Reference No. 53-22

Observations of Turbulent Mixing Processes
in a Tidal Estuary

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Abstract

An instrument has been developed to measure the turbulent variations of velocity and temperature in a river, and with it some observations have been made in the Kennebec Estuary of the vertical transport of heat and momentum. A comparison is made with the values determined from the average distribution of these properties by the method of Jacobsen. Only a small part of the turbulence transports momentum and heat; and the relation between the ratio $\mu_h / \mu_v$ and the Richardson number is found to be similar to that obtained by Jacobsen and Taylor.

Introduction

The mixing of fresh and salt water has previously been measured in estuaries by Jacobsen (1918) and Pritchard (1952) from the salinity and velocity distribution averaged over several tidal cycles. Pritchard's method is more refined, but Jacobsen's method illustrates the principle well. It is assumed that, when averaged over a tidal cycle, the only important terms in the horizontal equation of motion are the horizontal pressure gradient and the shearing stresses, and that the vertical equation is simply the hydrostatic equation. Then cross-differentiation eliminates the pressure term and

$$\frac{d^2 c}{d z^2} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x}$$

where $\mathcal{C}$ is the horizontal stress along the estuary, $z$ is measured vertically upward from the bottom, $g$ is the acceleration due to gravity.
and ρ is density. The assumptions amount to using steady flow equations for conditions that are actually periodic and can only be averaged over an integral number of periods. The stress is obtained by direct numerical integration with respect to z. Assumptions also have to be made about boundary conditions which enter as constants of integration. Similarly, the vertical flux of salt (or heat) is obtained by integration

$$\nabla = -\int_0^z u \frac{\partial s}{\partial x} \, dz$$

where s is salinity (or heat content). If observations of the time mean vertical gradients of velocity and salinity are also available the eddy coefficient μ_u of viscosity and μ_s of diffusivity may be computed:

$$\mu_u = \frac{\tau}{(\overline{du/\partial z})} \quad \mu_s = \frac{\nabla}{(\overline{ds/\partial z})}$$

Jacobsen (1918) carried out such computations, and Taylor (1931) showed that the ratio μ_s/μ_u is related to the Richardson number

$$g \frac{1}{\rho} \frac{\partial \rho}{\partial z} \left( \frac{\partial u}{\partial z} \right)^2$$

Because these computations are a rather indirect way of measuring the vertical turbulent transports of properties, it is appropriate to make measurements of the turbulent fluctuations themselves at various places and times in an estuary to compute the turbulent transfers directly from the formulae

$$\tau = -\rho \overline{u'w'} \quad \text{and} \quad \nabla = \rho \overline{T'w'c_p}$$
where the primes denote deviations from the mean, $\omega$ is vertical velocity, $T$ is temperature, and $C_p$ is specific heat. A strictly complete comparison of these direct measurements with those obtained by the Jacobsen method is of course impossible because a single ship can observe only at one place at a time, whereas the Jacobsen method averages over both space and time. The limited comparison possible, however, is interesting because it permits an inspection, for the first time, of the size, intensity, and period of the very eddy processes doing the transport.

**General Description of the Kennebec Estuary**

Three short cruises to the Kennebec Estuary were made at the suggestion of Dr. A. C. Redfield who suggested that on account of the geometrical simplicity of its shape, it might be an ideal location for various types of experiments. The valley is straight and of fairly uniform width and depth (Figure 1); only two sharp bends occur (Doubling Bends); and there is one very marked transition in width (at Bluff Head). Autumn discharge is about $290 \, m^3 \, sec^{-1}$; a tide of 3 m at the mouth produces currents of up to 2 m sec$^{-1}$ in the estuary. These produce considerable mixing. During the first short cruise, which was of an exploratory nature, some idea of the velocities and densities of the estuary were obtained. Two stations were occupied for a tidal cycle: one at the mouth, the other at Bath, about 20 km upstream. Velocity was measured with a Roberts meter, density by hydrometer. Figure 2 shows the main results: slight stratification, currents strong enough for easy propeller measurement. The estuary is somewhat more mixed vertically than Randersfjord, but
clearly should offer a good place to make vertical turbulent transport measurements.

