{1}{f\left(\frac{p_f}{p_0}\right)} \) will assume the same numerical value. From Eq. (6) it becomes apparent that the time \( \Delta t \) is then solely determined by the factor \( \frac{V_c}{A \cdot c} \)

This factor sets the time scale of the rapid decompression. It includes all constants of the system under consideration and is independent of the pressure conditions.

It is suggested that this term \( \frac{V_c}{A \cdot c} \) be given an identifying name and be called time-constant \( t_c \). A small time-constant means a short time of decompression, i.e., a fast decompression and vice versa. For example, a cabin of 500 cu. ft. and an area \( A \) of 1 sq. ft., together with speed of sound of 1,130 ft./sec. (680 F.) would yield a time-constant of 0.472 sec.
The theory, including other considerations such as various volume ratios of chambers in experimental decompression, subcritical and supercritical pressure ratios, has been elaborated upon more thoroughly in the appendix. It is shown in the appendix, that the total time $t_E$ of decompression can be expressed in a similar fashion as in Eq. (6). From Eq. (20) in the appendix, the time $t_E$ is

$$t_E = t_C \cdot P_1$$

$P_1$ is a function of the cabin pressure $P_{co}$ before decompression, and $P_{ao}$ the pressure of the ambient air. The term $P_1$ is described in the appendix and given in figure 1 as a function of $\frac{P_{co} - P_{ao}}{P_{co}}$.

There is one important conclusion to be drawn from Eq. (7) and figure 1. The value of $P_1$ does not depend upon the absolute value of the pressure difference $P_{co} - P_{ao}$, but depends only upon the ratio of this difference to the initial pressure $P_{co}$. A pressure difference of 200 mm. Hg at an initial pressure of 600 mm. Hg will bring about the same time of decompression as a difference of 100 mm. Hg at 300 mm. initial pressure. Or, another example, a pressure difference of 100 mm. Hg at 200 mm. Hg initial pressure yields a longer time of decompression than a pressure difference of 100 mm. Hg at 600 mm. Hg. The relative reduction of the initial pressure is the most important factor and not the absolute reduction.

Thus, the determination of the time of decompression is not difficult. If, for instance, cabin pressure $P_{co} = 600$ mm. Hg, ambient pressure $P_{ao} = 200$ mm. Hg, then $\frac{P_{co} - P_{ao}}{P_{co}} = \frac{600 - 200}{600} = \frac{400}{600} = 0.66$. The pertinent value of $P_1$ is found in figure 1 to be 2.10. If the time-constant is again $0.442$ sec, then the decompression time is

$$t_E = 0.442 \cdot 2.10 = 0.93$$

Figure 1 is strictly valid only for decompression with a constant back pressure. For decompression from a small chamber into a larger one, it would be necessary to
use a slightly different form of the function $P_1$, together with the same time-constant. It has been found, however, that the general function as shown in figure 1, is applicable if the final pressure $P_f$ is used instead of the initial $P_{f_0}$.

![Figure 1](image)

**Figure 1**

Pressure function $P_1$ for the total time $t_E$ of decompression.

As shown in the text, the total time of decompression $t_E = t_c \cdot P_1$. 

7
pressure $p_{ao}$. The final pressure $p_f$ is the pressure in both chambers after decompression and can easily be determined from the gas law to be

$$p_f = p_{ao} \frac{V_a}{V_a + V_c} + p_{co} \frac{V_c}{V_a + V_c}$$

(8)

if $V_a$ is the volume of the larger chamber and $V_c$ the volume of the smaller one. The pressure difference to be used in figure 1 is then $p_{co} - p_f$ instead of $p_{co} - p_{ao}$ and is given by

$$p_{co} - p_f = (p_{co} - p_{ao}) \frac{V_a}{V_a + V_c}$$

(9)

For all practical purposes this substitution is satisfactory with sufficient accuracy.

