Technical Report No. 97
PLASTIC STRESS-STRAIN RELATIONS BASED ON LINEAR LOADING FUNCTIONS
by
J. L. Sanders, Jr.

GRADUATE DIVISION OF APPLIED MATHEMATICS
BROWN UNIVERSITY
PROVIDENCE, R. I.
September, 1953

A11-97/20
ABSTRACT

The stress-strain relations of incremental theories of plasticity have recently been generalized by Koiter who introduced the use of any number of loading functions instead of only one. The present paper is concerned with incremental theory based on any number of linear loading functions. In this case the resulting plastic strain increments are integrable in a restricted sense and the stress-strain relations can be given in a form partially resembling deformation theories. The slip theory of Batdorf and Budiansky falls within the class of theories which are discussed in this paper.

1The results presented in this paper were obtained in the course of research sponsored by the Office of Naval Research under Contract N7onr-35801.

2Research Associate of Applied Mathematics, Brown University.
Introduction. Within the last few years the development of plastic stress-strain relations for strain hardening materials has received increasing attention from workers in the field of applied mechanics [1],[2],[3],[4].¹ A knowledge of the stress-strain curve for a material in simple tension or compression is not in itself sufficient for predicting the behavior of a material under more complex types of loading. At least two problems immediately present themselves when one considers loading which may involve all six components of stress. These are the generalization of the idea of a yield point to the idea of a yield surface in a stress space of six dimensions and the formulation of a plastic stress-strain relation for polyaxial loading.

During the history of the subject these problems have been attacked in two more or less distinct ways. One method of approach is to try to deduce the behavior of a bulk piece of metal from the behavior of the individual crystals making up the aggregate. A recent example of results obtained by pursuing this method is the slip theory of plasticity[2]. The other method of approach is to try to deduce from a few reasonable assumptions what the general form of the yield surface and the stress-strain relation must be without inquiring very deeply into the structure of metals but rather proceeding in a phenomenological manner[3].

Results obtained by one method often suggest research to be done by the other method which tends to unify the whole subject. The earliest known yield condition, the Tresca maximum shear

¹Numbers in brackets refer to the bibliography at the end of the paper.
condition \cite{5}, when expressed in geometrical form as a yield surface in stress space possesses "corners". The initial yield surface according to slip theory is the Tresca yield surface which arises as an envelope of planes in stress space. The yield surface after work hardening according to slip theory is also an envelope of planes and naturally possesses corners. Drucker \cite{3} allows for corners in his general discussion of yield surfaces from a phenomenological point of view. The relation between corners in the yield surface and linearity of the stress-strain relation was investigated experimentally by Stockton \cite{6}. The yield surface is closely connected with a function called the loading function which appears in the stress-strain relation. The geometrical expression of the loading function, the loading surface, is identical to the yield surface if there is only one loading function in the stress-strain relation; if the stress-strain relation is based on many loading functions the yield surface is the boundary of those points in stress space which represent elastic behavior with respect to all the loading surfaces. Obviously corners in the yield surface may arise even though all the loading surfaces are smooth. Stress-strain relations based on many loading functions were suggested by Koiter who also investigated some of their properties \cite{4}. Although slip theory was derived through physical reasoning Koiter states that it may be regarded as a theory of plasticity based on an infinite number of plane loading surfaces.

An interesting feature of slip theory is that the incremental form of its stress-strain relation is integrable in a
restricted sense which resembles deformation theories in this respect and is unlike most incremental theories. The present paper is concerned with the use of many linear loading functions in constructing stress-strain relations. It is shown that in some cases the resulting incremental stress-strain relation is integrable in a restricted sense. Means, suggested by the behavior of a single crystal, of including cross effect and Bauschinger effect in the stress-strain relation are also discussed.

A Single Plane Loading Surface. In the formulation of a plastic stress-strain relation it is commonly assumed that the strain may be separated into an elastic and a plastic part which are additive. This assumption is made in the following treatment and the plastic strain only is the subject of discussion. It is further assumed in the following that the plastic part of the strain may be separated into many parts each associated with a loading function.