**Second Cruise**

A second cruise was undertaken (R/V ASTERIAS) to test an instrument devised for measuring turbulent fluctuations in horizontal velocity and in inclination of flow. A propeller meter (von Arx, 1950) of the induction-coil type was used. The pulsed voltage output was rectified and smoothed by a condenser and then recorded on a Speedomax recorder. Inclination of the flow to the vertical was measured by a vane rotating on a horizontal axis ahead of its leading edge. The vane was attached to a potentiometer, the output of which was recorded on another Speedomax. The direction of flow in a horizontal plane was not measured, and because the ocean happened to be at the same temperature as the fresh water, no temperature fluctuations were present.

Various types of suspension were tried: (i) suspension by its flexible cable, (ii) fixed to a tripod resting on the bottom, (iii) fixed to a vertical mast projecting down into the water from the ship. The records obtained by methods (i) and (iii) show any rolling or pitching of the ship. The advantage of knowing the exact depth and position of the instrument in method (iii) was offset by the difficult problem of rigging it in a strong current. Method (ii) is confined to measurements at the bottom. Therefore method (i) was generally employed. In calm weather there does not appear to be any perceptible difference in records obtained by the three different methods.
After many instrumental difficulties, one velocity profile was obtained at Station 5C (depth 15 m) in perfectly calm weather on an ebb tide. A ten-minute record was taken at 20 cm depth, 3 m, 6 m, and 9 m, the meter being suspended on its cable. At depths below 9 m there was so little velocity that the meter did not register; a salinity reading showed that this was an example of a salt layer underlying the fast ebbing fresher water.

The records of horizontal velocity \( \bar{u} \) and inclination of flow \( \phi \) were analyzed as follows. A two-minute period was selected and the starting and end points identified in both records. Ordinates were scaled off each at three-second intervals and the mean values \( \bar{u} \) and \( \bar{\phi} \) computed. The variations ( \( u' = u - \bar{u} \) ) and ( \( \phi' = \phi - \bar{\phi} \) ) were taken for each ordinate, and the product \( uu'\phi' \) formed, which is proportional to the \( u'w' \) ( \( w' \) = instantaneous vertical velocity) if it is assumed that the mean flow is horizontal (e.g. \( \phi = 90^\circ \) from vertical) and that \( \phi' \) is small so that \( \sin \phi' = \phi' \). In fact, \( \phi' \) never exceeded 10°, and there is no evidence to suppose that the mean flow was inclined to the horizontal.

The mean value of \( uu'\phi' \) over the 40 ordinates was converted to the Reynold's stress \( \tau = -\rho uu'w' \) using the suitable constants for the velocity and angle scales. The computations were repeated twice for succeeding two-minute intervals and the values of \( \tau \) are tabulated in Table I.

The differences of \( \tau \) from the successive periods of computation show presumably how some of the momentum is transported vertically by eddies of scale too large to be observed in the two-minute period. However, the order of the stresses is about what might be expected as the wall
Table I

Station 5G - Second Cruise

Values of Shear Stress

<table>
<thead>
<tr>
<th>Depth cm</th>
<th>20</th>
<th>120</th>
<th>550</th>
<th>730</th>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>period</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{u}$ cm sec$^{-1}$</td>
<td>124</td>
<td>98</td>
<td>62</td>
<td>33</td>
</tr>
<tr>
<td>$\tau$ dynes cm$^{-2}$</td>
<td>6.7</td>
<td>16.1</td>
<td>8.3</td>
<td>3.2</td>
</tr>
<tr>
<td><strong>Second</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>period</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{u}$ cm sec$^{-1}$</td>
<td>127</td>
<td>107</td>
<td>51</td>
<td>37</td>
</tr>
<tr>
<td>$\tau$ dynes cm$^{-2}$</td>
<td>22.1</td>
<td>21.5</td>
<td>32.9</td>
<td>12.2</td>
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<tr>
<td><strong>Third</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>period</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\bar{u}$ cm sec$^{-1}$</td>
<td>120</td>
<td>100</td>
<td>67</td>
<td>41</td>
</tr>
<tr>
<td>$\tau$ dynes cm$^{-2}$</td>
<td>14.1</td>
<td>9.3</td>
<td>8.2</td>
<td>12.6</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>period</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{u}$ cm sec$^{-1}$</td>
<td>124</td>
<td>102</td>
<td>60</td>
<td>37</td>
</tr>
<tr>
<td>$\tau$ dynes cm$^{-2}$</td>
<td>14.2</td>
<td>15.6</td>
<td>16.4</td>
<td>9.3</td>
</tr>
</tbody>
</table>
stress of a stream 800 cm deep flowing over a very smooth surface at 120 cm sec\(^{-1}\). A point of some interest lies in the high stresses observed quite near the surface. It is clear that the stress at the very surface must be zero on this windless day; yet at only 20 cm below it was as high as at all other points through the fluid. Either the instrument must in some subtle way give a false reading, possibly due to its large size compared with its submergence, or some hitherto unsuspected complication is caused by the free surface.