In order to arrive at a desired pressure difference $p_{co} - p_f$, it is necessary to evacuate the large chamber to the pressure $p_{ao}$, which is lower than the pressure $p_f$. Figure 2 shows a nomogram for calculation of the pressure difference $p_f - p_{ao}$ for various ratios $\frac{V_c}{V_a}$. The procedure for calculation is given in the legend of figure 2.

An evaluation of figure 1 is made for a time-constant of 1 sec. and is presented in figure 3. It shows the time of decompression in seconds as it depends on initial pressure $p_{co}$ and ambient pressure $p_{ao}$. It also shows lines for constant pressure differentials $p_{co} - p_{ao}$. It may be noted that for a time-constant other than 1.0, the actual time of decompression changes accordingly. For example, a time-constant of 0.42 sec. and a time of decompression of 2 sec., as found in figure 3, would result in an actual time of decompression of $2 \times 0.42$ or 0.88 sec.

In many cases it is important to know the initial rate of the pressure change. As outlined in the appendix, this rate of pressure change can be determined as

$$\frac{dp_f}{dt} = -\frac{p_{ao} - p_f}{t_c} \cdot \frac{p_f}{p_2}$$

(10)
Figure 2

Nomogram for the calculation of the pressure difference \( P_f - P_{ao} \).

Example:
The desired final pressure difference is \( P_{co} - P_f = 200 \text{ mm. Hg.} \) The volume ratio is 0.25 or \( \frac{1}{4} \). By drawing a line from 200 on the left scale to 0.25 on the middle scale a pressure difference of 50 mm. Hg is found on the right scale. Therefore a rise in back pressure of 50 mm. Hg is to be expected and an initial pressure difference \( P_{co} - P_{ao} = 200 + 50 = 250 \text{ mm. Hg} \) would be chosen.
Eq. (10) shows that the initial pressure $p_{co}$ and the time-constant $t_c$ are the determining factors for the initial rate of pressure change. The term $P_2$ is again a function of the pressure difference $\frac{p_{co} - p_{ao}}{p_{co}}$ and is shown in figure 4.

In the supercritical range, the term $P_2$ becomes constant because the speed in the orifice does not increase if the pressure ratio is increased. The determination of the rate of pressure change with the help of figure 4 is done in the following fashion: Assuming an initial pressure of $p_{co} = 600$ mm Hg, ambient pressure 200, then $\frac{p_{co} - p_{ao}}{p_{co}} = 0.66$. From figure 4, $P_2$ is found to be 0.69. With a time-constant of 0.42 sec, the initial rate of pressure change is

![Figure 3](image)

**Figure 3**

Total time $t_c$ of decompression as a function of the initial pressure $p_{co}$ for a time-constant $t_c = 1.0$ sec.

Note: Solid lines indicate ambient pressure $p_{ao}$; dotted lines indicate pressure differentials $p_{co} - p_{ao}$. 
\[ \frac{dP_c}{dt} = -\frac{600}{0.442} \cdot 0.69 = -937 \text{ mm Hg/sec} \]

An evaluation of figure 4 for a time-constant of 1 sec. is shown on figure 5. If the time-constant has a value other than 1.0 then the initial rate of pressure change varies accordingly. For instance, if a rate of change of 300 mm. Hg/sec. is found for the time-constant of 1.0 sec. the corresponding rate of change for a time-constant of 0.442 sec. would be

\[ -300 \cdot \frac{1.0}{0.442} = -679 \text{ mm Hg/sec} \]

![Pressure function graph](image)

**Figure 4**

Pressure function \( P_2 \) for the initial rate of pressure change.

As shown in the text, the initial rate of pressure change is

\[ \frac{dP_c}{dt} = -\frac{P_0 \cdot P_2}{P_0} \]
The time of decompression, as used in the foregoing discussions, is determined by the fact that the cabin pressure $p_c$ becomes equal to the ambient pressure $p_a$, i.e., after a complete equalization of pressures. This time is definite and becomes infinite only if decompression occurs in a complete vacuum. As can be seen from figure 3, this time of decompression can become very great and especially so if decompression to low ambient pressure is involved. In experimental decompression it is sometimes difficult to evaluate the recordings with regard to the decompression time because the pressure $p_c$ approaches the ambient pressure $p_a$ quite slowly. In order to facilitate the evaluation, it has been suggested that another time interval be used to characterize the process of decompression, rather than the total time. The method most often suggested is to

![Figure 5](image)

**Figure 5**

Initial rate of pressure change as function of the initial pressure for a time-constant $t_c = 1.0$ sec.

