UNCLASSIFIED

AD NUMBER

AD001908

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution: No Foreign.

AUTHORITY

ONR ltr., 26 Oct 1977

THIS PAGE IS UNCLASSIFIED

Reproduced by
Armed Services Technical Information Agency
DOCUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

AD-

208

UNCLASSIFIED

AD No. 12022
ASTIA FILE COPY

REPORT NO. HE-150-98

TECHNICAL REPORT

UNIVERSITY OF CALIFORNIA
INSTITUTE OF ENGINEERING RESEARCH
BERKELEY, CALIFORNIA

THEORETICAL AERODYNAMIC PROPERTIES FOR AN INCLINED FLAT PLATE IN SLIP FLOW

By

S. F. Hack

SERIES NO. 20

ISSUE NO. 87

DATE March 21, 1952

CONTRACT N7-ONR-295-TASK 3
PROJECT NR 061-003
REPORT NO. HE-150-98
SERIES NO. 20-87
MARCH 21, 1952

JOINTLY SPONSORED BY
OFFICE OF NAVAL RESEARCH AND
OFFICE OF SCIENTIFIC RESEARCH

FLUID FLOW AND
HEAT TRANSFER
AT LOW PRESSURES
AND TEMPERATURES

THEORETICAL AERODYNAMIC PROPERTIES FOR AN INCLINED FLAT PLATE IN SLIP FLOW

By

S. F. MACK

FACULTY INVESTIGATORS:

R.G. FOLSOM, PROFESSOR OF MECHANICAL ENGINEERING

S.A. SCHAAF, ASSOCIATE PROFESSOR OF ENGINEERING SCIENCES

Approved by:

S.A. Schaaf

NOMENCLATURE

a = correction factor for linearized velocity due to angle of attack (see eq. 4.1)

b = chord of flat plate

c = sound speed

c.p. = center of pressure with respect to leading edge

C = aerodynamic coefficient based on one chord length

erfc = complementary error function defined by $\mathrm{erfc}\,(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-r^2}\,dr$

exp = exponential function defined by $\exp\,(z) = e^z$

H = $\frac{\nu b a}{\lambda^2 U_\infty}$ (see eq. 5.3)

J_∞ = $\sqrt{\frac{\nu \rho}{\lambda^2 U_\infty}} = \frac{\sqrt{Re_\infty}}{1.5 M_\infty}$ for air, proportional to reciprocal of rarefaction parameter $\frac{M_\infty}{\sqrt{Re_\infty}}$

K_M = moment correction factor (see eq. 5.6 and HYD 2720)

K_N = normal force correction factor (see eq. 4.11 and HYD 2719)

m = quantity in moment relation defined by eq. (5.2)

M = net moment about leading edge of plate (both sides), positive in c.c. rotation, per unit of span length

M_∞ = $\frac{U_\infty}{c}$, free stream Mach number

N = net force acting normal to plate (both sides) per unit of span length

p = pressure

Re_∞ = $\frac{b U_\infty}{\nu}$, free stream Reynolds number based on chord b

s = parameter of the Laplace transform

$\bar{T}$ = total force acting tangent to plate (both sides) per unit of span length

u = component of velocity parallel to plate

U_∞ = free stream velocity

U = velocity parallel to plate from linearized supersonic flow theory (see eq. 4.1)

v = component of velocity normal to plate

NOMENCLATURE (CONT'D)

x = coordinate along chord of plate

y = coordinate normal to plate

α = angle of attack, radians

β = slope of the displacement thickness function

γ = 1.4 for air, ratio of specific heats

δ^* = displacement thickness function

θ = $\frac{\nu x}{\lambda^2 U}$ non-dimensional distance

λ = $1.5 \frac{\nu}{c}$ (approx.) for air, molecular mean free path

μ = $\rho \nu$, mechanical viscosity

μ' = $\sin^{-1} 1/M_\infty$, Mach angle

ν = kinematic viscosity

ρ = gas density

Subscripts

i = 1, upper surface of plate; = 2, lower surface of plate

∞ = free stream condition

D = drag

L = lift

M = moment

N = normal to plate

T = tangential to plate

Superscripts

$'$ = velocity perturbations

$-$ = Laplace transforms

THEORETICAL AERODYNAMIC PROPERTIES FOR AN INCLINED FLAT PLATE IN SLIP FLOW

1.0 INTRODUCTION

The theory initiated by Ref. 1 concerns the drag coefficient for the flat plate at zero angle of attack in the slip-flow regime of aerodynamics. Experimental verification of the basic results of this theory was obtained in Ref. 2 in which measurements of the drag coefficients of flat plates were reported, covering the conditions $2.3 < M_\infty < 3.6$, $30 < Re_\infty < 1650$, and $0.08 < M_\infty/\sqrt{Re_\infty} < 0.39$ $(2.6 < \sqrt{Re_\infty}/M_\infty < 12.5)$.

It is the purpose of this analysis to investigate the aerodynamic properties of a thin flat plate at small positive angles of attack along the lines of that theory. Specifically this analysis studies the drag, lift, and moment on the flat plate operating in air under slip-flow conditions at very small angles of attack. The analysis was simplified by first assuming that ideal linearized supersonic flow conditions obtained near the plate, and then making first order corrections for the effects of viscosity and slip and for the angle of attack.

2.0 RESULTS

The results of the analysis are contained in the quantities C_T, the tangential force (i.e. skin friction) coefficient; K_N, the normal force correction factor; and K_M, the moment correction factor. The correction factors refer to the normal and moment coefficients of the flat plate as determined by ideal linearized supersonic flow theory. Graphs of C_T, K_N, and K_M versus $\sqrt{Re_\infty}/M_\infty$, the reciprocal of the rarefaction parameter, for $M_\infty = 2$, 3, and 4 are presented in HYD's 2718, 2719, and 2720 respectively.

The aerodynamic coefficients C_T, C_N, and C_M were evaluated as functions of α up to terms of the first order in α with the following results. The tangential force coefficient C_T is independent of α but varies almost directly with the rarefaction parameter in the slip-flow region. The normal force coefficient C_N is proportional to α and to K_N, but the factor K_N does not differ appreciably from unity at any rarefaction condition (in fact, by less than 20% provided $M_\infty \geq 3$). The moment coefficient C_M has almost the identical character of C_N. From these considerations there results that if α is of infinitesimal order, the drag coefficient C_D is independent of α and may be taken equal to C_T evaluated for zero angle of attack.

On the other hand, the lift coefficient C_L is proportional to α and depends strongly upon a negative contribution due to C_T which becomes increasingly significant at the higher-altitude operating conditions. However, it appears that the lift curve slope $dC_L/d\alpha$ at $\alpha = 0$,

HYD 2721, is always positive and tends decreasingly toward its minimum at the higher-altitude, higher-speed conditions. Also according to this analysis, the center of pressure falls just forward of, but essentially at, the half-chord position at all altitude conditions when $M_\infty \geqq 2$.