During this cruise, a survey was made to find the horizontal salinity gradients along the estuary. It was carried out on an ebb tide, starting at the furthest upstream station at \(1\frac{1}{2}\) hours after high water at the mouth, and arriving at the mouth of the estuary \(\frac{1}{2}\) hour before low water slack tide and \(6\frac{1}{4}\) hours after high water. The return was made on the flood tide. The data are presented in Figure 3, the conductivity of the water (mho cm\(^{-3}\)) being plotted against distance downstream, for the water at both the surface and 9 m below. The sudden increase of conductivity at Station 2 is due to salt water being discharged into the main river from the tributary on the left bank, which rapidly sinks to the bottom of the river; otherwise the curves for both depths are somewhat parallel, and the salinity increases rather uniformly toward the sea. It is interesting to observe that the two sharp bends (Doubling Bends) and narrow portion downstream from them (between Stations 3 and 5) do not appear to change the salinity gradient very much. This is surprising, because in this region the apparent turbulence is great, great "boils" of water rising to the surface giving characteristic circular patches of still water. Such turbulence is
absent elsewhere. It might be thought that these "boils" are doing a great deal of mixing of the heavier salt water below with the fresher water above; if in fact they were doing so, the vertical salinity (or conductivity) gradients would be reduced, and the 9 m conductivity would be more nearly the surface one. No such effect is noticeable and a possibility exists that visible turbulence in the form of "boils" does not necessarily cause salt mixing.

Third Cruise (R/V CARYN)

The object of this cruise was to continue the observations of turbulent transfer processes of salt and momentum, and to investigate if the sudden widening of the river at Bluff Head acts as a control section on the velocity and density stratification upstream of it.

The velocity and vertical direction meter used in the second cruise was used again, with an additional element to measure temperature fluctuations, which was a "thermistor" bead connected in a bridge circuit to another Speedomax recorder. As the ocean water was this time some 10°F warmer than the fresh river water, the temperature of the mixed waters in the estuary was a tracer of the salinity. A correlation, Figure 4, between the water temperature and density was obtained from samples taken both at the surface and below by a Nansen bottle. Density was measured by hydrometer.

The whole meter assembly was suspended on the cable, with a heavy "depressor" weight of about 19 kgm secured below from a davit which projected some 1.8 m amidships. There is, therefore, a possibility that the
meter was affected by the proximity of the ship's side when it was close to the surface. Also, in a swift current, and the meter well immersed, the 1.2 cm diameter cable vibrates at about a \( \frac{1}{4} \) second period with a surface amplitude of several cm. It is not known how the meter performance was affected. Some uncertainty was experienced in deciding the direction of flow under the surface, particularly at Station 5B where on the ebb there was separating flow from Bluff Head. A large drag or current cross suspended on a thin wire sometimes was used to show this direction relative to the ship.

Observations were made over approximately a whole tidal cycle at each of four stations 5, 5A, 5B, 5C as shown in the map, Figure 5. Stations 5B and 5C are immediately downstream of the enlargement of the river cross section, 5A is in the narrow section, and 5 is well downstream. The bottom is occasionally rocky but the soundings show that there are no great discontinuities, and there is no great change of depth at Bluff Head. The observations were done in windless weather, with the air temperature nearly the same as that of the water, except for those at Station 5 when there was a fresh breeze against the current and therefore there were waves on the water surface. The ship's motion was just perceptible on deck, but was very obvious on the records of the inclination vane, which were so obscured as to make them useless for the turbulence analysis. At each station a sounding was made at two hourly intervals, when the combined instrument was lowered from the surface in 3 m increments of cable length. At each increment a record was taken for about ten minutes, and the cable angle recorded, in order to know the depth of the instrument. During the
"surface" measurement (really with meter center 20 cm from surface) the surface velocity was independently measured by timing a floating object passing down the length of the ship, and the surface density and temperature observed.