*Example:*

- For $p_{500} = 500$ mm Hg, $p_{800} = 400$ mm Hg, $\frac{dpc}{dt} = -290$ mm Hg/sec.
- For $p_{500} = 500$ mm Hg, $p_{800} = 200$ mm Hg, $\frac{dpc}{dt} = -345$ mm Hg/sec.
measure the time which elapses until the initial pressure difference has been reduced to a certain fraction of its initial value. The introduction of this arbitrary fraction, however, is debatable.

It is therefore proposed that use be made of the initial rate of pressure change in order to arrive at a well-defined time which can be easily evaluated. For this purpose the initial rate is used as a constant rate throughout the rapid decompression. As shown on figure 6 the line of initial rate of change is extended until it intersects the ambient pressure $p_{ao}$. The point of intersection marks a time which is evidently related to the initial rate of pressure change and the pressure difference. As already mentioned, it has the additional advantage of convenient evaluation from recordings. This time may be called the constant rate time.

The constant rate time $t_R$ is given by

$$t_R = t_c \cdot P_3$$

with $P_3$ being a function of $\frac{p_{co} - p_{ao}}{p_{co}}$ as shown in figure 7.

An evaluation of figure 7 for practical purposes is presented in figure 8 for a
time-constant of 1.0 sec. It may be noted that for subcritical pressure ratios, the constant rate time is about half of the total decompression time.

Either the total or the constant rate time, or the initial rate of pressure change, can be used to determine factors which are inadvertently omitted in

![Pressure Function](image)

Figure 7

Pressure function $P_3$ for constant rate time $t_R$.

As shown in the text, the constant rate time $t_R = t_c \cdot P_3$. 
experiments or reports. For instance, it is possible to determine the time-constant if the pressures involved and the decompression time are known. The time-constant is then given by

$$t_c = \frac{t_p}{\rho}$$

On the other hand, it is possible to determine the pressure ratio involved if time-constant and decompression time are given. The general presentations of figures 1, 4, and 7 are best suited for such manipulations.

EXPERIMENTAL RESULTS

Experiments have been carried out on two different chamber arrangements at the USAF School of Aviation Medicine. The first one was of the so-called parasite type, i.e., a small, separate chamber with a duct connecting the small chamber with a large chamber. Three valves, which can be operated independently, allow

![Figure 8]

**Figure 8**

Constant rate time $t_c$ as function of the initial pressure for a time-constant $t_p = 1.0$ sec.
the connection between the two chambers to be opened suddenly. A detailed description of this setup is given in a separate report (6).

The second chamber arrangement used in the experiments was a high-altitude chamber with a small air lock. The door from the air lock to the chamber has a circular opening of 11 inches in diameter. This opening can be sealed off by a membrane which is then punctured for the rapid decompression. This latter arrangement will be referred to as the D-chamber, in this report.

Measurements were taken of the absolute pressure $p_c$ in the small chamber $V_c$ and of the pressure difference $p_c - p_a$ between the small and the large chamber. For the pressure recordings Statham strain gages were used with a pressure range of 15 p.s.i. and a natural frequency of 100 c.p.s. The temperature changes in the small chamber were recorded with iron-constantan thermocouples. A very small time lag of the thermocouples was obtained by the use of a wire with 0.0008-inch diameter. The recordings were made with suitable galvanometers and a photokymograph. No amplification was necessary. A typical recording is shown in figure 9.

![Figure 9](image.png)

Figure 9

Recording of changes in pressure and temperature as they occurred during rapid decompression.