It is convenient to phrase the discussion in geometrical terms and speak of loading surfaces, loading paths, strain increment vectors and so forth. Imagine a euclidean space with rectangular coordinates, one coordinate for each component of stress. Each stress state is represented by a point in this space. For convenience we also represent strain in the same space with each component of the strain acting as coordinate on the same axis as the corresponding component of stress. The loading history of a piece of strain-hardening material which has been subjected to a varying homogeneous state of stress is representable in this space
as a loading path. We now want to consider the contribution of a single plane loading surface to the plastic strain.

The stress-strain relation to be described in the following is consistent in that it satisfies all the mathematical conditions which have been discovered by phenomenological studies to apply to incremental stress theories of plasticity. The proof that in the particular case of a plane loading surface the incremental stress-strain relation is integrable is given in the appendix. The stress-strain relation in its integrated form is presented here.

Fig. 1 shows a symbolic representation of the loading plane and the loading paths of two hypothetical experiments. As loading proceeds from 0 to P the material behaves elastically. At P plastic straining begins and continues as long as the loading path is directed outward from the loading plane. Thus during the part Q'R of the loading path OPQ'R no plastic straining occurs. One may think of the loading plane being pushed out parallel to itself as loading proceeds. The total plastic strain suffered by this hypothetical material depends only on the distance the loading plane has been pushed out. Thus the total plastic strain is the same for either loading path OPQ or OPQ'R and is a function of d. The direction of the plastic strain vector is perpendicular to the loading plane.

Two Plane Loading Surfaces.

Several novel features of the use of many plane loading surfaces can be illustrated most simply by examining the case in
which there are only two plane loading surfaces \( f_1 \) and \( f_2 \). Fig. 2 is a schematic drawing which shows how stress space is divided into four regions by two plane loading surfaces. If the loading path remains in region I no plastic straining occurs. If the loading path only penetrates region II (or IV) then only one of the two planes is affected and we have the case treated in the last section. If however the stress path enters region III then both loading planes are affected as shown in Fig. 3.

Suppose the loading has reached a point such as \( P \) in Fig. 3 and now a small increment of stress \( d\sigma_{ij} \) is added to the load. There are several cases to consider depending on the direction of the stress increment vector. Fig. 4 shows the region of stress space in the neighborhood of \( P \). There are four regions around \( P \) into which the stress increment vector may extend. If \( d\sigma_{ij} \) extends into region 1 there is no additional plastic strain, consequently region 1 may be called the elastic region. If \( d\sigma_{ij} \) extends into either region 2 or 4 one of the two planes is affected and the other is not. If \( d\sigma_{ij} \) extends into region 3 both loading planes are affected so region 3 may be called the region of total loading. The strain increment is taken to be the sum of the contribution from \( f_1 \) and \( f_2 \) as though each acts independently.

The boundary of the elastic region is the yield surface which is shown in heavy lines in Fig. 4. In the case under consideration a "corner" has developed in the yield surface at the load point \( P \). If the load point were on any straight portion of the yield surface the strain increment vector would of course be
normal to the yield surface. At a corner such as in Fig. 4 there is no normal to the yield surface. However, the strain increment vector $\mathbf{d}\varepsilon_{ij}^p$ is the vector sum of the contribution from both loading surfaces. Therefore at a corner the strain increment vector has some determinate direction between the normal to $f_1$ and the normal to $f_2$ (see Fig. 5).

The mathematical expression of the stress-strain relation which we are considering contains the differentials of stress $d\sigma_{ij}$ to the first power only and thus is linear in some respects. If the stress increment vector $d\sigma_{ij}$ causes a plastic strain $d\varepsilon_{ij}^p$ then $2d\sigma_{ij}$ causes twice as much plastic strain, $2d\varepsilon_{ij}^p$. However $-d\sigma_{ij}$ might not cause any plastic strain at all because if the vector $d\sigma_{ij}$ falls in the region of total loading then $-d\sigma_{ij}$ falls in the elastic region. Hook's law for the elastic part of the strain is truly linear and does not possess this peculiarity. The plastic stress-strain relation is non-linear because the principle of superposition does not apply. A plastic stress-strain relation which involves even only one loading function is non-linear in this respect.