The velocity and temperature records were first examined to find how the velocity and temperature gradients changed during the tidal cycle. The mean velocity \( \bar{u} \) and temperature \( \bar{T} \) were estimated by eye for every ten-minute measurement, and were plotted as velocity and temperature profiles against depth in Figure 6. Here the horizontal abscissa is the time of the middle of the sounding relative to high water at Bluff Head, and the vertical lines in each profile have been placed at this time. All four stations are shown in the figure, and the velocities and temperatures are all plotted at the same scale. An uncertainty sometimes arises, and is shown by dotted lines when the surface velocity was repeated after a sounding (e.g. perhaps 1 hour after it was initially measured) and was found to have changed. This was, of course, noticeable near high and low water, and is well illustrated by a curious phenomenon noticed at Station 5 on an ebb tide. The meter was at 20 cm depth and registering about 60 cm per water speed, when a line of foam was noticed approaching the ship. As the line passed the meter, the velocity quite suddenly increased some 50% and the temperature fell, showing that a wedge of quite differently constituted water was passing (see Figure 7). Subsequent "fronts" or sudden increases of velocity were also noticed and these render doubtful the soundings at those times, e.g. Station 5 at +3\( 1/2 \) hours; Station 5B at -2\( 1/2 \) hours and +2 hours; Station 5C at +1\( 1/2 \) hours from high water.
The velocity profiles of Figure 6 show how flood begins first in the salt lower water, but ebb begins first at the surface. This is different from the phase lag due to friction alone (Lamb, 1932) where both flood and ebb begin at the bottom.

Analysis of Third Cruise Data

The records were further analyzed for their turbulence characteristics in the same manner as described for the records of the second cruise. In this case the heat transfer \( Q = \rho \omega' T' \) was computed from the temperature record \( T \) and \( \omega' = u \theta' \). A great deal of rather subjective selection was required to determine suitable parts of the record for these techniques. Records which had produced mean profiles of velocity or temperature of an \( S \) shape were rejected, for such profiles cannot be reliably used to determine gradients \( \frac{\partial u}{\partial z} \) and \( \frac{\partial T}{\partial z} \) with any certainty; those taken when there were surface waves present (and therefore the ship was moving) were rejected because the movement showed in the inclination vane and velocity records, thus giving some uncertainty; those which showed no variation in the temperature of the water were rejected, for without these fluctuations there will be no heat transfer; and records were rejected in which the mean velocity had altered in the course of taking the whole profile, such as that shown in Figure 7. When all these conditions were applied, only four profiles were regarded as wholly suitable, and three-minute portions of their records were analyzed for the depths shown in the accompanying Table II. The data of \( u', \theta' \) and \( T' \) were then examined by a statistical method to find the probability that

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1The test involved is that given by Fisher (1941), p. 202, who gives tables of the probability of a certain correlation by a random arrangement as a function of the number of observations.
Table II
Computed Turbulence Data