Note: The zero line for $p_c$ is not given because there is insufficient room on the recording paper to show the entire range.
In all, about 75 rapid decompressions were recorded over a wide range of pressures, with various combinations of the aforementioned valves, in order to have various time-constants.

The recordings were first evaluated with regard to the pressure-temperature relationship. In polytropic processes the absolute temperature $T$ and the pressure $p$ are related by

$$\frac{T_f}{T_{co}} = \left( \frac{p_f}{p_{co}} \right)^{\frac{n-1}{n}}$$

where $T_{co}$ is the absolute temperature before and $T_f$ after decompression, $n$ is the polytropic exponent. By plotting the temperature ratio $\frac{T_f}{T_{co}}$ against pressure ratio $\frac{p_f}{p_{co}}$ on double logarithmic paper, it is possible to determine the exponent $\frac{n-1}{n}$ from the slope and to find the polytropic exponent $n$. This has been done on figure 10. The least square slope indicates an exponent of $n = 1.16$, which is in the order expected, since the value should be between 1.0 (isothermal) and 1.4 (adiabatic). The value thus found was used to calculate the function $P_1$ as shown in figure 1.

It is realized that the polytropic exponent, as found in the described manner, actually is an average value over the time interval of the decompression rather
than an instantaneous one. However, in this discussion emphasis is on the average values, which are valid for the rapid decompression as a whole. The value of the polytropic exponent has an influence on the value of the speed of sound. With the experimentally found value $n = 1.16$ the speed of sound in ft./sec. is given by

$$C = 44.7 \cdot \sqrt{T_c + 459.4}$$

$T_c$ being the temperature in °F of the air in the cabin. Figure 11 shows the speed of sound as a function of cabin temperature. If high accuracy is not required, a value of 1,000 ft./sec. is most suitable. The polytropic exponent $n = 1.16$ yields a critical pressure ratio of $\frac{P_{co}}{P_{so}} = 1.75$. The pertinent value of $\frac{P_{co} - P_{so}}{P_{co}}$ is then $1 - \frac{1}{1.75} = 0.427$.

In evaluating the recordings of rapid decompression time, the procedure applied was as follows:

First, the time-constant of the system used was calculated, assuming a coefficient of orifice of 1.0. Then, the total time $t_E$ of decompression was

![Figure 11](image)

Speed of sound in rapid decompression as function of temperature.
measured from the pressure recordings, and the ratio \( \frac{t_E}{t_c} \) calculated. These values were then plotted against the pressure term \( \frac{P_{co} - P_f}{P_{co}} \). Figure 12 shows the results of all these experiments. Figure 12 also shows as a dotted line the theoretical values. It can be seen that the experimental values deviate from the theoretical ones, indicating that the effective cross section in the orifice differs from the geometrical one -- as used for calculating the time-constant. The deviation varies for the two different chambers which were used for the experiments. However, so far as the general trend is concerned, both the experiments and the theory are in good accord. This can be shown by taking into consideration a coefficient of orifice which compensates for the difference between geometrical and effective cross section. The agreement between theory and experiment is within reasonable limits if a coefficient of orifice of .25 is used for the parasite chamber and a coefficient of .95 for the D-chamber. See figure 13.

![Figure 12](image)

Figure 12

Ratio of \( \frac{t_E}{t_c} \) resulting from experiments. Dotted line indicates theoretical values; the values of the D-chamber deviate slightly.
Assuming the thermodynamic considerations in the theory of rapid decompression are correct, it is possible to determine the coefficient of the orifice by computing the ratio of the theoretical value of decompression time to the experimental values of decompression time. The experimental results were evaluated in that manner and coefficients of .20 and .30 were found for the parasite chamber and coefficients of .90 to close to 1.0 were found for the lock chamber. The smaller coefficient of the orifice for the parasite chamber appears plausible considering the long ducts with sharp bends and sudden changes in diameter.

Taking into account the orifice coefficients as found in the experiments, the time constants for the various arrangements are as follows:

**Parasite Chamber:**

Valve I \( t_c = 0.545 \text{ sec.} \)

Valves I and II \( t_c = 0.272 \text{ sec.} \)

![Figure 13](image)

The results of decompression experiments presented as relationship between \( \frac{t_c}{P_c} \) and \( \frac{P_{co} - P_f}{P_{co}} \).

