THE MEASUREMENT OF APERTURE TRANSMISSION COEFFICIENTS

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June 10, 1953

Technical Report No. 165

Cruft Laboratory
Harvard University
Cambridge, Massachusetts
The research reported in this document was made possible through support extended Cruft Laboratory, Harvard University, jointly by the Navy Department (Office of Naval Research), the Signal Corps, of the U. S. Army, and the U. S. Air Force under ONR Contract N5ori-76, T. O. 1.
The Measurement of Aperture Transmission Coefficients

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Abstract

The transmission coefficients of circular, elliptical, and square apertures in a plane conducting screen are determined from measurements of the far-zone scattered field in the direction of incidence. The measurements were carried out at K-band frequencies using an image-plane technique. Experimental values of the transmission coefficient are compared with the results of a number of theoretical formulations.

1. Introduction.

The analytical difficulties encountered in problems of diffraction by apertures and disks make it easier in many cases to obtain the quantities of physical interest by direct experimental measurement. For example, Andrews and Silver and Ehrlich have made many detailed measurements in the near zone of diffracting apertures and disks. Their results are in agreement with many theoretical predictions such as the behavior of the tangential component of the magnetic field in the aperture and the axial distribution of the fields. In the far zone, Aden and Sevick have measured the back scattering from spheres and coupled antennas by methods which can be extended easily to disks.

The transmission coefficients of apertures, however, have never before been measured. This report describes the results of such measurements, which were carried out at K-band frequencies. The equipment was originally built and used by Kodis for determining the near-zone diffraction patterns of cylinders and has been adapted to the measurement of far-zone scattered amplitudes. After calibration with circular apertures the apparatus is used to check the range of validity of approximate theories.
II.

Description of the Apparatus

A schematic diagram of the experimental arrangement used to measure transmission coefficients is shown in Fig. 1. In its original form the apparatus was designed and built for the investigation of diffraction by cylinders under conditions that approached the idealizations of theory as closely as possible. A detailed discussion of the means by which these conditions are realized can be found in Technical Report No. 105. The general features of the present problem are summarized briefly below.

Theoretical analyses generally make use of three assumptions;

1. The perforated screen is isolated in space.
2. The incident wave is plane.
3. The perforated screen has zero thickness.

In this experiment the isolation of the screen is approximated by mounting it vertically over a conducting image plane. The image plane, which is 96 wavelengths wide by 144 wavelengths long at \( \lambda = 1.25 \) cm, is large enough so that reflections from its edges are negligible. To make the incident wave nearly plane, at least over the region of the aperture, a remote point source is required. In this apparatus such a source is approximated adequately with a horn radiator (Fig. 2). The horn is driven by a 2K33 klystron through a length of K-band waveguide and provides the power gain required for measuring transmission coefficients. The thin screen is shown in Figs. 2 and 3. It is made of a chrome-plated steel sheet about 0.02 wavelength thick. The lower edge is fastened securely to the image plane; the upper edge is attached to a support, shown in Fig. 4, by means of which tension can be applied uniformly along the edge to keep the thin screen vertical and plane. The center section of the screen contains the half-aperture and is removable so that apertures of different sizes and shapes can be put in place with a minimum of effort.

With less than 20 milliwatts available from a 2K33 K-band oscillator
FIG. 1  MICROWAVE COMPONENTS FOR MEASUREMENT OF TRANSMISSION COEFFICIENT THROUGH APERTURES
FIG. 3  IMAGE PLANE, TRANSMITTING HORN, AND A DIFFRACTING SCREEN WITH A CIRCULAR APERTURE
FIG. 4 SUPPORT FOR THE DIFFRACTING SCREEN
very little energy is transmitted by a small aperture located far from the transmitting horn. Consequently, the measurement of its transmission coefficient, which is related to the far-zone field on the shadow side of the screen, requires a very sensitive detector and corresponding care in preventing leakage to the detector by paths other than through the aperture. A simple criterion is that with the aperture closed the leakage signal must be less than noise; the requirement is met as follows:

1. The screen containing the aperture is made large enough so that no detectable radiation is diffracted around any of its three free edges.
2. The fourth (bottom) edge of the screen is pinched tightly in a narrow slit in the image plane so that the joint between the two planes is not leaky.
3. The aperture insert is put into intimate contact with the rest of the diffracting screen by covering the overlapping joint with strips of aluminum foil two wavelengths wide and 0.0012 wavelength thick. These strips are sealed with conducting silver paint (cf. Fig. 5 through 8).