<table>
<thead>
<tr>
<th>Depth m</th>
<th>$u$ cm sec$^{-1}$</th>
<th>$T^\circ F$</th>
<th>$\tau$ dynes cm$^{-2}$</th>
<th>$\gamma F$ cal's cm$^{-2}$ sec$^{-1}$</th>
<th>Significance Test* for $\tau$</th>
<th>Significance Test* for $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>130</td>
<td>37.47</td>
<td>6.82</td>
<td>0.0049</td>
<td>$&lt; 1/100$</td>
<td>$&gt; 1/10$</td>
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<tr>
<td>2.13</td>
<td>126</td>
<td>37.63</td>
<td>25.3</td>
<td>0.250</td>
<td>$&lt; 1/100$</td>
<td>ca $1/20$</td>
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<tr>
<td>5.80</td>
<td>113</td>
<td>38.57</td>
<td>16.7</td>
<td>0.179</td>
<td>$&lt; 1/20$</td>
<td>ca $1/20$</td>
</tr>
<tr>
<td>8.20</td>
<td>116</td>
<td>38.91</td>
<td>17.1</td>
<td>-0.123</td>
<td>$&lt; 1/50$</td>
<td>ca $1/20$</td>
</tr>
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<td>Station 5A  2 hrs. before HW</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>3.0</td>
<td>115</td>
<td>37.96</td>
<td>32.9</td>
<td>0.102</td>
<td>$&lt; 1/10$</td>
<td>$&gt; 1/10$</td>
</tr>
<tr>
<td>6.1</td>
<td>97.9</td>
<td>37.93</td>
<td>20.5</td>
<td>0.0626</td>
<td>$&gt; 1/10$</td>
<td>$&gt; 1/10$</td>
</tr>
<tr>
<td>Station 5C  5 hrs. before HW</td>
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<tr>
<td>0.20</td>
<td>140</td>
<td>39.49</td>
<td>45.3</td>
<td>0.106</td>
<td>$&lt; 1/100$</td>
<td>$&lt; 1/100$</td>
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<tr>
<td>4.85</td>
<td>119</td>
<td>39.64</td>
<td>23.6</td>
<td>0.211</td>
<td>$&lt; 1/100$</td>
<td>$&lt; 1/100$</td>
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<td>8.5</td>
<td>85.0</td>
<td>39.70</td>
<td>62.2</td>
<td>0.192</td>
<td>$&lt; 1/100$</td>
<td>$&lt; 1/100$</td>
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<td>Station 5C  5 hrs. after HW</td>
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<td></td>
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</tr>
<tr>
<td>3.0</td>
<td>115</td>
<td>37.96</td>
<td>32.9</td>
<td>0.102</td>
<td>$&lt; 1/10$</td>
<td>$&gt; 1/10$</td>
</tr>
<tr>
<td>6.1</td>
<td>97.9</td>
<td>37.93</td>
<td>20.5</td>
<td>0.0626</td>
<td>$&gt; 1/10$</td>
<td>$&gt; 1/10$</td>
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<td>Station 5C  3½ hrs. after HW</td>
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</tr>
</tbody>
</table>

*Probability that $\tau$ and $\gamma$ are simply result of random correlation.
the computed values of $\tau$ and $\varphi$ could have appeared from a completely random set of values of $u'$, $w'$ and $\tau'$. Where this probability was better than $1/10$ the computed value of $\tau$ or $\varphi$ was considered to be so uncertain as not to warrant the drawing of deductions which are presented in Table III. This is tantamount to deciding that in the discarded cases there was so much turbulence present which did not transport momentum or heat that it masked the turbulence that was in fact transporting these quantities.

The deductions are made in groups of three depths, the velocity and temperature gradients being found from the extremes, and the transport of momentum and heat from the middle depths. Other methods of analysis are of course possible, which may in the last case reduce the anomaly of $\mu_s$ being larger than $\mu_w$ a situation which is physically difficult to explain. For example, in Table IIIA, the same deductions are made, but now the values of $\tau$ and $\varphi$ used to find $\mu_w$ and $\mu_s$ are the mean values from all the three depths. The Richardson number remains unchanged.

Kennebec Estuary Turbulent Transfer Coefficients by Method of Jacobsen

The data in Figures 3, 4, and 6 may be used to obtain average values of the turbulent exchange coefficients by the method of Jacobsen. As a working hypothesis $\frac{1}{\rho} \frac{\partial p}{\partial x}$ may be considered as independent of $z$. In order to take account of the integration constants, it may be assumed that $\tau = 0$ at $z = 0$, the bottom and $z = h$, the free surface, in which case $\tau$ is a maximum at mid-depth and
Table III
Deduced Quantities from Turbulence Analysis

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$\mu v$</th>
<th>$\mu s$</th>
<th>$\mu s / \mu v$</th>
<th>Stability $\rho \left( \frac{\partial U}{\partial z} \right)^2 / g \frac{\partial \rho}{\partial z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
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<td></td>
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<tr>
<td>2.13</td>
<td>830</td>
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<td>0.191</td>
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<td>5.80</td>
<td>1010</td>
<td>85</td>
<td>0.084</td>
<td>0.0525</td>
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<td>8.20</td>
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</tr>
<tr>
<td>Station 5A</td>
<td>2 hrs. before HW</td>
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</tr>
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<tr>
<td>4.85</td>
<td>356</td>
<td>830</td>
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<td>Station 5C</td>
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<tr>
<td>4.85</td>
<td>356</td>
<td>830</td>
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Table IIIA