All values are in good agreement with the theory (straight line) after being corrected for a proper coefficient of orifice.
Valves I, II, and III \[ t_c = 0.181 \text{ sec.} \]

**D-Chamber:**
\[ t_0 = 0.712 \text{ sec.} \]

It may be noted that the effective cross section is a point of uncertainty if it comes to reducing to practice the theory of rapid decompression since it has a strong influence on the time-constant. For cases like those occurring in aircraft, coefficients of 0.8 to 1.0 seem appropriate.

**Acknowledgment**

The authors wish to express their gratitude to Dr. R. W. Bancroft for his invaluable help and suggestions.

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APPENDIX

Notations:

- $P_c$: Pressure in small chamber
- $P_{co}$: Pressure in small chamber before decompression
- $\rho_c$: Air density in small chamber
- $\rho_{co}$: Air density in small chamber before decompression
- $V_c$: Volume of small chamber
- $m_c$: Air mass on $V_c$ before decompression
- $P_a$: Pressure in large chamber
- $P_{ao}$: Pressure in large chamber before decompression
- $\rho_a$: Air density in large chamber
- $\rho_{ao}$: Air density in large chamber before decompression
- $V_a$: Volume of large chamber
- $m_a$: Air mass in $V_a$ before decompression
- $P_f$: Final pressure in both chambers after decompression
- $\rho_A$: Air density in orifice
- $A$: Effective cross section of orifice
- $w$: Speed of air in cross section of orifice
- $c_o$: Speed of sound in $V_c$ before decompression
- $n$: Polytropic exponent
- $t$: Time
- $t_c$: Time constant $= \frac{V_c}{A \cdot c_o}$
- $t_E$: Time of decompression
- $\mu$: Mass ratio $= \frac{m_c}{m_a}$
The mass flow through the orifice is determined by

\[
\frac{dm_c}{dt} = -s_A \cdot A \cdot w
\]  

(1)

The mass element is given by

\[
dm_c = V_c \cdot ds_c
\]  

(11)

In polytropic processes

\[
s_c = s_{c_0} \cdot \left( \frac{p_c}{p_{c_0}} \right)^{\frac{1}{n_u}}
\]  

(11i)

\[
ds_c = \frac{1}{n_u} \cdot \frac{s_{c_0}}{p_{c_0}} \cdot \left( \frac{p_c}{p_{c_0}} \right)^{\frac{1-n_u}{n_u}} dp_c
\]  

(11iv)

For subcritical pressure ratios, i.e., \( \frac{p_{c_0}}{p_a} < 1.89 \)

the speed \( w \) is determined by

\[
w = \sqrt{\frac{2}{n-1} \cdot \frac{\dot{m}}{s_c} \cdot \frac{p_c}{p_{c_0}} \cdot \left[ 1 - \left( \frac{p_a}{p_c} \right)^{\frac{n-1}{n}} \right]}
\]  

(5)

Considering that the speed of sound is given by \( c_0 = \sqrt{\frac{n}{s_c} \cdot \frac{p_{c_0}}{p_{c_0}}} \)

and introducing Eq. (11) through (5) into (1) yields

\[
\frac{d\left( \frac{p_c}{p_{c_0}} \right)}{dt} \cdot \frac{V_c}{s_c} = \frac{k_c}{n-1} \cdot \frac{\dot{m}}{s_c} \cdot \left( \frac{p_c}{p_{c_0}} \right)^{\frac{n-1}{n}} \left[ 1 - \left( \frac{p_a}{p_c} \right)^{\frac{n-1}{n}} \right] - \left( \frac{p_a}{p_{c_0}} \right)^{\frac{n-1}{n}}
\]  

(6)

The pressure \( p_a \) is constant, if the decompression takes place into the open air. However, if the air flows from one chamber to the other then \( p_a \) is not constant.