Under these conditions no leakage signal can be detected by the receiving system, which consists of an antenna, a calibrated attenuator, and a spectrum-analyzer. The receiving antenna is a horn located 30 wavelengths from the diffracting aperture in the direction of incidence. It is designed for optimum directivity and flares 19 degrees in the E-plane and 22.5 degrees in the H-plane from standard K-band waveguide to a rectangular aperture 11 wavelengths wide and three wavelengths high (Fig. 2). From the horn the signal is transmitted by waveguide to a K-band spectrum-analyzer (TSK 2SE) through a precision variable attenuator. The combination of attenuator and spectrum-analyzer is used as a sensitive receiver.

III

Definition of the Transmission Coefficient

The analytic definition of the transmission coefficient of an aperture is
\[
\text{Re} \left[ \frac{\int_S \hat{\mathbf{r}} \cdot \mathbf{E}_n'(\mathbf{r}') \times \mathbf{H}^e_n(\mathbf{r}') \, dS'}{S |\mathbf{E}^\text{inc}(\mathbf{r}) \times \mathbf{H}^\text{inc}(\mathbf{r})|} \right],
\]

where \( S \) is the area of the aperture and the subscript \( n \) denotes the direction of propagation of the incident plane wave.

\[
\mathbf{E}^\text{inc}(\mathbf{r}) = \mathbf{E}_0 e^{i k \hat{n} \cdot \mathbf{r}}, \quad \mathbf{H}^\text{inc}(\mathbf{r}) = \hat{n} \mathbf{H}_0 e^{i k \hat{n} \cdot \mathbf{r}}
\]

Since the aperture field distribution cannot be measured easily, this definition is not very good in the operational sense. A better one is made available by the analysis given in Technical Report No. 164, which makes it possible to put equation (1), into a form involving only a far-zone electric field. First, however, by changing the order of the vector double product in the integrand of (1), it is possible to make use of the fact that in the aperture, \( \mathbf{H}^\text{inc}(\mathbf{r}) = \mathbf{H}^\text{inc}(\mathbf{r}) \).

The formula then simplifies to

\[
\text{Re} \left[ \frac{\int_S \hat{\mathbf{r}} \cdot \mathbf{E}_n'(\mathbf{r}') \cdot \hat{n} e^{-i k \hat{n} \cdot \mathbf{r}} \, dS'}{S \mathbf{E}_0} \right]
\]

The integral in (2) can be interpreted readily in terms of the far-zone electric field on the shadow side of the screen. The asymptotic form of this field is

\[
\mathbf{E}_n(\mathbf{r}) = \mathbf{f} \times A(\hat{\mathbf{r}}, \hat{n}) \frac{e^{i k r}}{r}, \quad r \to \infty
\]

where

\[
A(\hat{\mathbf{r}}, \hat{n}) = \frac{i k}{2 \pi} \int_S \hat{\mathbf{r}} \times \mathbf{E}_n'(\mathbf{r}') \exp(-i k \hat{\mathbf{r}} \cdot \mathbf{r}') \, dS'.
\]

Comparison of this expression with (2) shows that in terms of the scattered amplitude in the direction of incidence the transmission coefficient takes the simple form,

\[
t = \frac{2 \pi}{k \mathbf{E}_0} \text{Im} \left[ \hat{n} \cdot A(\hat{n}, \hat{n}) \right]
\]

With normal incidence \( \hat{n} = \hat{x}, \hat{n} = \hat{y} \), and equation (4) becomes

\[
t = \frac{2 \pi}{k \mathbf{E}_0} \text{Im} \left[ \hat{y} \cdot A(\hat{x}, \hat{y}) \right]
\]
Equation (5) can be written in terms of the scattered electric field by setting $\hat{r} = \hat{\boldsymbol{r}}\hat{\boldsymbol{r}}$ in (3) and forming the scalar product with the unit vector $\hat{\boldsymbol{r}}$. The result is

$$ \gamma \cdot A(\hat{\boldsymbol{z}}, \hat{\boldsymbol{r}}) = \frac{r\hat{\boldsymbol{r}} \cdot E(\hat{\boldsymbol{r}})}{e^{ikr}}. $$

Accordingly,

$$ t = \frac{2\pi r}{kE_0} |E_x(\hat{\boldsymbol{r}})| \sin(\vartheta_E - kr), $$

where $\vartheta_E$ is the phase angle of the complex quantity $E_x(\hat{\boldsymbol{r}})$.