The non-linearity for multiple loading surfaces is of a less trivial sort. Suppose for instance that $d\sigma_{ij}(C) = d\sigma_{ij}(A) + d\sigma_{ij}(B)$ and that $d\sigma_{ij}(A)$, $d\sigma_{ij}(B)$ and $d\sigma_{ij}(C)$ each acting by itself causes non-zero plastic strain increments $d\varepsilon_{ij}^p(A)$, $d\varepsilon_{ij}^p(B)$ and $d\varepsilon_{ij}^p(C)$ respectively. In general it does not follow that $d\varepsilon_{ij}^p(C) = d\varepsilon_{ij}^p(A) + d\varepsilon_{ij}^p(B)$. Fig. 6 illustrates a case in which
superposition does not hold. The stress increment $d\sigma_{ij}(A)$ is applied followed by $d\sigma_{ij}(B)$. Clearly the resulting plastic strain is not the same as that produced by $d\sigma_{ij}(C)$ acting alone because the loading plane $f_1$ is moved out more in the first instance than in the second. Even the order in which $d\sigma_{ij}(A)$ and $d\sigma_{ij}(B)$ are applied makes a difference. Experimental evidence for the existence of corners in the yield surface has recently been found[6].

Many Plane Loading Surfaces. There is no difficulty in generalizing the previous results by introducing any finite or infinite number of plane loading surfaces. If there are an infinite number of loading surfaces the stress-strain relation may involve an integration. The set of all those points in stress space which are elastic with respect to all the loading planes constitutes the elastic region. The boundary of the elastic region is the yield surface. Figs. 7 and 8 show symbolically the elastic region and yield surface when the number of plane loading surfaces is finite or infinite respectively.

As loading proceeds the loading planes get pushed out more or less depending on the loading path, but at any instant of loading there is a well defined yield surface. The property of integrability for a single plane loading surface leads to the following simple rule in the case of many plane loading surfaces:

The total plastic strain is the same for all loading paths which result in the same yield surface.
Fig. 9 illustrates a case in which two different loading paths A and B produce the same total plastic strain. If the loading had proceeded only as far as P the total strain for paths A and B would be different but continuing the loading to Q wipes out this difference.

The generalization of the notion "region of total loading" to the case of many plane loading surfaces can be done in the following way. Suppose loading has proceeded along a straight line loading path into the plastic range and reached a point P (Fig. 10). The yield surface is made up of part of the initial yield surface and of part of the envelope of the loading planes which pass through P and are tangent to the initial yield surface. The part of this conical envelope which extends outward from P bounds the region of total loading. All loading planes previously pushed out in reaching P are further pushed out by a stress increment \( \dot{\sigma}_{ij} \) whose vector drawn from P falls in the region of total loading.

The straight line loading path (radial loading) is a special case of a more general loading path traced out by a load point P which always proceeds into the region of total loading. For such loading paths (and only for such loading paths) the interesting result has been obtained that \( J_2 \) deformation theory coincides with a particular incremental theory based on an infinite number of plane loading surfaces. The details are left to a later paper.

Further Remarks on Plane Loading Surfaces. In all the foregoing discussion it has been assumed that the loading planes act entirely
independently. Fig. 11 illustrates a case in which the loading planes act interdependently. The loading plane $f_1$ has been pushed out to the position indicated by the dotted line. The other planes also move according to some law as a consequence of the motion of $f_1$. This type of behavior is certainly more complicated than independent action but the phenomenon of latent strain hardening of a single crystal suggests such behavior. In a metal crystal there is a certain preferred set of planes called slip planes so oriented with respect to the crystal lattice that slip is most likely to occur in them rather than in any others. Within these planes there are several preferred directions of slip. Thus there is some definite number of slip-plane slip-direction combinations or slip systems within a given crystal. When a given slip system has been activated it becomes more resistant to further slipping, that is it becomes strain hardened. However the slip systems do not strain harden independently. When any one slip system is strain hardened so are the others. The condition that the shear stress on a given slip system remain constant is that a certain linear combination of the stress components remain constant. The locus of points in stress space satisfying this condition is of course a plane. Perhaps a theory describing the macroscopic plastic strains of a single crystal could be formulated based on a finite number of plane loading surfaces acting interdependently. Even though it is not at all obvious how to take the step from a single crystal to a crystal aggregate the behavior of a single crystal is very suggestive.