3.0 TANGENTIAL FORCE

The friction force function $T(x)$ due to viscosity acts tangent to the plate surface and is defined for one surface of the plate and μ constant to be

$$T(x) = \mu \int_0^x \left(\frac{\partial u}{\partial y}\right)_{y=0} dx$$

The tangential force T_1 per unit of span length acting on the upper surface of the plate is obtained from eq. (.) of Ref. 2, part II, as

$$T_1 = \frac{\mu \lambda U_1^2}{\nu}\left[\exp\left(\frac{\nu b}{\lambda^2 U_1}\right) \operatorname{erfc}\left(\sqrt{\frac{\nu b}{\lambda^2 U_1}}\right) - 1 + \frac{2}{\sqrt{\pi}}\sqrt{\frac{\nu b}{\lambda^2 U_1}}\right] \tag{3.1}$$

A corresponding relation is found for T_2 on the lower surface.

The total shear force per unit of span length is then given by

$$T = T_1 + T_2 \tag{3.2}$$

Since U_1 and U_2 are functions of the angle of attack α, the quantity T is also. Hence for values of α near $\alpha = 0$, T may be written as

$$T(\alpha) = T(0) + T'(0)\alpha + \frac{1}{2} T''(0)\alpha^2 + \cdots$$

where the primes denote differentiation with respect to α. Only the first two terms are retained, and upon substitution of eq. (3.2), $T(\alpha)$ may be written as

$$T(\alpha) = T_1(0) + T_2(0) + \alpha\left(\frac{\partial T_1}{\partial U_1}\cdot\frac{dU_1}{d\alpha} + \frac{\partial T_2}{\partial U_2}\cdot\frac{dU_2}{d\alpha}\right)_{\alpha=0} \tag{3.3}$$

When $\alpha = 0$, it is readily seen that

$$T_1(0) = T_2(0), \quad \frac{\partial T_1}{\partial U_1} = \frac{\partial T_2}{\partial U_2}, \text{ and } \frac{dU_1}{d\alpha} = -\frac{dU_2}{d\alpha}$$

By employing these relations in eq. (3.3) it is found, as might have been expected, that up to terms of order α, $T(\alpha)$ reduces to

$$T(\alpha) = T_1(0) + T_2(0) = 2T_1(0)$$

Thus, the total tangential force on the plate is essentially constant over a small range of α near $\alpha = 0$. It is given for each unit of span length by

$$T = \frac{2\mu\lambda U_\infty^2}{\nu}\left[\exp\left(\frac{\nu b}{\lambda^2 U_\infty}\right)\operatorname{erfc}\left(\sqrt{\frac{\nu b}{\lambda^2 U_\infty}}\right) - 1 + \frac{2}{\sqrt{\pi}}\sqrt{\frac{\nu b}{\lambda^2 U_\infty}}\right] \qquad (3.4)$$

For a gas with $\gamma = 1.4$, it is known that $\lambda C = 1.5\,\nu$ (Ref. 3 p. 214, 82); hence,

$$\frac{\nu b}{\lambda^2 U_\infty} = \frac{Re_\infty}{225\,M_\infty^2} = J_\infty^2 \qquad (3.5)$$

Equation (3.4) may now be written in the more convenient form in the case of air:

$$T = \left\{\frac{1}{2}\rho U_\infty^2 b\right\}\left\{\frac{8}{3M_\infty J_\infty^2}\left[\exp(J_\infty^2)\operatorname{erfc}(J_\infty) - 1 + \frac{2J_\infty}{\sqrt{\pi}}\right]\right\} \qquad (3.6)$$

The quantity in the second set of braces is the tangential force coefficient C_T. A graph of C_T versus $\sqrt{Re_\infty}/M_\infty$ for $M_\infty = 2$, 3 and 4 is shown in HYD 2718.

4.0 NORMAL FORCE

In order to compute the force acting normal to the plate, the pressure coefficient on the plate is assumed the same as that on a body in ideal linearised supersonic flow, having the shape determined by the displacement thicknesses $\delta_1^*(x)$ and $\delta_2^*(x)$ on the upper and lower surfaces respectively. As will be shown in the case of slip flow at small angles of attack, this "displacement" body will have, essentially, a sharp leading edge (determined by $\beta(0) = 1/1.5\,M_\infty$ for a gas having $\gamma = 1.4$). This implies that no assumptions need be made concerning the presence of a normal shock wave at the leading edge of the displacement body, as may perhaps be necessary in the case of no-slip flow. As in the evaluation of the tangential force, all effects that may occur in the vicinity of the sides and trailing edge of the flat plate are omitted here. The shape of the displacement body is determined from the uniform parallel velocities U_1 and U_2, above and below

the plate respectively, of ideal linearised two-dimensional supersonic flow theory for the flat plate, as follows.

The velocity components (Ref. 4, ch. 11) contributing to the resulting ideal velocities U_1 and U_2 are shown in R.D 2717. U_1 and U_2 are uniform and parallel to the plate with

$$U_1 = U_\infty \cos\alpha + U_\infty \frac{\sin\alpha \sin(\mu'+\alpha)}{\cos(\mu'+\alpha)}$$

and

$$U_2 = U_\infty \cos\alpha - U_\infty \frac{\sin\alpha \sin(\mu'-\alpha)}{\cos(\mu'-\alpha)}$$

By means of elementary trigonometric relations and the relation $\sin\mu' = 1/M_\infty$, these reduce to

$$\left.\begin{aligned} U_1 &= \frac{U_\infty}{a_1}, \text{ where } a_1 = \cos\alpha - \frac{\sin\alpha}{\sqrt{M_\infty^2 - 1}} \\ \text{and} \quad U_2 &= \frac{U_\infty}{a_2}, \text{ where } a_2 = \cos\alpha + \frac{\sin\alpha}{\sqrt{M_\infty^2 - 1}} \end{aligned}\right\} \qquad (4.1)$$

Because the normal force is to be evaluated only up to terms of order α, the substitutions $\cos\alpha = 1$ and $\sin\alpha = \alpha$ appear warranted at this point, but, as will be seen, this leads to no essential simplification of the analysis.