<table>
<thead>
<tr>
<th>Depth m</th>
<th>( \mu n )</th>
<th>( \mu s )</th>
<th>( \mu s/\mu n )</th>
<th>Stability</th>
</tr>
</thead>
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<td>0.0408</td>
<td>0.0525</td>
</tr>
<tr>
<td>5.80</td>
<td>8.20</td>
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<tr>
<td>Station 5C 3(\frac{1}{2}) hrs. after HW</td>
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<td>665</td>
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</tbody>
</table>
\[ \tau_{\text{max}} = \frac{g}{\rho} \frac{\partial \rho}{\partial x} \frac{h^2}{8} \]

This parabolic distribution of \( \tau \) is of course an approximation to the much more complicated distribution shown by the turbulence analysis actually to exist. From Figure 3 estimates were made of

\[ \frac{1}{\rho} \frac{\partial \rho}{\partial x} \approx 8.3 \times 10^{-7} \text{ cm}^{-1}; \ h \approx 20 \text{ m} \]

and from Figure 6 \[ \frac{\partial u}{\partial x} \approx 3.3 \times 10^{-2} \text{ sec}^{-1} \]

Hence \( \tau = 4 \text{ dynes cm}^{-2} \) and \( \mu u = \frac{\tau}{(\partial u/\partial x)} = 120 \text{ cm}^2 \text{ sec}^{-1} \)

Similarly the mean temperature gradient is \( \frac{dT}{dx} = 3.3 \times 10^{-6} \text{ oF cm}^{-1} \), and \( \frac{dT}{dz} = 1/1200 \text{ oF cm}^{-1} \). The turbulent transport of heat at mid-depth is approximately \( \gamma = \frac{\varrho_c}{\varrho} \frac{\partial T}{\partial x} \) where \( \varrho_c \) is the mean discharge per unit width of the lower upstream moving heavy water. The value of \( \varrho_c \) can be roughly estimated from the conservation of salt. Inasmuch as the average density of the bottom layer is 1.010, and that of the surface layer about 1.008, \( \varrho_b \) must be about 4 times that of the fresh water alone, \( \varrho_0 \). As \( \varrho_0 \) is the river discharge (290 m\(^3\) sec\(^{-1}\)) divided by the average width of the channel (\( \approx 580 \text{ m} \)), thus \( \varrho_c = 2 \text{ m}^2 \text{ sec}^{-1} = 2 \times 10^4 \text{ cm}^2 \text{ sec}^{-1} \)

and \( \gamma = 0.07 \text{ oF cals cm}^{-2} \text{ sec}^{-1} \)

and \( \mu_s = \gamma / \frac{dT}{dz} = 78 \text{ cm}^2 \text{ sec}^{-1} \)
The above values are now combined to give

\[ \frac{\mu_s}{\mu_n} = 0.67 \text{ and } \frac{g}{\rho} \frac{\partial \rho}{\partial z}/(\mu_n) = 1.9 \]

as the mean mixing constants for the estuary.

Discussion of Turbulent Mixing Computations

The values of the ratio \( \frac{\mu_s}{\mu_n} \) obtained by our turbulent transfer measurements have been plotted logarithmically against the reciprocal of the Richardson number on the graph Figure 8 which is based on that given by Anderson and Munk (1948). The value found by the Jacobsen method for the Kennebec is plotted, as well as the original Taylor-Jacobsen values. It is clear from the figure that a stable system tends to inhibit mixing and so give a low \( \frac{\mu_s}{\mu_n} \), a result well known qualitatively. Conversely, if the velocity gradients are great and density gradient small, the heat (or salinity) and momentum are equally well mixed and \( \frac{\mu_s}{\mu_n} = 1.0 \). The mechanism of mixing of salt cannot therefore be the same as the mixing of momentum (Taylor, 1931). Although it is by no means original with us, the following physical explanation seems quite reasonable: that a great mass of heavy, slow, liquid is torn from a lower layer by turbulent pressure fluctuations; it then is ejected into the faster, higher liquid where it speeds up and so takes up momentum by pressure forces on it. During this time the outer surface is being frittered away and so some mixing of the mass is caused. However, in very stable systems not all the liquid is thus mixed, and the reduced mass falls back again to its parent liquid but now at an increased speed, where its momentum is absorbed. Inspection
of the velocity and temperature records shows that periods of eddies from 1 to 150 seconds could sometimes be seen, though it is possible that the longer periods may not have been adequately averaged in the 180 second period of the computations.