Fig. 12a shows a possible yield surface after an experiment performed on a material whose initial yield surface is
\[ J_2 = k^2. \] The material has been stressed in simple tension in the 
x direction beyond the elastic limit. The yield stresses for 
tension and compression in the \( y \) direction have been altered 
(cross effect) and the yield stress for compression in the \( x \) direc-
tion has also been altered. (Bauschinger effect)[7]. Fig. 12b 
shows the results of the same experiment performed on a hypothetical 
material which obeys a stress-strain relation based on an infinite 
number of plane loading surfaces which act independently. The 
initial yield surface for this material is also \( J_2 = k^2 \). No cross 
effect or Bauschinger effect in the sense of a lowering of the 
yield point in compression following loading in tension is exhibited 
by such a material. Fig. 12c shows the result of the same experi-
ment performed on another hypothetical material obeying the simple 
\( J_2 \) rule. The material of 12c could also be regarded as obeying a 
stress-strain relation based on an infinite number of plane loading 
surfaces in which the planes always move in such a way as to envel-
op a surface \( J_2 = \) constant. The property of integrability does not 
necessarily hold for stress-strain relations based on many plane 
loading surfaces which act interdependently. The properties of 
such stress-strain relations have only begun to be investigated, 
but the phenomenon of latent strain hardening exhibited by individ-
ual metal crystals as well as the evidence from experiments on 
polycrystalline materials suggests further investigations.

Many plate buckling experiments agree well with a 
theory in which the hypothetical material of Fig. 12b is taken as 
the mathematical abstraction of the real material. Unfortunately
other kinds of experiments show that this material is not acceptable as a good approximation to a real material. Recent investigations[8] on the stability of non-linear systems indicate that the classical buckling criterion is not applicable in the plate buckling problem. These investigations show that small and unavoidable initial imperfections can have a pronounced effect on the load carrying capacity of a plate and yet not cause much scatter in the test data. If these conclusions are correct plate buckling experiments are not decisive in the choice between various possible stress-strain relations.
APPENDIX

Consider the following stress-strain relation:

\[ d\epsilon^p_{ij} = G(f) \frac{\partial f}{\partial \sigma_{ij}} df \quad df > 0 \]
\[ = 0 \quad df \leq 0 \]  \hspace{1cm} (1)

in which \( f \), the loading function, is a function of stresses alone. Provided the surface \( f = \text{constant} \) be sufficiently smooth and non-convex toward the origin in stress space, this is the most general stress-strain relation involving only one loading function which belongs to the class of stress theories of plasticity. If \( f \) is a linear combination of the stresses then (1) may be integrated to give the components of total strain in terms of the stresses. The resulting relation is of the form:

\[ \epsilon^p_{ij} = H(f) \sigma_{ij} \]  \hspace{1cm} (2)

where \( dH = Gdf \) and \( \sigma_{ij} \) are direction numbers of the normal to the loading plane. This may be verified by taking the differential of (2) to obtain (1). Conversely the only integrable stress-strain relations of the form (1) are those for which \( f = \text{constant} \) is a plane surface in stress space. This may be proved by the following considerations. If \( d\epsilon^p_{ij} \) is integrable then there exists a family of surfaces in stress space such that \( \epsilon^p_{ij} \) is constant on each surface of the family. Suppose that a point moves on a surface \( f = \text{constant} \); then according to (1) \( df = 0 \) and \( d\epsilon^p_{ij} = 0 \) so the surface \( f = \text{constant} \) must also be a surface \( \epsilon^p_{ij} = \text{constant} \), hence
\( \varepsilon_{ij} \) is a function of \( f \) alone. This implies that \( \frac{\partial f}{\partial \varepsilon_{ij}} \) is a function of \( f \) alone, say:

\[
\frac{\partial f}{\partial \varepsilon_{ij}} = F_{ij}(f). \tag{3}
\]

Now \( \frac{\partial f}{\partial \varepsilon_{ij}} \) are the direction numbers of the normal to the surface \( f = \text{constant} \) and (3) implies that the normal has a constant direction; hence the surface \( f = \text{constant} \) is a plane. All that has been proved is that \( f \) must be a function of a linear combination of the stresses, however there is no loss in generality in taking \( f \) itself to be linear in the stresses since any additional arbitrariness may be absorbed into the factor \( G \) which occurs in (1).
Bibliography


FIG. 1

Single plane loading surface

direction of plastic strain vector

FIG. 2

Two plane loading surfaces (one active)