The displacement thickness $\delta^*(x)$ along one surface of the plate is determined by the velocity distribution $u(x,y)$ for that surface, as follows. A flat plate of chord b lying along the x-axis with the leading edge as origin is visualised, over which is flowing a uniform stream with velocity U parallel to the plate surface in the positive x-direction. The steady two-dimensional boundary layer equation for the velocity components u and v near the surface is (Ref. 5, p. 135)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \qquad (4.2)$$

This equation is linearized by considering a perturbation u on the uniform velocity U; viz.

$$u = U + u' \text{ with } |u'| << U$$

$$v = v' \text{ with } |v'| << U$$

Upon substituting these relations into eq. (4.2), expanding, and neglecting terms of second order in the perturbation, there results

$$U \frac{\partial u'}{\partial x} = \nu \frac{\partial^2 u'}{\partial y^2} \tag{4.3}$$

At the surface of the plate slip-flow theory for diffuse reflection gives the boundary condition (Ref. 1)

$$U + u' = \lambda \frac{\partial u'}{\partial y} \; ; \quad 0 \leqq x \quad , \quad y = 0 \tag{4.4}$$

Equation (4.3) with boundary condition (4.4) can be solved for the perturbation velocity $u'(x,y)$ quote conveniently by means of Laplace transforms defined by

$$\bar{f}(s) = \int_0^\infty e^{-sx} f(x)\, dx$$

The transforms of eqs. (4.3) and (4.4) are

$$U s \bar{u}' = \nu \frac{d^2 \bar{u}'}{d y^2} \tag{4.$\bar{3}$}$$

$$\frac{U}{s} + \bar{u}' = \lambda \frac{d \bar{u}'}{d y} \; ; \quad y = 0 \tag{4.$\bar{4}$}$$

Equation (4.$\bar{3}$) is an ordinary differential equation with boundary condition (4.$\bar{4}$) and has the general solution

$$\bar{u}' = C_1 \exp\left(\sqrt{\frac{Us}{\nu}}\, y\right) + C_2 \exp\left(-\sqrt{\frac{Us}{\nu}}\, y\right)$$

The requirement that u' and hence $\bar{u}'$ shall vanish for large y implies that $C_1 = 0$. The constant C_2 is determined by the boundary condition (4.$\bar{4}$); hence,

$$\bar{u}'(s, y) = \frac{-U\sqrt{\frac{\nu}{\lambda^2 U}} \exp\left(-\sqrt{\frac{Us}{\nu}}\, y\right)}{s\left(\sqrt{\frac{\nu}{\lambda^2 U}} + \sqrt{s}\right)} \tag{4.$\bar{5}$}$$

If required, the perturbation $u'(x,y)$ may be found by applying Ref. 6, p. 300, no. 86 to eq. (4.5).

The displacement thickness $\delta^*(x)$ defined by

$$\delta^*(x) = \int_0^\infty \left(1 - \frac{u}{U}\right) dy = \int_0^\infty - \frac{u'}{U}\, dy$$

has in view of eq. (4.5) the transform

$$\bar{\delta}^*(s) = \int_0^\infty \frac{\sqrt{\frac{\nu}{\lambda^2 U}}\, \exp\left(-\sqrt{\frac{Us}{\nu}}\, y\right)}{s\left(\sqrt{\frac{\nu}{\lambda^2 U}} + \sqrt{s}\right)}\, dy$$

$$= \frac{\frac{\nu}{\lambda U}}{s\sqrt{s}\left(\sqrt{\frac{\nu}{\lambda^2 U}} + \sqrt{s}\right)}$$

By means of Ref. 6, p. 297, no. 45, the displacement thickness along the plate surface is found to be

$$\delta^*(x) = \lambda \left[\exp\left(\frac{\nu x}{\lambda^2 U}\right) \operatorname{erfc}\left(\sqrt{\frac{\nu x}{\lambda^2 U}}\right) - 1 + \frac{2}{\sqrt{\pi}} \sqrt{\frac{\nu x}{\lambda^2 U}} \right] \qquad (4.6)$$

It may be of interest to point out that the displacement thickness in the case of slip flow is not parabolic as it is in the case of no-slip flow.

The slope $\beta(x)$ of the displacement thickness will be particularly required in evaluating the moment of the forces on the plate surface and is found directly from eq. (4.6) to be

$$\beta(x) = \frac{\nu}{\lambda U} \exp\left(\frac{\nu x}{\lambda^2 U}\right) \operatorname{erfc}\left(\sqrt{\frac{\nu x}{\lambda^2 U}}\right) \qquad (4.7)$$

Thus, $\beta(x)$ is a decreasing function of x with its maximum $\nu/\lambda^2 U$, occurring at the leading edge (i.e. $x = 0$). For air $\lambda c = 1.5\,\nu$; hence, $\beta(0) = 1/1.5M$. This implies that, for moderate Mach numbers, the nose of the displacement body appears sharp in the approximate theory being presented.

The flat plate at the small angle of attack α with respect to the uniform stream of velocity U_∞ will have on its upper surface the displacement thickness $\delta_1^*(x)$ given by eq. (4.6) upon substituting for U in eq. (4.6) the quantity U_1 of eq. (4.1):

$$\delta_1^*(x) = \lambda \left[\exp\left(\frac{U x \alpha_1}{\lambda^2 U_\infty}\right) \operatorname{erfc}\left(\sqrt{\frac{U x \alpha_1}{\lambda^2 U_\infty}}\right) - 1 + \frac{2}{\sqrt{\pi}} \sqrt{\frac{U x \alpha_1}{\lambda^2 U_\infty}} \right] \qquad (4.8)$$

Correspondingly, $\delta_2^*(x)$ is obtained for the lower surface. The slopes $\beta_1(x)$ and $\beta_2(x)$ are not large at $x = 0$ and decrease fairly rapidly with x; hence, the approximations $\beta_1(x) = \tan^{-1}\beta_1(x)$ and $\beta_2(x) = \tan^{-1}\beta_2(x)$ are valid over most of the plate.

An expression for the net force N per unit of span length acting normal to the plate surfaces is now found from Ref. 4, pp. 198-200, as

$$N = \frac{1}{2} \rho_\infty U_\infty^2 \int_0^b \frac{2}{\sqrt{M_\infty^2 - 1}} \left[\beta_2(x) - \beta_1(x) + 2\alpha \right] dx$$

$$= \frac{1}{2} \rho_\infty U_\infty^2 \left\{ \frac{4\alpha b}{\sqrt{M_\infty^2 - 1}} + \frac{2}{\sqrt{M_\infty^2 - 1}} \left[\delta_2^*(b) - \delta_1^*(b) \right] \right\} \qquad (4.9)$$