This term of the transmission coefficient makes it evident that it can be determined experimentally from a measurement of the amplitude and phase of $E_x$ in the far zone of the aperture. Unfortunately, with the small amount of power available the phase measurement is not feasible, but reasonable values of $t$ can still be obtained by measuring $|E_x|$ and estimating $\vartheta_E$ from the theoretical results of Technical Report No. 164. It is found that over most of the frequency range of interest $\sin(\vartheta_E - kr)$ is nearly unity as shown in Fig. 9. This value checks with the optical limit of the exact theory.

The Measuring Procedure

With the definitions just given, the measurement of transmission coefficients for normal incidence is reduced to the problem of measuring the far-zone electric field directly behind the aperture. Since this field is small and frequency sensitive, certain precautions are necessary.

In the first place, the radiation frequency must be known and held constant throughout a series of measurements. This can be accomplished with sufficient accuracy by measuring the guide wavelength on a slotted line and monitoring the frequency continuously with the reference pip of the spectrum-analyser. In addition, the small signal strength makes it necessary to tune all transmitting and receiving components as carefully as possible.

The actual measurement of the amplitude of the far-zone electric field for each aperture is a relatively simple matter. The diffracted wave that is incident upon the receiving horn is proportional to $|E_x|$ at a point directly behind the aperture. The resulting signal is transmitted through a section
of waveguide containing a precision attenuator to a spectrum-analyzer where it is displayed on the oscilloscope (Fig. 10). By adjusting the attenuator so that with each aperture, the signal is constant in amplitude, the relative magnitude of $E_x$ can be determined from the attenuator settings at each value of $ka$. The relative transmission coefficients of a group of similar apertures are then readily calculated from equation (7), and they can be compared with theoretical results after being normalized to the limiting value of unity for large values of $ka$.

The apertures for which this measuring procedure was carried out are shown in Figs. 11 through 14.

V.

Comparison of Results

The experimental values of the transmission coefficient for circular apertures are compared in Fig. 15 to the results of the exact theory. Good agreement is obtained over almost the entire frequency range considered. The largest error occurs at $ka = 9$, and is probably due to the fact that over a large aperture the incident wave from a point source is no longer plane to the required degree of approximation.

Since the exact theory has not yet been worked out for elliptical apertures, the experimental results are compared to those obtained by the variational and Kirchhoff approximations.\textsuperscript{1, 2} The curves are shown in Figs. 16 and 17. The remarkable agreement between the experimental result and the variational approximation probably indicates that the single-component trial field resembles the true field in the aperture quite closely when the incident electric field is polarized along the major axis of the ellipse. The contrast with the Kirchhoff approximation hardly needs to be emphasized; this approximation tends toward the correct result only for large values of $ka$.

Because of analytical difficulties, the only available theoretical result for the transmission coefficient of a square aperture is the Kirchhoff approximation. Unfortunately, the experimental result is also inaccurate because the factor $\sin(\theta_E - kr)$ is unknown. The theoretical curve together with the experimentally determined values, is shown in Fig. 18. No valid comparison can be
\[ \theta_A = \theta_e - k\tau \]

**Fig. 9** The phase angle of \( A(\perp, \perp) \), the far zone scattered amplitude computed by variational methods.
FIG. II  CIRCULAR APERTURE
FIG. 12  ELLIPTICAL APERTURES OF ECCENTRICITY $\sqrt{3}/2$
FIG. 13 ELLIPTICAL APERTURES OF ECCENTRICITY $\frac{8}{3}$
FIG. 14  SQUARE APERTURES
FIG. 15 TRANSMISSION COEFFICIENT OF CIRCULAR APERTURES FOR NORMAL INCIDENCE OF PLANE ELECTROMAGNETIC WAVES
FIG. 16 TRANSMISSION COEFFICIENT OF ELLIPTICAL APERTURES FOR NORMAL INCIDENCE OF ELECTROMAGNETIC WAVES
FIG. 17 TRANSMISSION COEFFICIENT OF ELLIPTICAL APERTURE FOR NORMAL INCIDENCE OF PLANE ELECTROMAGNETIC WAVES
made except at large values of \(ka\) where the phase factor is expected to be unity. From the results for circular and elliptical apertures, however, it can be inferred that for small values of \(ka\), the measured coefficients should be smaller than those shown.

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