FIG. 3

Two plane loading surfaces (both active)
FIG. 4
Loading from a corner

FIG. 5
Strain increment vector at a corner

FIG. 6
Non-linear effects at a corner

fan of directions in which the strain increment vector must lie
FIG. 7
Finite number of plane loading surfaces

FIG. 8
Infinite number of plane loading surfaces

FIG. 9
Illustration of restricted path independence
FIG. 10

FIG. 11

Interdependent plane loading surfaces
Yield surfaces as envelopes of infinitely many plane loading surfaces.

a) cross effect and Bauchinger effect
b) no cross effect or Bauchinger effect
c) Isotropic work hardening
## APPROVED DISTRIBUTION LIST FOR UNCLASSIFIED TECHNICAL REPORTS

**Issued by**

BROWN UNIVERSITY  
Contract N7onr-358, T.O. 1  
NR O41 032

Office of Naval Research  
Washington 25, D. C.

<table>
<thead>
<tr>
<th>Code</th>
<th>Branch (Code)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-2</td>
<td>Mathematics Branch (Code 432)</td>
</tr>
<tr>
<td>M-1</td>
<td>Mechanics Branch (Code 438)</td>
</tr>
<tr>
<td>M-1</td>
<td>Physics Branch (Code 421)</td>
</tr>
<tr>
<td>M-1</td>
<td>Metallurgy Branch (Code 423)</td>
</tr>
</tbody>
</table>

M-2  
Commanding Officer  
Office of Naval Research Branch Office  
150 Causeway Street  
Boston, Massachusetts

M-1  
Commanding Officer  
Office of Naval Research Branch Office  
346 Broadway  
New York, New York

M-1  
Commanding Officer  
Office of Naval Research Branch Office  
844 North Rush Street  
Chicago 11, Illinois

M-1  
Commanding Officer  
Office of Naval Research Branch Office  
1000 Geary Street  
San Francisco 9, California

M-1  
Commanding Officer  
Office of Naval Research Branch Office  
1030 East Green Street  
Pasadena 1, California

M-18  
Officer-in-Charge  
Office of Naval Research  
Navy #100  
Fleet Post Office  
New York, New York

M-9  
Director  
Naval Research Laboratory  
Washington 20, D. C.  
Attn: Scientific Information Division

<table>
<thead>
<tr>
<th>Code</th>
<th>Branch (Code)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-2</td>
<td>Library (Code 2021)</td>
</tr>
<tr>
<td>M-1</td>
<td>Applied Mathematics Branch (Code 3830)</td>
</tr>
<tr>
<td>M-1</td>
<td>Shock and Vibrations Section (Code 3850)</td>
</tr>
<tr>
<td>M-1</td>
<td>Structures Branch (Code 3860)</td>
</tr>
</tbody>
</table>
Bureau of Ships
Department of the Navy
Washington 25, D. C.

M-2 Attn: Code 364 (Technical Library)
R-1 Code 423 (Underwater Explosion Research)
M-1 Code 442 (Scientific Section, Design)

David Taylor Model Basin
Carderock, Maryland
M-2 Attn: Library
M-1 Structural Mechanics Division

Naval Ordnance Laboratory
White Oak, Silver Spring 19, Maryland
M-2 Attn: Library

Bureau of Aeronautics
Department of the Navy
Washington 25, D. C.

M-1 Attn: AER-TD-414
R-1 Materials Branch
R-1 Design Elements Division

Bureau of Yards and Docks
Department of the Navy
Washington 25, D. C.

R-2 Attn: Director, Research Division

Commander
Norfolk Naval Shipyard
Norfolk, Virginia
M-1 Attn: Technical Library (Code 243A)
M-1 UERD (Code 290)

Superintendent
Aeronautical Structures Laboratory
Building 600, Naval Air Experimental Station
Philadelphia 12, Pennsylvania
R-1 Attn: Experimental Structures Section

Office, Assistant Chief of Staff, G4
The Pentagon
Washington, D. C.
M-1 Attn: Research and Development Division

M-1 The Chief, Armed Forces Special Weapons Project
Department of Defense
P. O. Box 2610
Washington, D. C.