As in the case of the tangential force T, only the first two terms of the expansion of $N = N(\alpha)$ in a power series in α about $\alpha = 0$ are retained. Thus, $N(\alpha) = N(0) + \alpha N'(0) = \alpha N'(0)$ because the constant term $N(0)$ vanishes upon putting $\alpha = 0$ in eq. (4.9). In view of

$$\left[\frac{\partial \delta_1^*(b)}{\partial \alpha}\right]_{\alpha=0} = \left[\frac{\partial \delta_1^*(b)}{\partial \alpha_1} \cdot \frac{\partial \alpha_1}{\partial \alpha}\right]_{\alpha=0} = \frac{U b}{\lambda U_\infty} \exp\left(\frac{U b}{\lambda^2 U_\infty}\right) \operatorname{erfc}\left(\sqrt{\frac{U b}{\lambda^2 U_\infty}}\right) \left(\frac{-1}{\sqrt{M_\infty^2 - 1}}\right)$$

and

$$\left[\frac{\partial \delta_2^*(b)}{\partial \alpha}\right]_{\alpha=0} = \left[\frac{\partial \delta_2^*(b)}{\partial \alpha_2} \cdot \frac{\partial \alpha_2}{\partial \alpha}\right]_{\alpha=0} = \frac{U b}{\lambda U_\infty} \exp\left(\frac{U b}{\lambda^2 U_\infty}\right) \operatorname{erfc}\left(\sqrt{\frac{U b}{\lambda^2 U_\infty}}\right) \left(\frac{1}{\sqrt{M_\infty^2 - 1}}\right)$$

the normal force, eq. (4.9), becomes

$$N = \left\{\frac{1}{2}\rho_\infty U_\infty^2 b\right\}\left\{\frac{4\alpha}{\sqrt{M_\infty^2-1}}\left[1 + \frac{U}{\lambda U_\infty\sqrt{M_\infty^2-1}}\exp\left(\frac{Ub}{\lambda^2 U_\infty}\right)\operatorname{erfc}\left(\sqrt{\frac{Ub}{\lambda^2 U}}\right)\right]\right.$$

By means of eq. (3.5), N may be written in the case of air as

$$N = \left\{\frac{1}{2}\rho_\infty U_\infty^2 b\right\}\left\{\frac{4\alpha}{\sqrt{M_\infty^2-1}}\left[1 + \frac{2}{3M_\infty\sqrt{M_\infty^2-1}}\exp(J_\infty^2)\operatorname{erfc}(J_\infty)\right]\right\} \qquad (4.10)$$

The quantity in the second set of braces is the normal force coefficient C_N of the flat plate including viscosity and slip effects. The quantity $4\alpha/\sqrt{M_\infty^2-1}$ is the normal force coefficient for the flat plate in the ideal two-dimensional supersonic flow (Ref. 4, p. 200). Thus, the factor

$$K_N = 1 + \frac{2}{3M_\infty\sqrt{M_\infty^2-1}}\exp(J_\infty^2)\operatorname{erfc}(J_\infty) \qquad (4.11)$$

represents a correction to the ideal normal force coefficient to account for viscosity and slip. A graph of K_N vs. $\sqrt{Re_\infty}/M$ for $M_\infty = 2$, 3, and 4 is shown in HYD 2719.

5.0 MOMENT

The center of moment is taken as the leading edge of the plate (x = 0) and the moment is considered positive if the aerodynamic forces tend to rotate the plate in a counterclockwise direction. The tangential stress contributes nothing to the moment because the plate thickness was assumed to be zero. The moment per unit of span length is then given by (Ref. 4, p. 201)

$$M = \frac{1}{2}\rho_\infty U_\infty^2 \int_0^b \frac{2x}{\sqrt{M_\infty^2-1}}\left[2\alpha + \beta_2(x) - \beta_1(x)\right] dx$$

$$= \left\{\frac{1}{2}\rho_\infty U_\infty^2 b^2\right\}\left\{\frac{2}{\sqrt{M_\infty^2-1}}\left[\alpha + m_2(\alpha) - m_1(\alpha)\right]\right\} \tag{5.1}$$

where for convenience the following designations were made:

$$m_i(\alpha) = \frac{1}{b^2}\int_0^b x\beta_i(x,\alpha)\,dx; \qquad i = 1, 2. \tag{5.2}$$

By means of eqs. (3.5), (4.1), (4.7), and the substitution $\theta = J_\infty^2 a_i \frac{x}{b}$, eq. (5.2) becomes

$$m_i(\alpha) = \frac{\lambda}{b}\,\frac{1}{J_\infty^2 a_i}\int_0^{J_\infty^2 a_i} \theta \exp(\theta)\,\mathrm{erfc}(\sqrt{\theta})\,d\theta$$

This may be integrated by parts with the result

$$m_i(\alpha) = \frac{\lambda}{bH_i}\left[(H_i-1)\exp(H_i)\,\mathrm{erfc}(\sqrt{H_i}) + 1 + \frac{2\sqrt{H_i}}{\sqrt{\pi}}\left(\frac{H_i}{3}-1\right)\right] \tag{5.3}$$

where $H_i = J_\infty^2\, a_i$ and a_i is given in eq. (4.1).

As in the preceding cases, only the first two terms of the series expansion of $M = M(\alpha)$ in a power series in α about $\alpha = 0$ are retained. The constant term is seen to vanish, since in eq. (5.1) $\beta_1(x) = \beta_2(x)$ when $\alpha = 0$.

In view of the relations

$$\left(\frac{\partial m_1}{\partial \alpha}\right)_{\alpha=0} = \left(\frac{\partial m_1}{\partial H_1}\,\frac{\partial H_1}{\partial a_1}\,\frac{\partial a_1}{\partial \alpha}\right)_{\alpha=0} = \frac{-J_\infty^2}{\sqrt{M_\infty^2-1}}\left(\frac{\partial m_1}{\partial H_1}\right)_{\alpha=0}$$

$$\left(\frac{\partial m_2}{\partial \alpha}\right)_{\alpha=0} = \left(\frac{\partial m_2}{\partial H_2}\,\frac{\partial H_2}{\partial a_2}\,\frac{\partial a_2}{\partial \alpha}\right)_{\alpha=0} = \frac{J_\infty^2}{\sqrt{M_\infty^2-1}}\left(\frac{\partial m_2}{\partial H_2}\right)_{\alpha=0}$$

and

$$\left(\frac{\partial m_1}{\partial H_1}\right)_{\alpha=0} = \left(\frac{\partial m_2}{\partial H_2}\right)_{\alpha=0}$$

the moment given by eq. (5.1) becomes, up to order α,

$$M = \left\{\frac{1}{2}\rho_\infty U_\infty^2 b^2\right\}\left\{\frac{2\alpha}{\sqrt{M_\infty^2-1}}\left[1+\frac{2J_\infty^2}{\sqrt{M_\infty^2-1}}\left(\frac{\partial m_1}{\partial H_1}\right)_{\alpha=0}\right]\right\} \qquad (5.4)$$