It is obvious that

\[
dm_c = -dm_c
\]  

(vii)

or

\[
V_a \cdot ds_a = -V_c \cdot ds_c
\]
Integrating Eq. (vii) and introducing Eq. (iii) results in

$$\frac{P_e}{P_{co}} = \frac{P_{o}}{P_{co}} \left[ 1 + \mu \left[ 1 - \left( \frac{P_{o}}{P_{co}} \right)^{\frac{1}{\nu}} \right] \right]^{\nu} \quad (viii)$$

Finally

$$\frac{\alpha(\frac{P_{o}}{P_{co}})}{\alpha(\frac{t}{t_c})} = -\frac{2}{\nu+1} \left( \frac{P_{o}}{P_{co}} \right)^{\frac{1}{\nu+1}} \left[ 1 + \mu \left( 1 - \left( \frac{P_{o}}{P_{co}} \right)^{\frac{1}{\nu}} \right) \right] \left( \frac{P_{o}}{P_{co}} \right)^{\frac{\nu}{\nu+1}} \left[ \mu - \frac{1}{\nu} \right]^{\frac{\nu+1}{\nu+1}} \quad (ix)$$

is obtained. To facilitate integration the following substitution is made

$$\frac{P_{o}}{P_{co}} = (1 - \beta)$$

which is then expanded into a power series. It was found that accuracy was still satisfactory if all terms of orders higher than $\beta^2$ were omitted. By this way Eq. (ix) is reduced to

$$\frac{d\beta}{d(\frac{t}{t_c})} = -\frac{2}{\nu+1} \left( \frac{P_{o}}{P_{co}} \right)^{\frac{1}{\nu+1}} \sqrt{a - b \beta - c \beta^2} \quad (x)$$

with

$$a = 1 - \left( \frac{P_{o}}{P_{co}} \right)^{\frac{\nu+1}{\nu}}$$

$$b = (\nu+1) - 2 \left[ 1 - \frac{\nu+1}{\nu} \left( \frac{P_{o}}{P_{co}} \right)^{\frac{\nu+1}{\nu}} \right] \mu$$

$$c = -\frac{1}{2} (\nu+1) (\nu-2) + 2 (\nu-1) \mu - \frac{1}{2} (\nu+1) \left( \frac{P_{o}}{P_{co}} \right)^{\frac{\nu+1}{\nu}}$$

Integration of (x) yields

$$\frac{1}{t_c} \cdot \arcsin \left( \frac{1 + 2 \frac{\beta}{t_c}}{\sqrt{1 + 4 \frac{\beta^2}{t_c^2}}} \right) + C = \sqrt{\frac{1}{\nu+1}} \left( \frac{P_{o}}{P_{co}} \right)^{\frac{1}{\nu}} \times \frac{t}{t_c} \quad (xi)$$
C being the constant of integration, which is determined by setting

\[ \beta = 0 \quad \text{for} \quad t = 0 \]  

(xii)

The time \( t_E \) for decompression is found by setting \( \frac{\partial \beta}{\partial \left( \frac{t}{t_c} \right)} = 0 \) in Eq. (x).

If \( \frac{\partial \beta}{\partial \left( \frac{t}{t_c} \right)} = 0 \), then from (x)

\[ \sqrt{a - \alpha \beta - c \cdot \beta^2} = 0 \]  

(xiii)

Taking into account the conditions (xii) and (xiii) the time \( t_E \) of decompression finally is obtained

\[ t_E = t_c \cdot \sqrt{\frac{a}{2}} \cdot \frac{1}{\left( \frac{p_a}{p_c} \right)^{\frac{a-1}{a}}} \cdot \arctan \sqrt{\frac{a-c}{c}} \]  

(xiv)

This relation is valid for pressure ratios smaller than the critical. For greater ratios the speed in the orifice is independent of the back pressure \( p_a \) and is given by

\[ w = \sqrt{2 \cdot \frac{a-1}{a+1} \cdot \frac{p_c}{p_c}} \]  

(xv)