U. S. Army Arsenal
Watertown 72, Massachusetts
M-1 Attn: Dr. R. Beewukses
M-1 Mr. J. Bluhm
Frankford Arsenal
Pitman-Dunn Laboratory
Philadelphia 37, Pennsylvania
\[M-1\] Attn: Dr. Herbert I. Fusfeld

Picatinny Arsenal
Dover, New Jersey
\[M-1\] Attn: Dr. L. Gilman

Commanding General
Wright Air Development Center
Wright-Patterson Air Force Base
Dayton, Ohio
\[M-1\] Attn: WCACD

Department of Commerce
Office of Technical Service
Washington 25, D. C.
\[M-1\] Attn: Library Section

National Advisory Committee for Aeronautics
1724 F. Street NW
Washington 25, D. C.
\[M-1\] Attn: Chief of Aeronautical Intelligence

National Advisory Committee for Aeronautics
Langley Aeronautical Laboratory
Langley Field, Virginia
\[M-1\] Attn: Library

National Advisory Committee for Aeronautics
Lewis Flight Propulsion Laboratory
Cleveland Airport
Cleveland 11, Ohio
\[M-1\] Attn: Library

National Bureau of Standards
Washington, D. C.
\[M-1\] Attn: Dr. W. R. Ramberg

Director of Research
Sandia Corporation
Albuquerque, New Mexico
\[M-1\] Attn: Dr. S. C. Hight

Brooklyn Polytechnic Institute
85 Livingston Street
Brooklyn, New York
\[R-1\] Attn: Dr. N. J. Hoff
\[R-1\] Dr. H. Reissner
\[M-1\] Dr. P. G. Hodge, Jr.
\[M-1\] Dr. F. S. Shaw (Dept. Aero. Engr. & Appl. Mech.)

Brown University
Providence 12, Rhode Island
\[M-1\] Attn: Chairman, Graduate Division of Applied Mathematics
Stanford University
Stanford, California
R-1 Attn: Dr. L. Jacobsen
M-1 Dr. A. Phillips, Dept. of Mechanical Engineering
R-1 Dr. J. N. Goodier

Stevens Institute of Technology
Hoboken, New Jersey
R-1 Attn: Dr. E. G. Schneider

Swarthmore College
Swarthmore, Pennsylvania
M-1 Attn: Capt. W. P. Roop
R-1 Dr. S. T. Carpenter

University of Texas
Austin 12, Texas
R-1 Attn: Dr. A. A. Topractosoglou

University of Utah
Salt Lake City, Utah
M-1 Attn: Dr. H. Eyring

Washington State College
Pullman, Washington
R-1 Attn: Dr. B. Fried

Wheaton College
Norton, Massachusetts
R-1 Attn: Dr. H. Geiringer

Aerojet, Inc.
Azusa, California
R-1 Attn: F. Zwicky

Aluminum Company of America
New Kensington, Pennsylvania
M-1 Attn: R. L. Templin
M-1 H. N. Hill, Aluminum Research Laboratory

Armstrong Cork Company
Lancaster, Pennsylvania
R-1 Attn: J. W. Scott

Bell Telephone Laboratories
Murray Hill, New Jersey
R-1 Attn: C. Herring
R-1 D. P. Ling
R-1 W. P. Mason

Corning Glass Company
Corning, New York
R-1 Attn: J. T. Littleton
R-1  Professor G. Wästlund  
Cement & Concrete Research Institute  
Royal Institute of Technology  
Stockholm 70, SWEDEN

M-1  Professor John E. Goldberg  
Department of Structural Engineering  
Purdue University  
Lafayette, Indiana

R-1  Hilad F. Hanna  
Ph.D.  
Massachusetts Institute of Technology  
Cambridge 39, Massachusetts

R-1  Dr. W. Freiberger  
Department of Supply  
Aeronautical Research Laboratories  
Box 4331 GPO  
Melbourne, AUSTRALIA

M-1  Professor B. W. Shaffer  
Department of Mechanical Engineering  
New York University  
New York 53, New York

M-1  Professor G. Sachs  
Division of Metallurgical Research  
Engineering & Science Campus  
East Syracuse 4, N. Y.

R-1  Professor Aziz Ghali  
Head, Structural Analysis Department  
Fonad University  
Giza, EGYPT

M-1  Professor Hugh Ford  
Mechanical Engineering Department  
Imperial College of Science & Technology  
London, S.W.7  
ENGLAND

R-1  Dr. R. H. Wood  
D.S.I.R. Building Research Station  
Garston, Watford, Herts  
ENGLAND