The quantity $\left(\frac{\partial m_1}{\partial H_1}\right)_{\alpha=0}$ may be calculated directly from eq. (5.3) and upon substitution into eq. (5.4) together with the relation

$$\frac{\lambda}{b} = \frac{2}{3M_\infty J_\infty^2} \quad \text{for a gas with } \gamma = 1.4,$$

yields the final expression for the moment about the leading edge in the case of air:

$$M = \left\{\frac{1}{2}\rho_\infty U_\infty^2 b^2\right\}\left\{\frac{2\alpha}{\sqrt{M_\infty^2-1}}\left[1+\frac{4}{3J_\infty^4 M_\infty\sqrt{M_\infty^2-1}}\left\{\left(J_\infty^4-J_\infty^2+1\right)\exp\left(J_\infty^2\right)\operatorname{erfc}\left(J_\infty\right)-\frac{2J_\infty}{\sqrt{\pi}}\left(\frac{J_\infty^2}{3}-1\right)-1\right\}\right]\right\} \qquad (5.5)$$

The quantity $\frac{2\alpha}{\sqrt{M_\infty^2-1}}$ is the moment coefficient for the flat plate according to ideal linearized supersonic theory with the leading edge as moment center. Thus, the factor

$$K_M = 1 + \frac{4}{3J_\infty^4 M_\infty\sqrt{M_\infty^2-1}}\left\{\text{as in eq. (5.5)}\right\} \qquad (5.6)$$

represents a correction to be applied to the ideal moment coefficient to account for viscosity and slip. A graph of K_M vs. $\sqrt{Re_\infty}/M_\infty$ for $M_\infty = 2$, 3, and 4 is shown in HYD 2720.

6.0 AERODYNAMIC PROPERTIES FOR SMALL α

The functions C_T, K_N, and K_M are sufficient to describe the usual aerodynamic properties of the flat plate operating in air at given values of α, M_∞, and $\sqrt{Re_\infty}/M_\infty$. The drag coefficient C_D and the lift coefficient C_L are given by

$$C_D = C_T \cos\alpha + C_N \sin\alpha \cong C_T \tag{6.1}$$

and

$$C_L = C_N \cos\alpha - C_T \sin\alpha \cong \left(\frac{4 K_N}{\sqrt{M_\infty^2 - 1}} - C_T \right) \alpha \tag{6.2}$$

when terms only up to order α are retained (i.e. α infinitesimal). The graphs of $\left(\frac{dC_L}{d\alpha}\right)_{\alpha=0}$, HYD 2721, show it to be between 0.41 and 2.31 for all $\sqrt{Re_\infty}/M_\infty$ provided $2 \leqq M_\infty \leqq 4$.

The moment coefficient with respect to the leading edge is given by

$$C_M = \frac{2\alpha K_M}{\sqrt{M_\infty^2 - 1}} \tag{6.3}$$

The center of pressure, c.p., with reference to the leading edge is given by

$$c.p. = \frac{b}{2} \frac{K_M}{K_N} \tag{6.4}$$

In the ideal linearized theory the center of pressure falls at the half-chord position. Calculations of c.p. according to eq. (6.4) show it to be slightly forward of that location; in fact,

$$0.98 \frac{b}{2} \leqq c.p. \leqq \frac{b}{2} \quad \text{whenever} \quad M_\infty \geqq 2 \quad \text{and} \quad \sigma_\infty \geqq 0.$$

Thus, the center of pressure remains essentially at the half-chord position when the plate is operating under slip-flow conditions.

The asymptotic forms of C_T, K_N, and K_M for $\sqrt{Re_\infty}/M_\infty \to 0$ and $\sqrt{Re_\infty}/M_\infty \to \infty$ are listed below. These are useful for

computation purposes, but should not be construed as being valid in the free-molecule and continuum-flow regions respectively.

	$\frac{\sqrt{Re_\infty}}{M_\infty} \sim 0;\ J_\infty \sim 0$	$\frac{\sqrt{Re_\infty}}{M_\infty} \sim \infty;\ J_\infty \sim \infty$
C_T	$\frac{8}{3M_\infty}\left(1-\frac{4J_\infty}{3\sqrt{\pi}}\right)$	$\frac{16}{3\sqrt{\pi}\,J_\infty M_\infty}$
K_N	$1+\frac{2\left(1-\frac{2J_\infty}{\sqrt{\pi}}\right)}{3M_\infty\sqrt{M_\infty^2-1}}$	$1+\frac{2}{3\sqrt{\pi}\,J_\infty M_\infty\sqrt{M_\infty^2-1}}$
K_M	$1+\frac{2\left(1-\frac{12J_\infty}{5\sqrt{\pi}}\right)}{3M_\infty\sqrt{M_\infty^2-1}}$	$1+\frac{4}{9\sqrt{\pi}\,J_\infty M_\infty\sqrt{M_\infty^2-1}}$

7.0 CONCLUSIONS

1. The flat-plate slip-flow theory of Ref. 1 has been extended to include the aerodynamic coefficients C_D, C_L, and C_M at very small angles of attack.

2. The coefficient C_D at small angles of attack may be taken equal to the skin-friction coefficient as presented in Ref. 2, part II.

3. The skin-friction stresses tend to reduce C_L at high altitude, high-speed conditions.

4. The moment coefficient C_M will not be significantly greater in slip flow than $2\alpha / \sqrt{M_\infty^2 - 1}$, its value derived from ideal linearized two-dimensional supersonic continuum-flow theory.

5. In general, except for the skin friction coefficient, the corrections to be applied to the aerodynamic formulas of linearized supersonic theory, which account for viscosity and slip, will not be significant in the slip-flow region of aerodynamics. The correction factors involved vary between 1 and 1.06 when $2 \leq M_\infty \leq 4$ and $2.5 \leq \sqrt{Re_\infty}/M_\infty \leq 12.5$, conditions typical of operation in the No. 3 Wind Tunnel (Ref. 7).

MARCH 21, 1952 — S. F. MACK

R E F E R E N C E S

1) S.A. Schaaf - "A Note on the Flat Plate Drag Coefficient" - Univ. of Calif. Engineering Research Report HE-150-66, February, 1950.

2) F.S. Sherman - "Skin Friction in Supersonic Low Density Gas Flow" - Univ. of Calif. Engineering Research Reports HE-150-70, Part I, July, 1950, and HE-150-81, Part II, March, 1951.

3) L.B. Loeb - "Kinetic Theory of Gases" - McGraw-Hill Book Company, New York, 1927.