The comparatively good agreement of our values of $\mu_s/\mu_u$ with those obtained by Jacobsen is perhaps a vindication of the latter method, dependent as it is on a not entirely justifiable averaging process over a tidal cycle. It seems from the turbulence analysis that in part of this estuary at some states of tide the mixing of salt and fresh water is good and $\mu_s/\mu_u = 1.0$; and that at times the stratification is pronounced, when $\mu_s/\mu_u < 1.0$ (see Figure 8). It is gratifying, therefore, to find that the apparent mean value of $\mu_s/\mu_u$ and Richardson's number, found by Jacobsen's method, lies between the sounding analyzed which had been classified as well stratified, and the sounding which had weak stability and was therefore well mixed. Although the numerical data for the Jacobsen method are approximate, the value is not changed much relative to those found by the turbulence analysis if other approximations are taken; and we make the very tentative conclusion that to a certain degree the mixing and stability of an estuary are rather linear, and that average values seem to have some significance relative to spot values.

It will be seen from Table II that high stresses are again found near the surface. Whether this effect is a false one, due purely to the instrument's limitations or the method of suspension, remains to be seen. At any rate, it is another reason for further study of the surface layers of an open stream.
We think that further study on the mixing processes in estuaries can develop along two distinct lines.

1. Further, more detailed observations are required on the gross features of the salinity and velocity patterns in estuaries with different sorts of control sections. More refined methods are required of determining velocity and density gradients quickly, so that the essentially unsteady tidal flow of the estuary may be sampled at all depths at nearly the same time. This requirement is probably to be met by having several meters registering at the same time, at different depths, though these meters need only give mean velocities and not the turbulent fluctuations.

2. The small scale turbulent details of the flow require study to know more exactly the mixing of the fluids under given mean velocity and density gradients. This study involves measurements both close to the bottom and to the free surface, and requires instruments with a quick enough response to register the turbulent fluctuations of the current. It is desirable, again, to have a number of instruments at different depths operating simultaneously, though this will undoubtedly add to the complications of the experiment. We think that difficulties due to non-steady conditions would be eliminated if the experiments were done in a non-tidal, steady flow channel, in effect a large flume, if such an estuary could be found.
References


FIRST CRUISE. DENSITY AND VELOCITY CHANGES IN A TIDAL CYCLE.

STA. II

DENSITY

0800 1000 1500 2000

TIME

H.W. BATH

STA. VII

DENSITY

1700 2000 0000 0400

TIME

H.W. MOUTH OF RIVER

STA. II

EAST

1000 1500

VELOCITY (KNOTS)

H.W. BATH

STA. VII

EAST

1600 2000 0000 0400 0800

VELOCITY (KNOTS)

H.W. MOUTH OF RIVER
LONGITUDINAL SALINITY PROFILE OF ESTUARY

SPEC. COND. mho cm⁻³

DISTANCE DOWNSTREAM

0 SCALE 5 km.
CORRELATION BETWEEN TEMPERATURE AND DENSITY OF MIXED WATER

FIG. 4
TIME RELATIVE TO HIGH WAT

STATION 5A

DEPTH SCALE M

SPEED SCALE

TEMPERATURE SCALE

VELOCITY PROFILES
UNCERTAINTIES
TEMP. PROFILES

FLOOD TO LEFT OF VERTICAL, EBB TO RIGHT
VERTICAL IS 39°F

KENNEBEC RI
DEC. 8-9
INCREASE OF SPEED AND TEMPERATURE ON PASSING OF FOAM LINE

VELOCITY RECORD

TEMPERATURE RECORD

- 70
- 60
- 50
- 40

VELOCITY SCALE

CM. SEC\(^{-1}\)

-40.0°F
-39.8°F
-39.6°F

1 MINUTE

TIME

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