Introducing (xv) into Eq. (i) leads to

\[ \frac{d\left( \frac{p_c}{p_c} \right)}{d\left( \frac{t}{t_c} \right)} = -u \cdot \left( \frac{a}{a+1} \right)^{\frac{a}{a+1}} \cdot \left( \frac{p_c}{p_c} \right) \cdot \sqrt{\left( \frac{p_c}{p_c} \right)^{\frac{a-1}{a}}} \]  

(xvi)

which is easily integrated to be

\[ t = t_c \cdot \frac{2}{a-1} \cdot \left( \frac{a+1}{a} \right)^{\frac{a+1}{2(a-1)}} \left[ \frac{1}{\left( \frac{p_c}{p_c} \right)^{\frac{a-1}{a}}} - 1 \right] \]  

(xvii)

The supercritical flow exists until the pressure ratio \( \frac{p_c}{p_c} \) has reached the critical value \( p_{cr} \). During this supercritical phase the pressure \( p_a \) in \( V_a \) rises. When the critical ratio is approached the pressure \( p_c \) becomes
The time \( t_E \) required for evacuation in the supercritical phase is then

\[
\frac{\rho_c}{\rho_{c0}} = \rho_{cr} \cdot \frac{\rho_{ae}}{\rho_{c0}} \cdot \left[ \frac{1 + \mu}{1 + \mu \left( \frac{\rho_{ae}}{\rho_{c0}} \right)^{\eta/\alpha}} \right]^{\frac{\alpha-1}{\alpha}}
\]

The time \( t_E \) required for evacuation in the supercritical phase is then

\[
t_E = t_c \cdot \frac{d}{4 - 1} \cdot \left( \frac{\rho_{cr}}{\rho_{c0}} \right) \cdot \left[ \left( \frac{1}{\rho_{ae}} \right) \cdot \left[ \frac{1 + \mu \left( \frac{\rho_{ae}}{\rho_{c0}} \right) \eta/\alpha}{1 + \mu} \right]^{\frac{\alpha-1}{\alpha}} - 1 \right]
\]  

(xviii)

For the following subcritical phase Eq. (xiv) becomes applicable. However, there is one fact to be considered. During the decompression in the supercritical phase, the temperature in \( V_c \) has dropped. It is therefore necessary to account for this in the computation of the factors \( a, b, c, \) and \( \mu \). The modified factors will be called \( a^*, b^*, c^*, \) and \( \mu^* \) and are determined by

\[
a^* = \left( \frac{1}{\rho_{ae}} \right) \cdot \left[ 2 - (\alpha + 1) \cdot \left( \frac{1}{\rho_{ae}} \right)^{\eta/\alpha} \right] \cdot \mu^*
\]

\[
b^* = (\alpha - 1) - \left[ 2 - (\alpha + 1) \cdot \left( \frac{1}{\rho_{ae}} \right)^{\eta/\alpha} \right] \cdot \mu^*
\]

\[
c^* = -\frac{1}{2} (\alpha - 1)(\alpha - 2) + 2 (\alpha - 1) \cdot \mu^* - \left[ -\frac{1}{2} (\alpha - 1)(\alpha - 2) \cdot \left( \frac{1}{\rho_{ae}} \right)^{\eta/\alpha} \right] \cdot \mu^*
\]

\[
\mu^* = \mu \cdot \left[ \rho_{cr} \cdot \frac{\rho_{ae}}{\rho_{c0}} \right]^{\eta/\alpha}
\]

It is also necessary to modify the value of the speed of sound from \( c_o \) to \( c_o^* \) in order to make possible the use of the same time constant for both phases. It is

\[
c_o^* = c_o \cdot \left[ \rho_{cr} \cdot \frac{\rho_{ae}}{\rho_{c0}} \right]^{\frac{\eta}{2\alpha}} \cdot \left[ \frac{1 + \mu}{1 + \mu \cdot \left( \frac{\rho_{ae}}{\rho_{c0}} \right)^{\eta/\alpha}} \right]^{\frac{\alpha-1}{2\alpha}}
\]