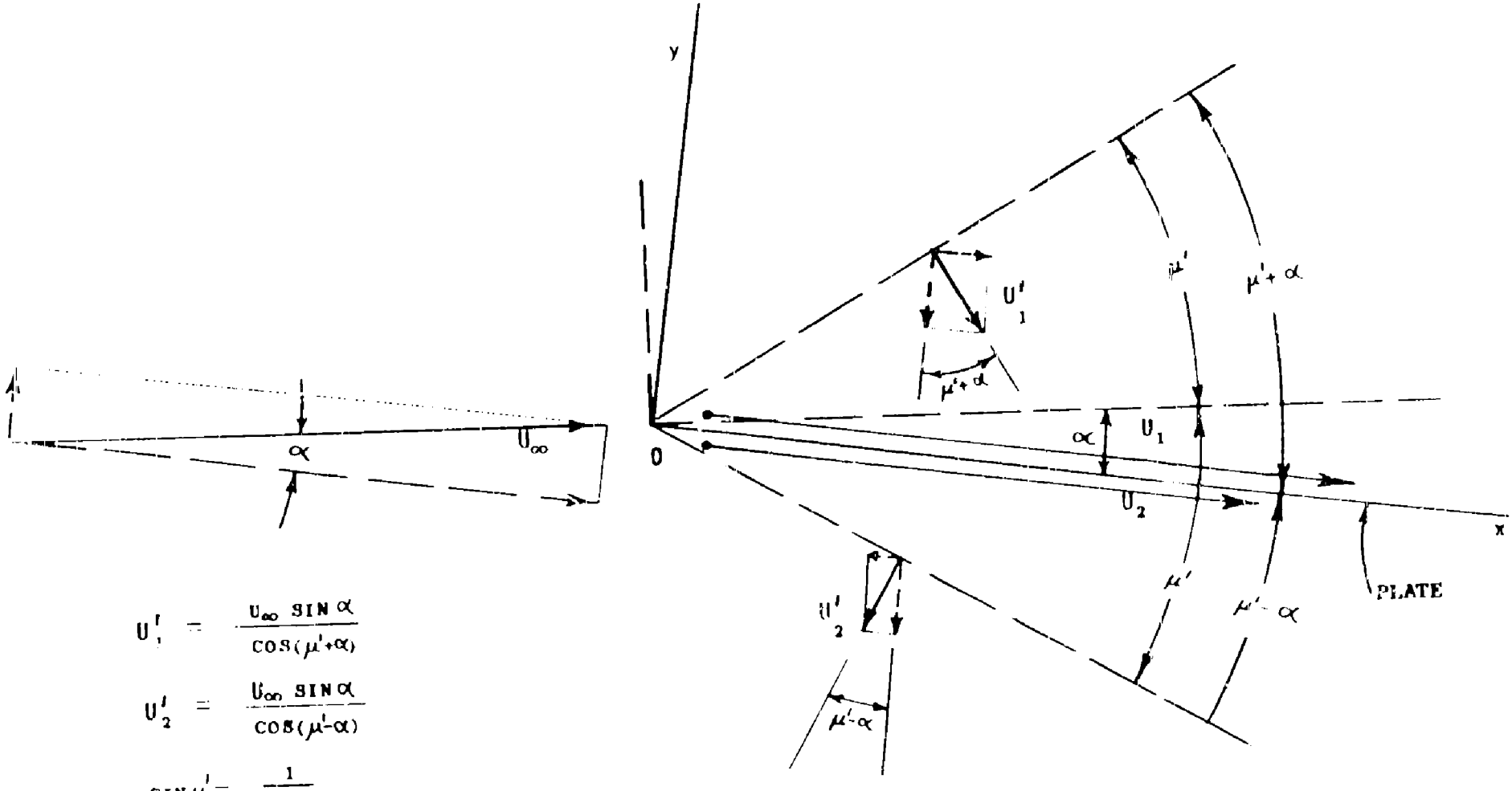
4) A.M. Kuethe and J.D. Schetzer - "Foundations of Aerodynamics" - John Wiley & Sons, Inc., New York, 1950.

5) S. Goldstein - "Modern Developments in Fluid Dynamics" - Oxford Univ. Press, London, 1938.

6) R.V. Churchill - "Modern Operational Mathematics in Engineering" - Mc-Graw Hill Book Co., Inc., New York, 1944.

7) S.A. Schaaf, D.O. Horning and E.D. Kane - "Design and Initial Operation of a Low Density Supersonic Wind Tunnel" - Univ. of Calif. Engineering Projects Report HE-150-62, August, 1949. Also presented at Heat Transfer and Fluid Mechanics Institute, Berkeley, June 1949, and published by ASME.

* * * * * * * * * * * * * * * * * * * *



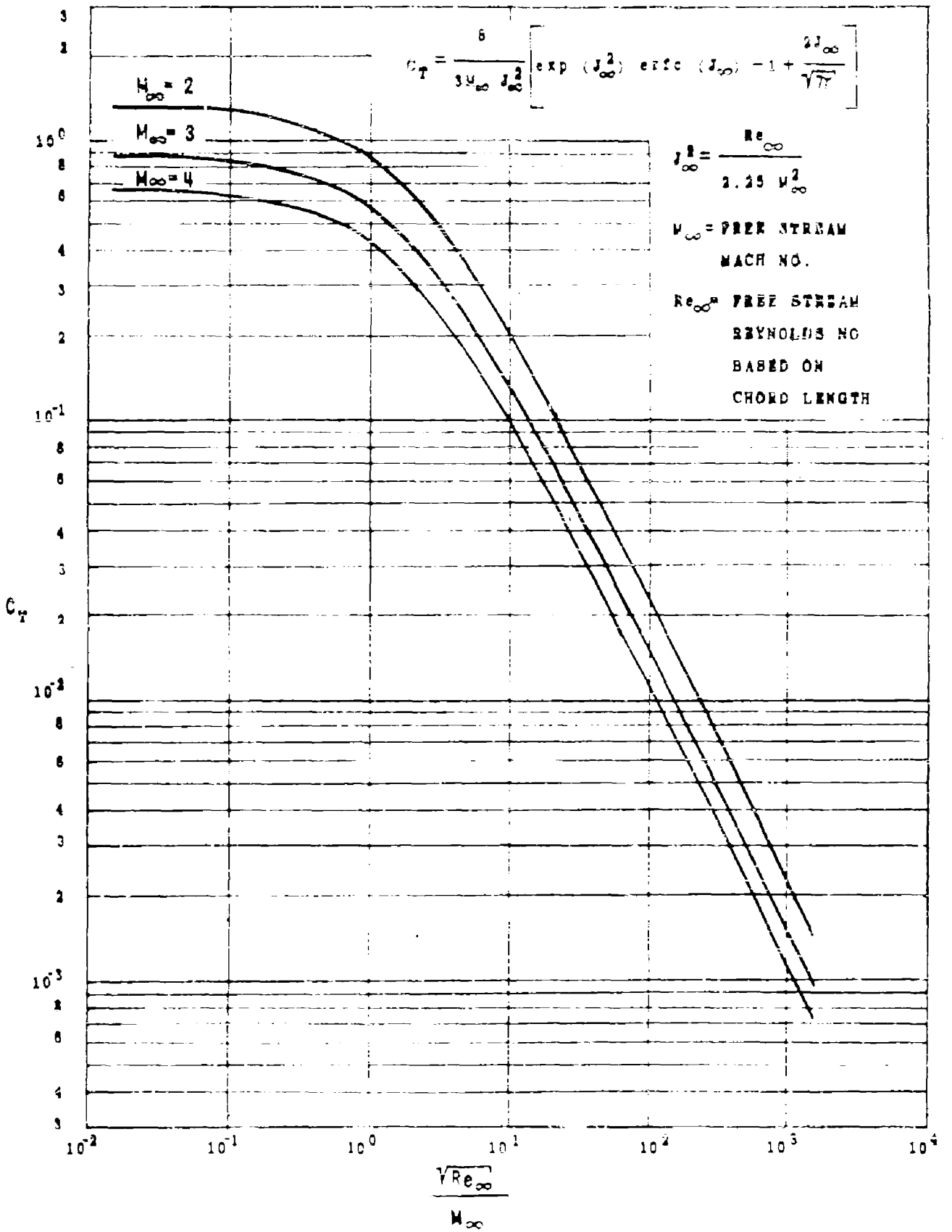
$$U'_1 = \frac{U_\infty \sin\alpha}{\cos(\mu'+\alpha)}$$

$$U'_2 = \frac{U_\infty \sin\alpha}{\cos(\mu'-\alpha)}$$

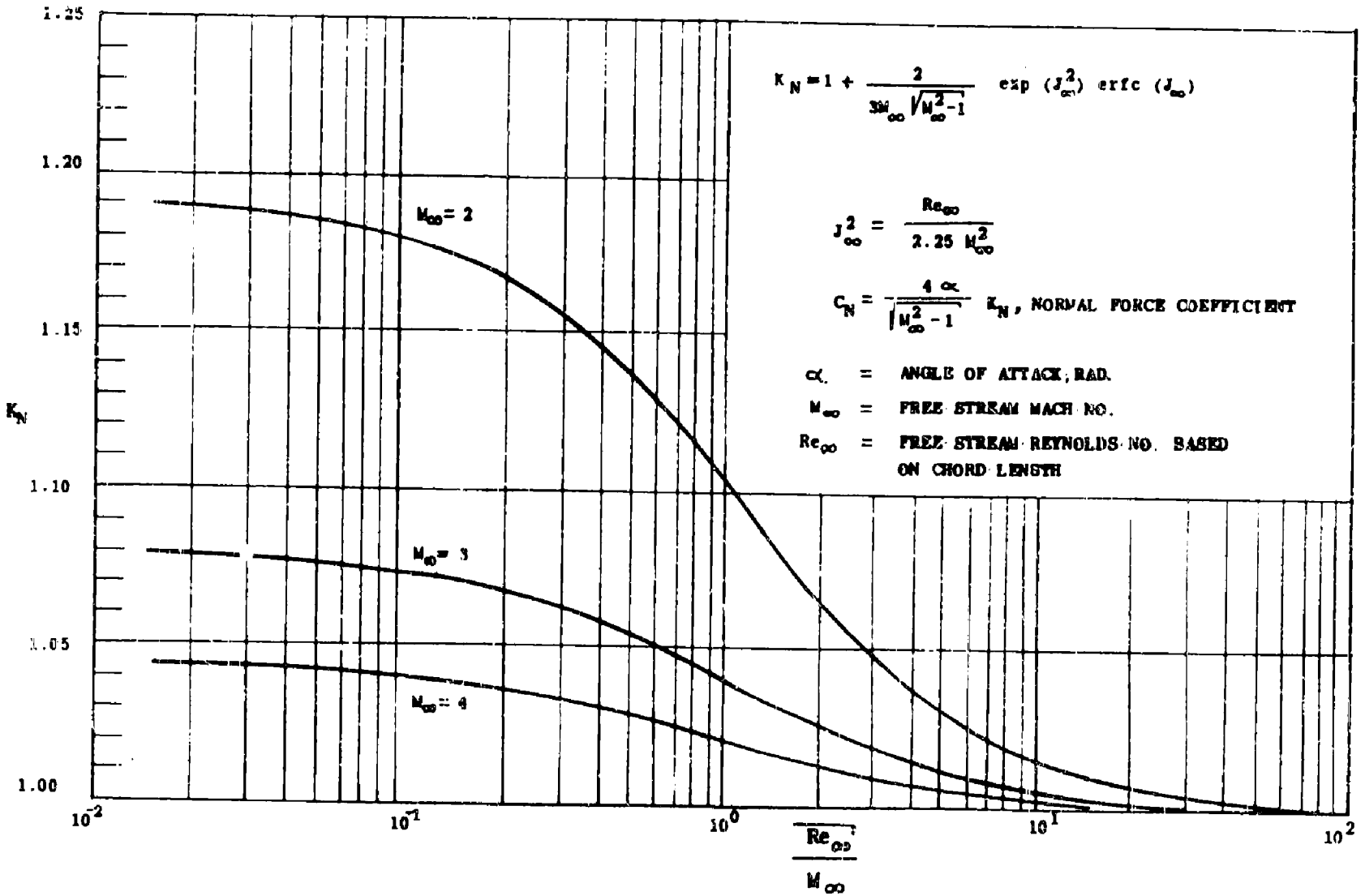
$$\sin\mu' = \frac{1}{M_\infty}$$

VELOCITY COMPONENTS OF IDEAL LINEARIZED SUPERSONIC THEORY

FOR THE FLAT PLATE

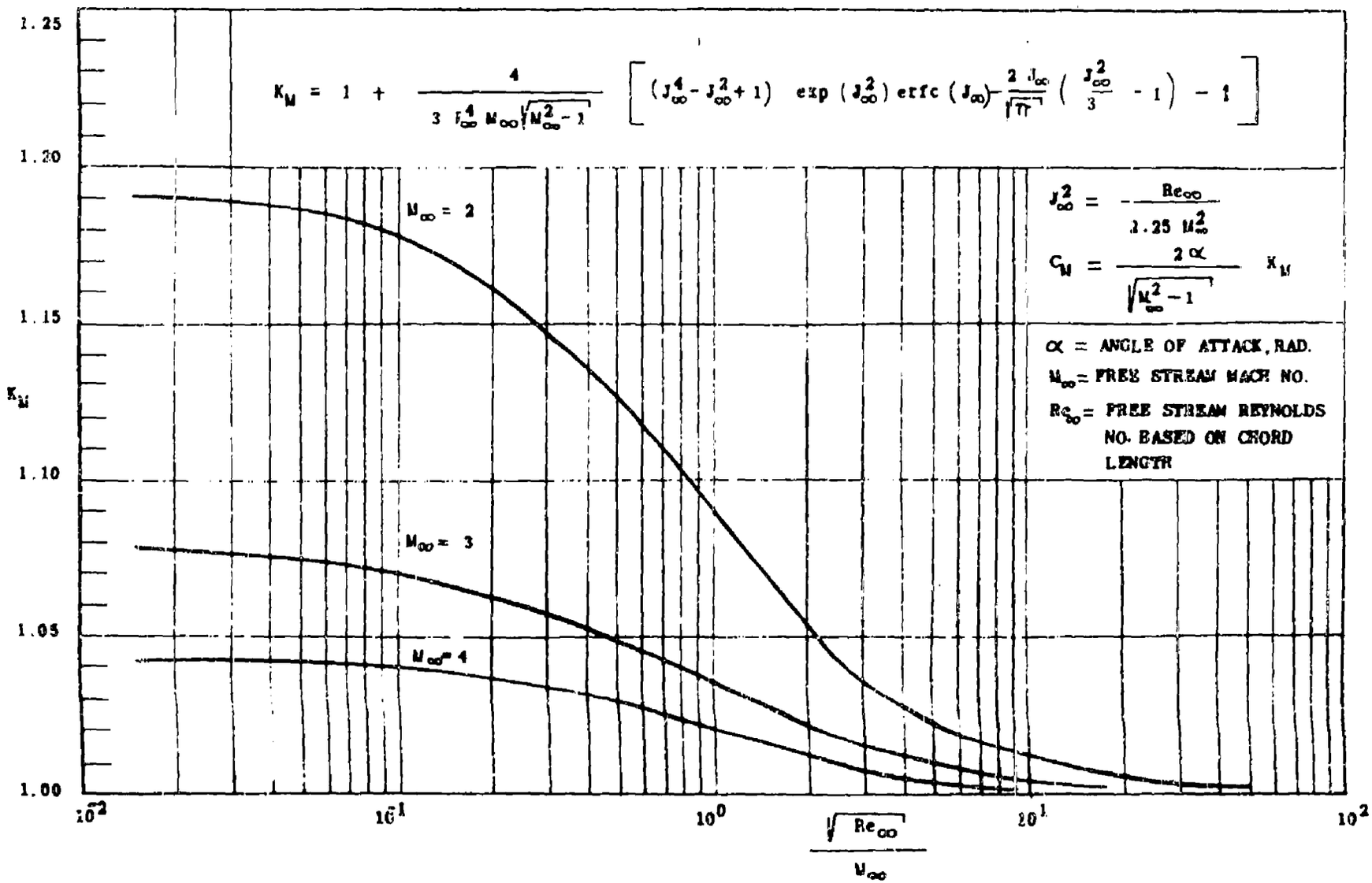


TANGENT FORCE COEFFICIENT FOR FLAT PLATE IN AIR

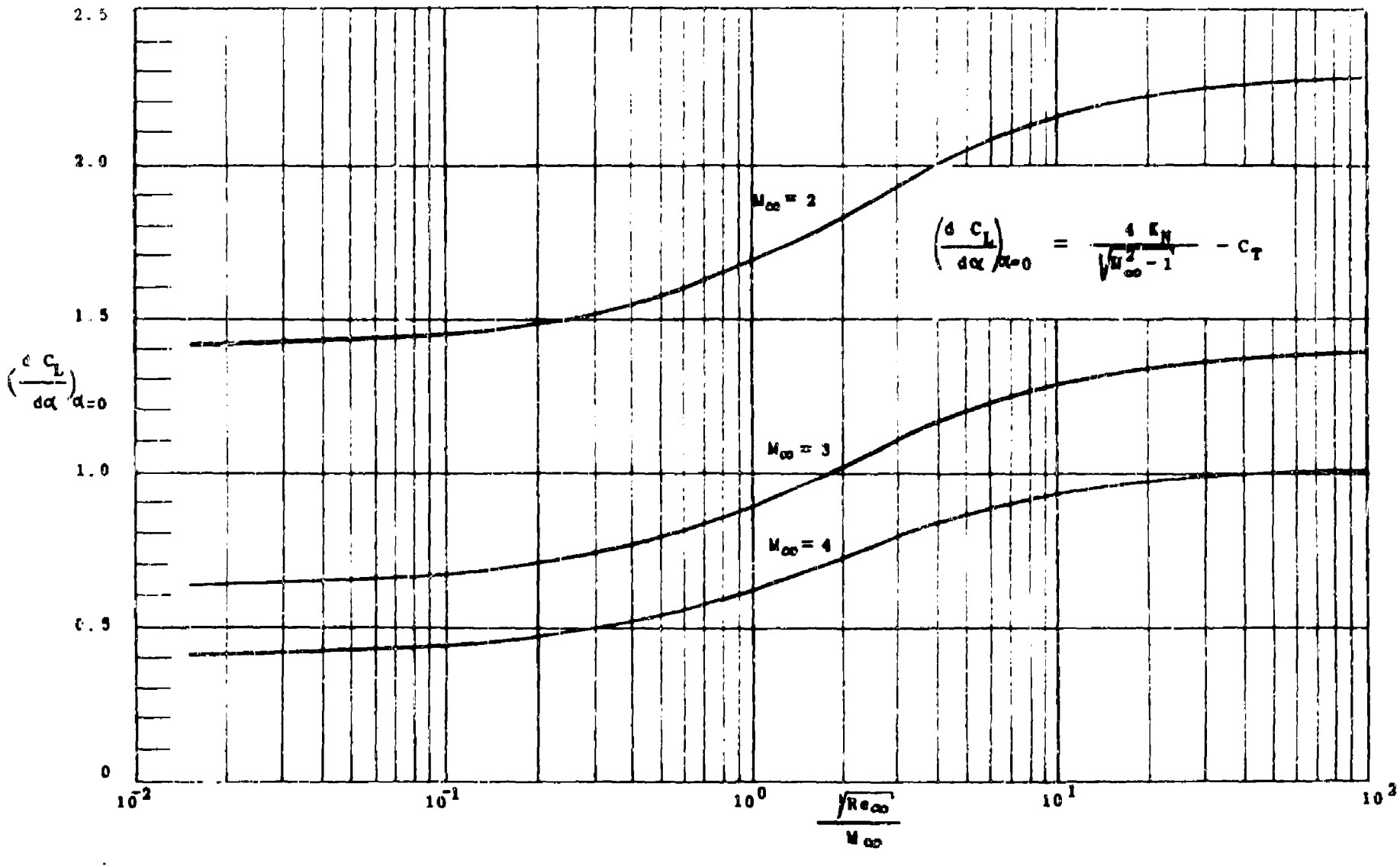


NORMAL FORCE CORRECTION FACTOR FOR FLAT PLATE IN AIR

HYD 2719-150-0



MOMENT CORRECTION FACTOR FOR FLAT PLATE IN AIR



LIFT CURVE SLOPE AT $\alpha = 0$ FOR FLAT PLATE IN AIR

Reproduced by
Armed Services Technical Information Agency
DOCUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

AD-

1908

UNCLASSIFIED