The total time of decompression for an initial pressure ratio greater than the critical is then found to be

\[
t_E = t_c \cdot \frac{d}{4 - 1} \cdot \left( \frac{\rho_{cr}}{\rho_{c0}} \right) \cdot \left[ \left( \frac{c_a}{c_o^*} \right)^{\frac{\eta}{2\alpha}} \cdot \frac{1}{\left( \frac{\rho_{ae}}{\rho_{c0}} \right)^{\eta/\alpha}} \cdot \left[ \frac{1 + \mu}{1 + \mu \cdot \left( \frac{\rho_{ae}}{\rho_{c0}} \right)^{\eta/\alpha}} \right]^{\frac{\alpha-1}{2\alpha}} \right]^{\frac{\alpha}{\alpha - 1}}
\]  

(xix)
Eq. (xix) can be written
\[ t_2 = t_0 \cdot P_i \]

\( P_i \) being a function of \( p_{co}, p_{ao} \) and \( \mu \). For \( \mu = 0 \) i.e. decompression into open air \( P_i \) is given by
\[
P_i = \frac{1}{\mu-\epsilon} \left( R \right) \left[ \frac{2}{R \cdot \epsilon} \right]^{\frac{\mu}{\epsilon}} \left[ 1 + \left( \frac{2}{\epsilon} \right)^{\frac{\mu}{\epsilon}} \cdot \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{\epsilon}{\mu} - 1} \right] \]

Eq. (xx) is plotted in figure 1 against the pressure difference \( \frac{p_{co} - p_{ao}}{p_{ao}} \)

In many cases, it is important to know the initial rate of pressure change. This rate can be determined by setting \( \frac{P_{ci}}{P_{co}} \) in Eq. (ix) and Eq. (xvi). The initial rate of pressure change can then be expressed by
\[
\frac{dP_{ci}}{dt} = - \frac{P_{co}}{t_c} \cdot P_2
\]
The term \( P_2 \) is in the subcritical range
\[
P_2 = u \cdot \sqrt{\frac{2}{\mu-\epsilon}} \cdot \left( \frac{p_{ao}}{p_{co}} \right)^{\frac{\mu}{\epsilon}} \cdot \sqrt{1 - \left( \frac{p_{ao}}{p_{co}} \right)^{\frac{\mu}{\epsilon}}} \]
and in the supercritical range
\[
P_2 = u \cdot \left( \frac{1}{\mu+1} \right)^{\frac{1}{1+\mu}}
\]
Figure 4 shows \( P_2 \) as a function of \( \frac{p_{co} - p_{ao}}{p_{co}} \)

The initial rate of pressure change can be used to determine the constant rate time \( t_2 \) (see text). It is assumed that the initial rate is maintained throughout the entire decompression until the pressure \( p_0 \) is equal to the pressure \( p_a \). See figure 6. The initial rate time is then determined by
that is

\[ t_R = - \frac{P_c - P_2}{\frac{dP_c}{dt} P_c} = t_c \cdot \frac{P_c - P_2}{P_c} \cdot \frac{1}{P_3} = t_c \cdot P_3 \]

with

\[ P_3 = \frac{P_c - P_2}{P_c} \cdot \frac{1}{P_2} \]

\( P_3 \) is shown in figure 7.
MEMORANDUM FOR DTIC-OCQ
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2513 Kennedy Circle
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SUBJECT: Changing the Distribution Statement on a Technical Report

This letter documents the requirement for DTIC to change the distribution statement from “C” to “A” (Approved for public release; distribution is unlimited.) on the following technical report:
AD Number AD0020374, XC-SAM, PHYSICS AND ENGINEERING OF RAPID DECOMPRESSION: A GENERAL THEORY OF RAPID DECOMPRESSION.

If additional information or a corrected cover page and SF Form 298 are required please let me know. You can reach me at DSN 240-6019 or my e-mail address is sherry.mathews@brooks.af.mil.

Thank you for your assistance in making this change.

SHERRY Y. MATHEWS
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Previously AFIOH STINFO Officer